# Constraints on space–time torsion from Hughes–Drever experiments

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## Abstract

The coupling of space–time torsion to the Dirac equation leads to effects on the energy levels of atoms which can be tested by Hughes–Drever type experiments. Reanalysis of these experiments carried out for testing the anisotropy of mass and anomalous spin couplings can lead to the till now tightest constraint on the axial torsion by $K \leqslant 1.5 \times 10^{-15}$ m$^{-1}$. © 1997 Elsevier Science B.V.

## 1. Introduction

The geometrical frame for general relativity is a Riemannian space–time. Within this frame one can calculate solar system effects and finds within an accuracy of $10^{-4}$ that all predictions of GR are confirmed by experiment. The equivalence principle which is at the basis of the geometrisation of gravity is tested even to much better accuracy (for a review see Ref. [1]). However, on theoretical grounds this geometrical frame may be too narrow, and there are indeed many reasons to consider a more general geometrical structure as mathematical description of physical space–time. One very prominent generalisation is the Riemann–Cartan geometry which (i) is the most natural generalisation of a Riemannian geometry by allowing a non-symmetric metric-compatible connection, (ii) treats spin on the same level as mass as it is indicated by the group theoretical analysis of the Poincaré group, and (iii) arises in most gauge theoretical approaches to general relativity, as e.g. in the Poincaré-gauge theory [2–4] or supergravity [5]. However, till now there is no experimental evidence for torsion. On the other hand, from the lack of effects which may be due to torsion one can calculate estimates on the maximal strength of the torsion fields. This is the purpose of this Letter: We first calculate that torsion in principle influences the experimental outcome of Hughes–Drever type experiments (for a review on these experiments see Ref. [1]). Since no effects were observed we get from the accuracy of these experiments

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upper bounds on the torsion strength. Therefore, by means of a reinterpretation of the Hughes–Drever type experiments we obtain the till now most stringent upper bounds on the torsion strength.

While torsion does not influence the behaviour of macroscopic bodies [7,6] it acts on the evolution of spin degrees of freedom and can in principle be measured by determining the precession of an elementary spin [8]. This spin–torsion interaction also modifies in first order of $\hbar$ the trajectory of an elementary particle. Turning around the way of reasoning, it is also possible to establish torsion by allowing the spin to behave in a way not predicted by general relativity [10,11]. The effect of torsion on spin can also influence the outcome of an interference experiment with neutrons [12,13]. This fact and the lack of any experimental evidence for a coupling has been used in Ref. [13] to pose an upper bound of $\leqslant 10^{-7}$ m$^{-1}$ on the strength of torsion fields. Another estimate [14] relates the strength of the coupling of spinning matter to torsion to the density of polarised particles. We will show that by means of a reinterpretation of Hughes–Drever type experiments we can restrict torsion to $\leqslant 10^{-15}$ m$^{-1}$. This of course does not mean that torsion does not exist or does not play an important role in our understanding of gravitation.

In the following we first derive the non-relativistic limit of the Dirac equation in a Riemann–Cartan space-time. The resulting Pauli equation with additional coupling to the axial torsion vector is used to derive the Hamiltonian for the energy levels of a bound two-particle system. This determines in the non-relativistic regime the energy levels of a nucleus consisting of a core and one valence proton which is the physical system usually taken to perform Hughes–Drever type experiments.

A Riemann–Cartan space-time consists of a metric $g_{\mu\nu}$ ($\mu,\nu = 0,\ldots,3$) and a metric-compatible connection $\Gamma^{\sigma}_{\mu\nu} = \{^{\sigma}_{\mu\nu}\} - K_{\mu\nu}{}^{\sigma}$ where $\{^{\sigma}_{\mu\nu}\}$ is the usual Christoffel connection and $K_{\mu\nu}{}^{\sigma}$ is the contorsion tensor related to the torsion $S_{\mu\nu}{}^{\sigma}$ through $S_{\mu\nu}{}^{\sigma} = \Gamma^{\sigma}_{[\mu\nu]} = -K_{\mu\nu}{}^{\sigma}$. We introduce tetrads $h_a^{\mu}$ ($a,b = 0,\ldots,3$) through $g_{\mu\nu}h_a^{\mu}h_b^{\nu} = \eta_{ab}$ with $\eta = \mathrm{diag}(-+++)$.

