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Tests of the weak equivalence principle for charged particles in space

H. Dittus, C. Lämmerzahl *

Centre of Applied Space Technology and Microgravity (ZARM), University of Bremen, Am Fallturm, D-28359 Bremen, Germany

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Abstract

The Weak Equivalence Principle states that in a gravitational field all structureless point-like particles follow the same path. The Weak Equivalence Principle is confirmed for neutral bulk matter with an accuracy of $5 \cdot 10^{-12}$ and for neutral quantum matter on a level of 10^{-9}, rare tests for charged matter have been carried out. The experiment for freely falling electrons carried out by Witteborn and Fairbank [Witteborn, F.C., Fairbank, W.M. Experimental comparison of the gravitational force on freely falling electrons and metallic electrons. Phys. Rev. Lett., 19, 1049, 1967] with an accuracy of 0.1 is the only one cited in literature. A brief report and re-analysis of this experiment is given and the gravity induced disturbing electrical fields are discussed. We show that experimentation under conditions of strongly reduced residual accelerations (near-weightlessness) will reduce the disturbing effects considerably and will improve the results for free fall tests with charged particles by orders of magnitude.

Keywords: Equivalence principle; Charge; Weightlessness condition; Witteborn–Fairbank experiment

1. Introduction

The Weak Equivalence Principle (WEP) states that all point-like structureless particles fall along the same path within a gravitational field. If general relativity is correct, the WEP holds for all forms of matter and antimatter, but most experiments are carried out with neutral matter only. The reason is that electromagnetic stray fields influence gravitational experiments with charged particles or magnetized matter very strongly and must be shielded carefully. Here, we are concerned with the WEP for charged matter. Until now, there is only one single experiment to test charged matter carried out by Witteborn and Fairbank (1967). The reported accuracy was 0.09. Stray electric fields and gravity-induced electric fields in solids stemming from the Schiff–Barnhill effect (Schiff and Barnhill, 1966) and the DMRT effect (Dessler et al., 1968) caused major errors.

There are at least two reasons to consider the WEP for charged particles: (1) search for hypothetical anomalous couplings between charge and gravity which may signal remnants of quantum gravity and (2) start-up for enhanced tests of the Weak Equivalence Principle for antimatter. Tests for antimatter (e.g. for positrons or anti-protons) could be carried out in a similar way than for charged particles. We refer to an experimental proposal by Huber et al. (2001). There are reasons for a violation of the WEP for antimatter. Some theories predict anomalous behavior of antimatter. Models of quantum gravity usually lead to additional gravitational spin-0 and spin-1 fields. While for ordinary matter the gravitational field of the Earth is attractive, it may be repulsive for antimatter (Goldman et al., 1986). Thus, antimatter behaves differently than antimatter in generalized theories of gravity. Nevertheless, there are also arguments against violations of the Weak Equivalence Principle for antimatter (Nieto and Goldman, 1991). A more detailed discussion of the theoretical approaches stimulating the experimental proofs are given in (Dittus and Lämmerzahl, 2002; Dittus et al., 2004).

* Corresponding author.
E-mail addresses: dittus@zarm.uni-bremen.de (H. Dittus), laemmerzahl@zarm.uni-bremen.de (C. Lämmerzahl).

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In the following, we at first discuss a possible and consistent test theory and briefly review the Witteborn–Fairbank experiment, describe its functionality and its principal problems. Then, we discuss the advantages to carry out this kind of experiment in a near-weightlessness environment and report the results of our analysis.

2. Consistent test theory

Though it is not influencing the description of the experiments, we want to set up a simple frame and introduce notions which enable to describe consistently tests of the Weak Equivalence Principle for charged particles. The anticipated result is that the well-known Eötvös coefficient

$$\eta_{\mathrm{E}} = \left| 2 \cdot \frac{(m_{\mathrm{g}}/m_{\mathrm{i}})_1 - (m_{\mathrm{g}}/m_{\mathrm{i}})_2}{(m_{\mathrm{g}}/m_{\mathrm{i}})_1 + (m_{\mathrm{g}}/m_{\mathrm{i}})_2} \right| \tag{1}$$

in its usual definition (here the indices g and i are related to the gravitational and the inertial mass, respectively, of two test masses (1 and 2) of different composition) should be split into a charge-independent part describing a violation of the WEP for neutral particles (or due to the mass only) and a part which scales with the charge-to-mass ratio. If the WEP is violated due to charge, then the inertial and/or gravitational mass of a particle should possess a small contribution due to the charge of that mass. Therefore, we split the inertial and gravitational masses