## 2. Dirac equation in Riemann–Cartan space–time

The Dirac matrices $\gamma^{\mu}$ are defined by the Clifford algebra $\gamma^{(\mu}\gamma^{\nu)} = g^{\mu\nu}$ and are connected with the standard Minkowski Dirac matrices $\gamma^a$ fulfilling $\gamma^{(a}\gamma^{b)} = \eta^{ab}$ by $\gamma^{\mu} = h_a^{\mu}\gamma^a$. In Dirac representation ($m = 1,2,3$)

$$\gamma^{(0)} = -\mathrm{i}\beta, \qquad \gamma^m = -\mathrm{i}\beta\alpha^m \tag{1}$$

with

$$\alpha^m = \begin{pmatrix} 0 & \sigma^m \\ \sigma^m & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{2}$$

$$\Sigma^m = \begin{pmatrix} \sigma^m & 0 \\ 0 & \sigma^m \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{3}$$

We use the Dirac equation which is derived from a minimally coupled Dirac Lagrangian [15,2,8] (for a recent review see also Ref. [9])

$$0 = \mathrm{i}\hbar\gamma^{\mu}D_{\mu}\psi + \frac{\mathrm{i}}{2}K_{\rho\mu}{}^{\rho}\gamma^{\mu}\psi + mc\psi = \mathrm{i}\hbar\gamma^{\mu}\overset{\{\}}{D}_{\mu}\psi - \hbar K_{\mu}\gamma_5\gamma^{\mu}\psi + mc\psi, \tag{4}$$

where

$$D_{\mu}\psi = \partial_{\mu}\psi + \Gamma_{\mu}\psi \tag{5}$$

with the spinorial representation of the anholonomic connection

$$\Gamma_{\mu} = \tfrac{1}{4}D_{\mu}h_a^{\nu}h_{\nu}^b\gamma_b\gamma^a = \tfrac{1}{4}\overset{\{\}}{D}_{\mu}h_a^{\nu}h_{\nu}^b\gamma_b\gamma^a - \tfrac{1}{4}K_{\mu\sigma}{}^{\nu}h_a^{\sigma}h_{\nu}^b\gamma_b\gamma^a. \tag{6}$$

$\overset{\{\}}{D}_\mu$ and $\overset{\{\}}{\Gamma}_\mu$ is the Christoffel part of the covariant derivative and connection, respectively. $K_\mu = \frac{1}{6}\epsilon_{\mu\sigma}{}^{\nu\rho}K_{\nu\rho}{}^\sigma$ is the axial part of the space–time torsion.

## 3. Non-relativistic limit: Pauli equation in Riemann–Cartan space–time

In order to carry through a non-relativistic approximation of the Dirac equation coupled to metric and torsion we first perform a Newtonian approximation of Riemann–Cartan theory. Along the lines of Ref. [16] we expand the metric with respect to the Newtonian potential in a quasi-Newtonian coordinate system ($dx^0 = c\,dt$, $i, j = 1, 2, 3$)

$$g_{00} = -1 + 2\frac{U}{c^2}, \tag{7}$$

$$g_{0i} = 0, \tag{8}$$

$$g_{ij} = \left(1 + 2\frac{U}{c^2}\right)\delta_{ij}. \tag{9}$$

The corresponding tetrads are

$$h^0_{(0)} = 1 + \frac{U}{c^2}, \tag{10}$$

$$h^i_{(0)} = 0, \tag{11}$$

$$h^0_m = 0, \tag{12}$$

$$h^i_m = \left(1 - \frac{U}{c^2}\right)\delta^i_m \tag{13}$$

from which one can calculate the matrices $\gamma^\mu$. The Riemannian part of the spinorial representation of the anholonomic connection is

$$\overset{\{\}}{\Gamma}_0 = -\frac{1}{2c^2}\alpha^i\partial_i U, \tag{14}$$

$$\overset{\{\}}{\Gamma}_i = -\frac{i}{2c^2}\epsilon_{ij}{}^k\Sigma^j\partial_k U. \tag{15}$$

The additional axial torsion is taken to be approximately constant at the position of the experiment.

We insert the metric, tetrads and connection into the Dirac equation and solve it with respect to $(\partial/\partial t)\psi$ where we neglect squares of the Newtonian potential and products of $U$ and torsion,

$$i\hbar\frac{\partial}{\partial t}\psi = -i\hbar c\left(1 - 2\frac{U}{c^2}\right)\alpha^i\partial_i\psi - \frac{i\hbar}{2c}\alpha^i\partial_i U\psi + \hbar c K_{(0)}\gamma_5\psi - \hbar c K_i\Sigma^i\psi + \left(1 - \frac{U}{c^2}\right)\beta mc^2\psi. \tag{16}$$

In order to perform a Foldy–Wouthuysen transformation of the Hamiltonian (compare Ref. [17]) we split the Hamiltonian into even and odd parts:

$$H = \mathcal{O} + \mathcal{E} + \beta mc \tag{17}$$

with

$$\mathcal{O} = -i\hbar c\left(1 - 2\frac{U}{c^2}\right)\alpha^i\partial_i - \frac{i\hbar}{2c}\alpha^i\partial_i U + \hbar c K_{(0)}\gamma_5, \tag{18}$$

$$\mathcal{E} = \left(1 - \frac{U}{c^2}\right)\beta mc^2 - \hbar c K_i \Sigma^i \tag{19}$$

and get

$$H'\varphi = \beta\left(mc^2 + \frac{\mathcal{O}^2}{2mc^2} - \frac{\mathcal{O}^4}{8m^3c^6}\right)\varphi + \mathcal{E}\varphi - \frac{1}{8m^2c^4}[\mathcal{O},[\mathcal{O},\mathcal{E}]]\varphi - \frac{i}{8m^2c^4}[\mathcal{O},\dot{\mathcal{O}}]\varphi$$

$$= \beta\left[mc^2\varphi + \left(-\frac{\hbar^2}{2m}\Delta\varphi - \frac{\hbar}{m}K_{(0)}\Sigma^i i\hbar\partial_i\varphi\right)\right] - U\beta m\varphi - \hbar c K_i\Sigma^i, \tag{20}$$

where we neglected all relativistic corrections and squares and derivatives of the torsion. Projection onto the large components and elimination of the rest energy gives the Pauli equation in a Riemann–Cartan space–time

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\Delta\psi - \frac{\hbar}{m}K_{(0)}\sigma^i i\hbar\partial_i\psi - mU\psi - \hbar c K_i\sigma^i\psi. \tag{21}$$

(The occurrence of $c$ in the last term just means that $K_i$ has the dimension $1/\text{length}$.) The coupling to the electromagnetic field is accomplished by

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi - e\phi\psi - \frac{e\hbar}{2mc}H_i\sigma^i\psi - mU\psi - \frac{\hbar}{m}K_{(0)}\sigma^i i\hbar\nabla_i\psi - \hbar c K_i\sigma^i\psi = H\psi \tag{22}$$

with $\nabla_i = \partial_i - (ie/\hbar c)A_i$. The coupling to rotation $\Omega_i$ can be introduced either by starting with the appropriately modified metric components (7) and (8) (see Ref. [18]) or by performing a unitary transformation of the quantum states obeying (21) to a rotating frame [19] (see also Ref. [20] and references therein). In each case one gets the additional coupling term $\Omega_i(L^i + \frac{1}{2}\hbar\sigma^i)$ where $L^i = \epsilon^{ijk}x^j(-i\hbar\nabla_k)$ is the angular momentum operator. (Since we are treating the non-relativistic limit without second quantisation, we encounter no problems which may arise from a non-renormalisability of a Fermi type coupling when inserting into the Dirac equation the field equations for torsion within the Einstein–Cartan theory [2].)