$$m_{\mathrm{i}} = m_{\mathrm{i}}^0 \left(1 + \kappa_{\mathrm{i}} \frac{q}{m^0_{\mathrm{i}}} \right) \quad \text{and} \quad m_{\mathrm{g}} = m_{\mathrm{g}}^0 \left(1 + \kappa_{\mathrm{g}} \frac{q}{m_{\mathrm{g}}^0} \right), \tag{2}$$

where κ_{i} and κ_{g} parametrize the hypothetical influence of charge on the inertial and gravitational mass, and $m_{\mathrm{i,g}}^0$ are the charge-independent parts of the inertial and gravitational masses of the particles. This enables us to define a modified Eötvös coefficient

$$\widetilde{\eta}_{\mathrm{E}} = 2 \frac{\ddot{x}_2 - \ddot{x}_1}{\ddot{x}_2 + \ddot{x}_1} \approx \eta_E + (\kappa_{\mathrm{g}2} - \kappa_{\mathrm{i}2}) \frac{q_2}{m_2^0} - (\kappa_{\mathrm{g}1} - \kappa_{\mathrm{i}1}) \frac{q_1}{m_1^0}. \tag{3}$$

If one of the test particles is neutral (e.g. $q_2 = 0$), Eq. (3) can be simplified to

$$\widetilde{\eta}_{\mathrm{E}} = \eta_{\mathrm{E}} + \delta\kappa \frac{q}{m^0}. \tag{4}$$

A charge-induced violation of the Weak Equivalence Principle is encoded in the parameter $\delta\kappa = \kappa_{\mathrm{g}} - \kappa_{\mathrm{i}}$. Only if the electromagnetic energy of the charged particles contributes in the same way to inertial and gravitational mass, there will be no violation. To first order approximation, charge-induced violations are independent of any "bare" violation described by Eq. (1). Consequently, a comparison of a charged with a neutral particle gives an estimate for the total $\widetilde{\eta}_{\mathrm{E}}$. Only if we change the charge of the test particle, we can make statements about the contributions η_{E} and $\delta\kappa$ to $\widetilde{\eta}_{\mathrm{E}}$. For a single charged particle, like the electron or a positron, η_{E} and $\delta\kappa$ may compensate. For ions with different ionisation grades, one can get information about both parts of $\widetilde{\eta}_{\mathrm{E}}$.

3. Discussion of the Witteborn–Fairbank experiment

3.1. Analysis of gravity-dependent electrical fields

To our knowledge, the experiment carried out by Witteborn and Fairbank (1967) is the only one dedicated to measure the net force on electrons freely falling in a copper tube. The experimental set-up consisted of a vacuum tank cooled down to 4.2 K in liquid helium bath and a vertical copper "drift" tube inside the vacuum tank to shield stray electrical fields. The drift tube was about 1 m high and had a diameter of 5 ± 0.0003 cm. Electrons became emitted by a cathode at the bottom of the drift tube and moved along the symmetry axis of the tube and had been forced to do so by a magnetic field of a coaxial solenoid. After passing the tube, the electrons had been detected by an electron multiplier. Short burst emission enabled to measure the mean flight time along the way from the cathode to the detector. In the case that only gravity is acting, the maximum flight time, defined by the time electrons need to arrive at the detector with zero velocity, can easily be calculated to

$$t_{\max} = \sqrt{\frac{m_{\mathrm{i}}}{m_{\mathrm{g}}} \cdot \frac{2h}{g}}, \quad \text{with} \quad \nabla U = -g, \tag{5}$$

where h is the drift length and g the gravitational acceleration; m_{i}, and m_{g} are inertial and gravitational mass of the test particle, respectively (here: the electron). For known h and measured $t_{\max}$, one can determine the ratio of inertial to gravitational mass of an electron. A comparison with other electrons or with neutral particles gives the Eötvös coefficient.

However, the purely gravitational case cannot be realized in a ground-based experiment. Beside many disturbing effects and several stray fields like patch effects, magnetic stray fields, thermoelectric fields, rest gas scattering effects, and radiation pressure (see Darling et al. (1992) for a complete error analysis of the Witteborn–Fairbank experiment), there are at least three electric fields present with major impacts to the experiment:

1. The Schiff–Barnhill field: Schiff and Barnhill (1966) calculated that electrons bound in the metallic (copper) shield of the drift tube create an electric field

$$E_{\mathrm{SB}} = \frac{m_{\mathrm{ge}}}{e^-} g, \quad |E_{\mathrm{SB}}| \approx 5 \cdot 10^{-11}\ \mathrm{V/m}, \tag{6}$$

which balances the electrons in the metallic shield against gravity. Here, and in the following, the subscript e is related to the electron with charge e.