## 4. The two-particle-Hamiltonian

The two-particle Hamiltonian is given by the sum of two Hamiltonians of the above form. We restrict from the very beginning to the following case given by the experiment: A nucleus consisting of a core with vanishing total angular momentum ($J = 0$) and a valence proton with spin $S = \frac{1}{2}$ and some angular momentum $L$. The wave function is a function of two position variables and an angular momentum variable $\psi = \psi_{J,M_J}(x_1, x_2, t)$. Then

$$H = H_1 + H_2 + V \tag{23}$$

with

$$H_1 = -\frac{\hbar^2}{2m_1}\nabla_1^2 - e_1\phi - m_1 U, \tag{24}$$

$$H_2 = -\frac{\hbar^2}{2m_2}\nabla_2^2 - e_2\phi - m_2 U - \mu_2 H_i\sigma^i - \frac{\hbar}{m_2}K_{(0)}\sigma^i i\hbar\nabla_{2i} - \hbar c K_i\sigma^i, \tag{25}$$

where $V = V(x_2 - x_1)$ is some binding potential, $\mu_2 = e_2\hbar/2m_2 c$ and $\nabla_{1m}$ denotes the $U(1)$-covariant derivative with respect to the coordinate $x_1^m$. The quantisation axis is defined by the external magnetic field $H_i$.

We introduce the relative coordinate $x$ and a center-of-mass coordinate $X$,

$$x := x_2 - x_1, \qquad X := \frac{m_1}{m_1 + m_2} x_1 + \frac{m_2}{m_1 + m_2} x_2 \tag{26}$$

and insert the corresponding coordinate transformation into the Hamiltonian (23). We also split the electromagnetic potentials $A_i$ and $\phi$ into a part due to external sources $A_i^e$, $\phi^e$ and a part due to the charge of the other particle $A_j^i$, $\phi^i$: $A_i = A_i^e + A_i^i$, $\phi = \phi^e + \phi^i$. We absorb the electromagnetic field which is created by the particles itself and which is not connected with a derivative into a modified potential $V'(x)$ which depends on the relative coordinate only. Then the transformed two-particle Pauli equation reads (in the following we omit the index $J, M_J$ characterising the wave function)

$$H\varphi = -\frac{\hbar^2}{2m}\Delta_X\varphi - \frac{\hbar^2}{2m_{\text{red}}}\Delta_x\varphi + \frac{i\hbar}{c}\left(\frac{e_2}{m_2}A_i^e(x_2) - \frac{e_1}{m_1}A_i^e(x_1)\right)\frac{\partial\varphi}{\partial x^i} + \frac{i\hbar}{mc}[e_1 A_i^e(x_1) + e_2 A_i^e(x_2)]\frac{\partial\varphi}{\partial X^i}$$

$$+ \frac{i\hbar}{c}A_j^i(x)\frac{e}{m}\frac{\partial\varphi}{\partial X^j} + \frac{i\hbar}{c}A_j^i(x)\left(\frac{e_2}{m_2} - \frac{e_1}{m_1}\right)\frac{\partial\varphi}{\partial x^j} - \mu_2 H_i\sigma^i\varphi + m_2 U(x_2)\varphi + m_1 U(x_1)\varphi$$

$$- \hbar c K_i\sigma^i\varphi - i\frac{\hbar^2}{m_2}K_{(0)}\sigma^i\left(\frac{m_2}{m}\frac{\partial}{\partial X^i} + \frac{\partial}{\partial x^i}\right)\varphi + V'(x)\varphi + e_1\phi^e(x_1)\varphi + e_2\phi^e(x_2)\varphi, \tag{27}$$