2. The so-called DMRT field introduced by and named after Dessler et al. (1968) is caused by lattice deformation effects of the metallic (copper) shield in the gravitational field of the Earth. It is also proportional to the gravitational acceleration and be calculated to

$$E_{\mathrm{DMRT}} = \gamma \frac{m_{\mathrm{ga}}}{e^-} g, \tag{7}$$

where m_{ga} is the gravitational mass of an atom of the metallic shield, the copper drift tube. γ is a material-dependent parameter to be determined by solid state physics. It is remarkable that $E_{\rm DMRT}$ is stronger than $E_{\rm SB}$ by a factor of 10^4, but have not been observed by Witteborn and Fairbank in their original experiment. Only in a later experiment by Lockart et al. (1977) it has been proven, that below the helium boiling temperature of 4.2 K this field apparently disappears completely. This curious effect became explained by Darling et al. (1992): The helium rest gas builds a molecular layer on the Cu-walls of the drift tube, and at temperature near helium boiling, large temperature gradients arise and cause a thermoelectric field antiparallel to $E_{\rm DMRT}$.

3. The external uniform field $E_a < 2.5 \cdot 10^{-10}$ V/m directed parallel to the drift tube and applied in order to decelerate the electrons. The field is needed to vary the flight time of the electrons and to find the maximum time of flight. Considering these additional fields, Eq. (5) must be modified:

$$t_{\rm max} = \sqrt{\frac{2h}{\frac{m_{\rm g}}{m_{\rm i}} g - \frac{q}{m_{\rm i}}(E_{\rm SB} + E_{\rm DMRT} - E_a)}} = \sqrt{\frac{2h}{\eta_{\rm WF} g - \frac{q}{m_{\rm i}} E_a}}. \tag{8}$$

q is the charge of the test particle. With Eqs. (6) and (7) $\eta_{\rm WF} = 1/m_{\rm i} \cdot (m_{\rm g} - (q/e)(m_{ge} - m_{\rm ga}))$ can be defined as an Eötvös coefficient specifically for the Witteborn–Fairbank experiment. It is easy to see that for the Witteborn–Fairbank experiment with an $E_{\rm DMRT} = 0$ due to the cryogenic effect described above:

$$\eta_{\rm WF} = 0 \quad \text{for} \quad q = e \quad \text{and} \quad m = m_e. \tag{9}$$

Therefore, for experiments with electrons it is not possible to make any statements about a relation between inertial and gravitational mass. For any other particle, we obtain

$$\eta_{\rm WF} \approx \frac{m_{\rm g} - m_{\rm ge}}{m_{\rm i}} \neq \frac{m_{\rm g}}{m_{\rm i}}, \tag{10}$$

which shows that the ratio between inertial and gravitational mass is dependent on the unknown gravitational mass of the electron.

3.2. Interpretation by Witteborn and Fairbank

The statement derived from Eq. (9) seems to be a strong argument against the experiment. However, the statement is valid only if the Schiff–Barnhill-field is of the form given in Eq. (6), that is, if the gravitational acceleration of the electrons inside the metal structure of the drift tube $g_{\rm bulk}$ is the same as for free electrons $g_{\rm free}$. If we distinguish between these two accelerations, as it was done by Witteborn and Fairbank (1967), we obtain from Eq. (8):

$$t_{\rm max} = \sqrt{\frac{2h}{\frac{m_{ge}}{m_{ie}}(g_{\rm free} - g_{\rm bulk}) + \frac{e}{m_{ie}}(E_{\rm DMRT} + E_a)}}. \tag{11}$$

If we set $E_{\rm DMRT} = 0$, then by measuring $t_{\rm max}$ and by varying E_a

$$\frac{m_{ge}}{m_{ie}}(g_{\rm free} - g_{\rm bulk}) \leqslant (0.13 \pm 0.47) \cdot 10^{-11}\ \text{eV/m} \tag{12}$$

could be determined for electrons. This sets the upper limit for the Eötvös coefficient for charged matter if one interprets it in the sense of

$$\eta = 2\frac{g_{\rm free} - g_{\rm bulk}}{g_{\rm free} + g_{\rm bulk}} \leqslant 0.09. \tag{13}$$

Nevertheless, our analysis shows that Witteborn and Fairbank (1967) did not really proof the WEP, but instead measured the equality of accelerations of bound and free electrons. Therefore, the significance of their result is rather limited. Tests of the WEP for charged matter can only be performed for particles different from electron.