where we defined the total charge $e := e_1 + e_2$, the total mass $m := m_1 + m_2$, the reduced mass $m_{\text{red}} = m_1 m_2/m$, used the Coulomb gauge for the external electromagnetic potential, and neglected squares of the electromagnetic potential and products of torsion with the electromagnetic potential. We approximate

$$\frac{e_2}{m_2}A_i^e(x_2) - \frac{e_1}{m_1}A_i^e(x_1) \approx \frac{e_{\text{red}}}{m_{\text{red}}}A_i^e(X) + \frac{2c}{\hbar}\mu_{\text{red}}x^k\nabla_k A_i^e(X), \tag{28}$$

$$e_1 A_i^e(x_1) + e_2 A_i^e(x_2) \approx e A_i^e(X) + e_{\text{red}}x^k\nabla_l A_i^e(X), \tag{29}$$

$$e_1\phi(x_1) + e_2\phi(x_2) \approx e\phi(X) + e_{\text{red}}x^i\nabla_i\phi(X), \tag{30}$$

$$m_1 U(x_1) + m_2 U(x_2) \approx mU(X) + \tfrac{1}{2}m_{\text{red}}x^k x^l\nabla_k\nabla_l U(X), \tag{31}$$

where we introduced the "reduced charge"

$$e_{\text{red}} := m_{\text{red}}\left(\frac{e_2}{m_2} - \frac{e_1}{m_1}\right) \tag{32}$$

and the "reduced Bohr's magneton"

$$\mu_{\text{red}} := \frac{\hbar}{2mc}\left(m_1\frac{e_2}{m_2} + m_2\frac{e_1}{m_1}\right). \tag{33}$$

(In the usually considered case $m_1 \to \infty$ (very heavy nucleus) we get $e_{\text{red}} \to e_2$ and $\mu_{\text{red}} \to e_2\hbar/2m_2 c = \mu_2$.) If we complete the partial derivatives to covariant derivatives, neglect squares of the Maxwell potential, and neglect the internal vector potential $A_j^i(x)$ which is of the order $c^{-1}$, then we get

$$H\varphi = -\frac{\hbar^2}{2m}\nabla_X^2\varphi - \frac{\hbar^2}{2m_{\text{red}}}\Delta_x\varphi + \frac{e_{\text{red}}}{mc}x^i\nabla_i A_j^e(X)i\hbar\frac{\partial\varphi}{\partial X^j} + 2\mu_{\text{red}}x^k\nabla_k A_j^e(X)i\frac{\partial\varphi}{\partial x^j} - \mu_2 H_i\sigma^i\varphi + V'(x)\varphi$$

$$+ e\phi^e(X)\varphi + e_{\text{red}}x^i\nabla_i\phi^e(X)\varphi + mU(X)\varphi + \tfrac{1}{2}m_{\text{red}}x^k x^l\nabla_k\nabla_l U(X)\varphi$$

$$- i\frac{\hbar^2}{m_2}K_{(0)}\sigma^i\left(\frac{m_2}{m}\frac{\partial}{\partial X^i} + \frac{\partial}{\partial x^i}\right)\varphi - \hbar c K_i\sigma^i\varphi \tag{34}$$

with $\nabla_{Xi} = \partial / \partial X^i - (ie/\hbar c) A_i^e(X)$. This is the final form of the Hamilton operator expressed with respect to the relative coordinates $x$ and the center-of-mass coordinates $X$.