4. Advantages of a near weightlessness environment

4.1. Idealized case

As we have seen in the previous section, the free fall experiments with charged particles are mainly influenced by gravity-induced electric fields. It seems to be obvious to ask how the experiments could be carried out if gravity could be mostly compensated in a near weightlessness environment on a satellite. In the following discussion, we again will only focus on the gravity-induced electric fields. For a discussion of all non-gravity-induced errors we refer to the comprehensive analysis of Darling et al. (1992). For a detailed analysis of a space experiment, the various accelerations appearing in Eq. (8) have to be carefully analyzed, though they are the same on Earth. A complicating circumstance is that we have three kinds of particles involved, the charged test particle, the ions of the metallic shield (subscript a), and the electrons bound in the shield (subscript e), and for each pair of these particles a violation of the WEP may occur. In addition, the Schiff–Barnhill field and the DMRT-field are not given absolutely, but depend on relative motions of the various particles. The accelerations $(m_{\rm g}/m_{\rm i})g$, $(m_{\rm ge}/m_{\rm ie})g$, and $(m_{\rm ga}/m_{\rm ia})g$ are the accelerations of the charged test particle, of the bound electrons, and of the ions in the shield, respectively, as described in the rest frame of the gravitating Earth. In order to calculate the Schiff–Barnhill effect in near-weightlessness which may result from a violation of the WEP between electrons and atoms of the metallic shield, we consider the forces and the equations of motion of the electrons and atoms of the metallic shield in the reference system of the Earth. These are given by

$$F_{\rm a} = m_{\rm ia}\ddot{x}_a = m_{\rm ga} g, \quad F_e = m_{\rm ie}\ddot{x}_e = m_{\rm ge} g. \tag{14}$$

In order to analyze a near-weightlessness experiment, we have to calculate F_e in the rest system of the metallic shield. Therefore, we perform a coordinate transformation to the frame which accelerates with the metallic shield with respect to the Earth system. That is, we look for new coordinates $x^* = f(x)$, so that $\mathrm{d}^2x/\mathrm{d}t^2 = 0$, which is given by $x^* = x - \frac{1}{2}(\mathrm{d}^2x/\mathrm{d}ht^2)t_2$ for constant acceleration. We get for the force on the electron

$$\begin{aligned} m_{\mathrm{ie}}\ddot{x}_e^* &= m_{\mathrm{ie}} - \frac{\mathrm{d}}{\mathrm{d}t^2}\left(x_e - \frac{1}{2}\ddot{x}_a t^2\right) \\ &= eE^*_{\mathrm{SB}} = \left(m_{\mathrm{g}e} - m_{\mathrm{ie}}\frac{m_{\mathrm{g}a}}{m_{\mathrm{ia}}}\right)\cdot g \end{aligned} \tag{15}$$

The reason for the last equality is that the force on the electron has to be balanced by the Schiff–Barnhill field in the rest frame of the atoms defined by the last expression in Eq. (15). Now we can write Eq. (8) in the rest frame of the metallic shield:

$$t_{\max} = \sqrt{\frac{m_{\mathrm{i}q}h}{m_{\mathrm{g}q}x^* + \frac{q}{e}\left(m_{\mathrm{g}e} - m_{\mathrm{i}e}\frac{m_{\mathrm{ga}}}{m_{\mathrm{ia}}}\right)g - E_a}}, \tag{16}$$

where the subscript q is related to the test particle of charge q. If we express x^* with respect to the Earth reference system as $(\mathrm{d}^2x^*/\mathrm{d}t^2) = (\mathrm{d}^2x/\mathrm{d}t^2) - (\mathrm{d}^2x_a/\mathrm{d}t^2)$, we obtain

$$\begin{aligned} t_{\max} &= \sqrt{\frac{h}{\frac{m_{\mathrm{g}q}}{m_{\mathrm{i}q}} - \frac{m_{\mathrm{ga}}}{m_{\mathrm{ia}}} + \frac{q}{e}\frac{m_{\mathrm{ie}}}{m_{\mathrm{i}q}}\left(\frac{m_{\mathrm{g}e}}{m_{\mathrm{i}e}} - \frac{m_{\mathrm{ga}}}{m_{\mathrm{ia}}}\right)\cdot g + \frac{q}{m_{\mathrm{i}q}}E_a}} \\ &= \sqrt{\frac{h}{\delta_q - \delta_a - \frac{q}{e}\frac{m_{\mathrm{g}e}}{m_{\mathrm{i}q}}(\delta_a - \delta_e)\cdot g + \frac{q}{m_{\mathrm{i}q}}E_a}} \\ &= \sqrt{\frac{h}{\widetilde{\eta}_{\mathrm{E}}\cdot g + \frac{q}{m_{\mathrm{i}q}}E_a}} \end{aligned} \tag{17}$$

where we also used Eq. (14). The expression $\widetilde{\eta}_E$ can be interpreted as a modified Eötvös factor which consists of the difference of the ratios of gravitational and inertial masses of test particles and the atoms in the metallic shield with an additional term consisting of the difference of gravitational and inertial masses of the electrons and the atoms in the metallic shield weighted with the ratio of their charge-to-mass ratios. g multiplied by $\widetilde{\eta}_{\mathrm{E}}$ describes an effective acceleration.