Next we extract from this total Hamiltonian that Hamiltonian which describes the energy levels of this bound system by freezing the center-of-mass motion and keeping the center-of-mass coordinate of the atom at $X = 0$. In addition, we specialise to $\phi^e = 0$ and gauge away constant terms so that we get

$$H\varphi = -\frac{\hbar^2}{2m_{red}}\Delta_x\varphi + 2\mu_{red}x^k\nabla_k A_i^e(0)i\hbar\frac{\partial\varphi}{\partial x^i} - \mu_2 H_i\sigma^i\varphi + V'(x)\varphi$$

$$+ \tfrac{1}{2}m_{red}x^k x^l\nabla_k\nabla_l U(0)\varphi - i\frac{\hbar^2}{m_2}K_{(0)}\sigma^i\frac{\partial}{\partial x^i}\varphi - \hbar c K_i\sigma^i\varphi. \tag{35}$$

Again we neglected squares of the vector potential. We take a constant external magnetic field: $A_i^e(X) = \tfrac{1}{2}\epsilon_{ilk}H^l X^k$. Then the Hamiltonian giving the energy levels is

$$H_E = -\frac{\hbar^2}{2m_{red}}\Delta_x - \mu_{red}H^l\delta^{ij}\epsilon_{ilk}x^k i\hbar\frac{\partial}{\partial x^j} - \mu_2 H_i\sigma^i - \frac{\hbar^2}{m_2}K_{(0)}\sigma^i i\frac{\partial}{\partial x^i}$$

$$+ \tfrac{1}{2}m_{red}x^k x^l\nabla_k\nabla_l U(0) - \hbar c K_i\sigma^i + V'(x). \tag{36}$$

If we consider rotation we have to add $\Omega_i(l^i + \tfrac{1}{2}\hbar\sigma^i)$ where $l^i$ is the angular momentum with respect to the relative coordinates. The spin-rotation term has been discussed by Mashhoon [19,20] and the $\Omega_i l^i$ term by Silverman [21]. We get various parts for this Hamiltonian describing the energy levels of a bound system:

$$H_E = H_0 + H_{em} + H_{Newton} + H_{torsion} \tag{37}$$

with

$$H_0 = -\frac{1}{2m_{red}}\Delta + V'(x), \tag{38}$$

$$H_{em} = -H_i\left(\frac{\mu_{red}}{\hbar}l^i + \mu_2\sigma^i\right), \tag{39}$$

$$H_{Newton} = \tfrac{1}{2}m_{red}x^k x^l\nabla_k\nabla_l U(0), \tag{40}$$

$$H_{torsion} = -\frac{\hbar^2}{m_2}K_{(0)}\sigma^i i\frac{\partial}{\partial x^i} - \hbar c K_i\sigma^i. \tag{41}$$

For the electric proton–nucleus interaction we have $V'(x) = -Ze^2/x$. For the nuclear proton–nucleus interaction we have to take some appropriate model for the potential of the nucleus, e.g. the harmonic oscillator potential or Wood–Saxon potential. Note that there are no Einsteinian effects due to the acceleration $\nabla U$. This is in agreement with the equivalence principle: The effect of gravitational acceleration can be cancelled by a transformation to a suitable accelerated frame and therefore does not influence the energy levels. $H_0$ describes the atom without external fields, $H_{em}$ the Zeeman effect. The third Hamiltonian is the usual gravitational interaction with the Newtonian part of the Riemannian space-time curvature. The last term describes the coupling to torsion under consideration. The first term amounts to a spin–momentum coupling which will lead to second order effects only. This is the generalised Pauli equation for the energy levels in a Riemann–Cartan space-time.

## 5. Comparison with experiment

We use the above Hamiltonian to calculate the Zeeman splitting of energy levels in an atom in the presence of torsion. We describe the case which is considered in usual Hughes–Drever type experiments (see e.g. Ref. [1])

namely an atomic nucleus which consists of a $J = 0$ core and a valence proton with angular momentum $L = 1$. Our quantisation axis for the spin is given by the external magnetic field $H_i$ (we can now use $\mu_{\text{red}} \approx \mu_2 = \mu_B$). We are going to calculate the shifts in the energy levels due to the interaction Hamiltonian (41) describing non-Einsteinian effects. We use first order perturbation theory. The unperturbed states $|J, M_J\rangle$ are given by

$$|\tfrac{3}{2}, \tfrac{3}{2}\rangle = \begin{pmatrix} |1,1\rangle \\ 0 \end{pmatrix}, \qquad |\tfrac{3}{2}, \tfrac{1}{2}\rangle = \begin{pmatrix} \sqrt{\tfrac{2}{3}}|1,0\rangle \\ \sqrt{\tfrac{1}{3}}|1,1\rangle \end{pmatrix},$$

$$|\tfrac{3}{2}, -\tfrac{1}{2}\rangle = \begin{pmatrix} \sqrt{\tfrac{1}{3}}|1,-1\rangle \\ \sqrt{\tfrac{2}{3}}|1,0\rangle \end{pmatrix}, \qquad |\tfrac{3}{2}, -\tfrac{3}{2}\rangle = \begin{pmatrix} 0 \\ |1,-1\rangle \end{pmatrix}. \tag{42}$$

The interaction Hamiltonian under consideration (41) has the structure $(\mathcal{A}_k^i p_i + \mathcal{A}_k)\sigma^k$. We neglect effects due to $\nabla_i \nabla_j U$ since these effects give energy shifts smaller than $10^{-40}$ eV which is too small to be detectable. We get for the corresponding expectation values

$$\langle \tfrac{3}{2}, \tfrac{3}{2}|\mathcal{A}_k^i p_i + \mathcal{A}_k|\tfrac{3}{2}, \tfrac{3}{2}\rangle = \langle 1,1|\mathcal{A}_z|1,1\rangle, \tag{43}$$

$$\langle \tfrac{3}{2}, \tfrac{1}{2}|\mathcal{A}_k^i p_i + \mathcal{A}_k|\tfrac{3}{2}, \tfrac{1}{2}\rangle = \tfrac{2}{3}\langle 1,0|\mathcal{A}_z|1,0\rangle - \tfrac{1}{3}\langle 1,1|\mathcal{A}_z|1,1\rangle, \tag{44}$$

$$\langle \tfrac{3}{2}, -\tfrac{1}{2}|\mathcal{A}_k^i p_i + \mathcal{A}_k|\tfrac{3}{2}, -\tfrac{1}{2}\rangle = \tfrac{1}{3}\langle 1,-1|\mathcal{A}_z|1,-1\rangle - \tfrac{2}{3}\langle 1,0|\mathcal{A}_z|1,0\rangle, \tag{45}$$

$$\langle \tfrac{3}{2}, -\tfrac{3}{2}|\mathcal{A}_k^i p_i + \mathcal{A}_k|\tfrac{3}{2}, -\tfrac{3}{2}\rangle = -\langle 1,-1|\mathcal{A}_z|1,-1\rangle, \tag{46}$$

where we used that the expectation value for expressions linear in the momentum vanishes. The transition frequencies turn out to be

$$\hbar\omega(\tfrac{3}{2} \to \tfrac{1}{2}) = \tfrac{4}{3}\langle 1,1|\mathcal{A}_z|1,1\rangle - \tfrac{2}{3}\langle 1,0|\mathcal{A}_z|1,0\rangle, \tag{47}$$

$$\hbar\omega(\tfrac{1}{2} \to -\tfrac{1}{2}) = \tfrac{4}{3}\langle 1,0|\mathcal{A}_z|1,0\rangle - \tfrac{1}{3}\langle 1,1|\mathcal{A}_z|1,1\rangle - \tfrac{1}{3}\langle 1,-1|\mathcal{A}_z 1,-1\rangle, \tag{48}$$

$$\hbar\omega(-\tfrac{1}{2} \to -\tfrac{3}{2}) = \tfrac{4}{3}\langle 1,-1|\mathcal{A}_z|1,-1\rangle - \tfrac{2}{3}\langle 1,0|\mathcal{A}_z|1,0\rangle. \tag{49}$$