If we take electrons as test particles ($q = e$, $m_{\mathrm{i}q} = m_{\mathrm{ie}}$, $m_{\mathrm{g}q} = m_{\mathrm{g}e}$), then $\widetilde{\eta}_{\mathrm{E}} = 0$ results. As in the corresponding experiment on Earth, no violation of the WEP may be observable, even if electrons, in comparison with other particles, would violate the WEP.

In order to obtain an Eötvös coefficient for a charged test particle, we can either (1) compare the acceleration of two different test particles or (2) simply compare the test particle with the metal shield which is also freely falling. For practical reasons, the second case is the more interesting and is discussed in the following. Although Eq. (17) is no relation from which we can derive uniquely the searched quantity $\delta_q - \delta_a$, we can conclude that $\widetilde{\eta}_{\mathrm{E}} \sim \delta_q - \delta_a$, within a small error due to the fact that the last term defining $\widetilde{\eta}_{\mathrm{E}}$ is very small and can be neglected. This last term would show, as the first one, a violation of the WEP, but would be of the order 10^{-3} for protons as test particles. Consequently, in a first approximation, by measuring $a_{\mathrm{eff}} = \widetilde{\eta}_{\mathrm{E}}\cdot g$ for one kind of charged particle different form an electron one can test the validity of the WEP in direct comparison to the neutral metallic shield. Because any experiment testing the WEP for charged matter needs a metallic shield, this calculation holds not only for a Witteborn–Fairbank approach, but is valid in general.

4.2. Errors from microgravity conditions

Finally, we calculate the theoretical improvements of space tests. If we set the residual acceleration level on the experimental platform to δa, then E_{SB} and E_{DMRT} change to δE_{SB} and δE_{DMRT} and Eq. (17) must be modified to

$$t_{\max} = \sqrt{\frac{h}{\widetilde{\eta}_{\mathrm{E}}\cdot g + \frac{q}{e}\frac{m_{\mathrm{g}e}+\gamma m_{\mathrm{ga}}}{m_{\mathrm{i}}}\cdot\delta a + \frac{q}{m_{\mathrm{i}q}}E_a}}. \tag{18}$$

The second term of the denominator in Eq. (18) is the microgravity induced error of the first term of the denominator one aims to measure. Thus,

$$\delta\widetilde{\eta}_{\mathrm{E}} = \frac{q}{e}\frac{m_{\mathrm{g}e} + \gamma m_{\mathrm{ga}}}{m_{\mathrm{i}}}\frac{\delta a}{g} \tag{19}$$

is the microgravity induced error in the Eötvös coefficient. If we calculate with $\delta a = 10^{-5}g$, as it may be realistic for experiments carried through on the on ISS, and assume $\gamma = 0.5\cdot 10^{-6}$, we attain $\delta\widetilde{\eta}_{\mathrm{E}} = \pm 10^{-5}$ for positron and $\delta\widetilde{\eta}_{\mathrm{E}} = \pm 10^{-9}$ for protons and anti-protons. The error limits will become even smaller for smaller residual accelerations as well as for larger test particle masses.

5. Conclusion

An experiment in space with the Witteborn–Fairbank set-up may be well suited to test the Weak Equivalence Principle for charged matter. The measured quantity directly gives to first order the Eötvös coefficient for the test particle in its free fall behavior in comparison to the metallic shield. Since in all conceivable set-ups for tests with charged particles one needs an electric shield, our analysis holds in general. Possible other set-ups may be carried out with ion interferometry or charged particles in a magnetic trap. In any case, the induced errors which are mainly due to the physics of the shield add up to the same error for all charged matter experiments to measure the Eötvös coefficient. Of course, all other error sources play the same role as for experiments carried out on Earth and are not necessarily negligible. In particular, the patch effect is still unknown, and even if it can be depressed as the E_{DMRT}-field by cooling the system to helium boiling temperature, it is the most limiting factor for accuracy. Nevertheless, a near-weightlessness environment would reduce the error effectively.

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