The matrix elements are $\langle 1,1|\mathcal{A}_k|1,1\rangle = \langle 1,0|\mathcal{A}_k|1,0\rangle = \langle 1,-1|\mathcal{A}_k|1,-1\rangle = \mathcal{A}_z$ so that we get an equal shift $\hbar\omega = \tfrac{2}{3}\hbar c K_z$ for all three transition frequencies. The search for such a shift during the change of the $z$-axis with respect to the orthogonal nonrotating Newtonian coordinate system amounts to a Hughes–Drever experiment. If space–time torsion will be detected it will lead to a diurnal shift of the Zeeman singlet line. However, present experiments (see Refs. [22–24]) did not detect any effects. Indeed, the experimental setup of Chupp et al. [22] uses two kinds of atoms, $^{21}$Ne and $^3$He, where the latter serves as magnetometer standard. Both kinds of atoms are subject to the same magnetic field which can be controlled to an accuracy of $\delta B \leqslant 10^{-10}$ G. Since both atoms possess different $g$-factors, $g(^{21}\text{Ne}) = -0.6619\mu_B$ and $g(^3\text{He}) = -2.1276\mu_B$ where $\mu_B$ is Bohr's magneton of a nucleon, the Zeeman lines are different. During the experiment the energy difference of these two Zeeman frequencies,

$$E(^3\text{He}) - E(^{21}\text{Ne}) = [g(^3\text{He}) - g(^{21}\text{Ne})](\mu_B B - \hbar c K_z), \tag{50}$$

can be recorded. Here $K_z$ is a function of the orientation of the quantisation axis. If this axis is fixed to the surface of the earth it can exhibit a diurnal time dependence. The accuracy of the experiments can be described in terms of an effective variation in the magnetic field $B$, $\delta B \leqslant 10^{-10}$ G. Consequently, if the influence of torsion $\hbar c K_z$ is larger than $\mu_B \delta B$, then an observable effect would occur. If one is going to redo this experiment, searches for the above described effect and observes a null-result, then we are lead to the following estimate on torsion,

$$K_z \leqslant \frac{1}{\hbar c}\mu_B \delta B \leqslant 1.5 \times 10^{-15} \text{ m}^{-1}. \tag{51}$$

From this new version of an already performed Hughes–Drever like experiment we may get the up to now best estimate for the space-components of the axial part of a hypothetical space–time torsion. It is not possible to test other parts of the torsion tensor since the Dirac equation couples to the axial part only. One needs higher spin equations for a coupling to the trace and the traceless part of the torsion tensor.

Originally Hughes–Drever type experiments are designed to search for possible anisotropies of space–time, or, equivalently, anisotropies of the mass of quantum systems. These parts will lead to a splitting of the singlet line to a triplet line. Therefore, it is possible to distinguish between effects due to mass anisotropy and torsion: while the first cause leads to a splitting of the singlet line, the torsion shifts the whole line spectrum in the same way.

Also the experiments designed to search for an anomalous spin–coupling [25] can be used for estimating the strength of the torsion coupling. This experiment has been analysed by Mashhoon [20] in order to show that it tests indirectly the spin–rotation coupling. Since space–time torsion couples to the spin in the same way as rotation (36), one also can draw the same conclusions regarding torsion. One arrives at similar estimates as above.

Although $K_{(0)}$ leads to second order effects only in the energy shift, it influences the center-of-mass motion via a spin–momentum coupling (compare Eq. (34)). Such a coupling can be tested with atom beam interferometry. Using a spin flip as described in Ref. [19] or in Ref. [26] for testing other spin–momentum couplings, we get the phase shift

$$\delta\phi = K_{(0)}\Delta l, \tag{52}$$

where $\Delta l$ is the distance between splitting and recombination of the atomic beam. Note that this phase shift is nondispersive and does not depend on the interaction time. If we take an absolute accuracy $\delta\phi \leqslant 10^{-2}$ and $\Delta l = 1$ m and assume a null experiment, then we get the estimate $K_{(0)} \leqslant 10^{-2}$ m$^{-1}$ for the time component of the axial torsion. The above phase shift which comes from a spin–momentum coupling due to the existence of torsion can be distinguished through its mass independence from a similar phase shift due to a violation of local Lorentz invariance [26].

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