Technical Matter Wave Optics

- Robert`s Axiom: Only errors exist*
- Berman`s Corrolary: One man`s error is another man`s data

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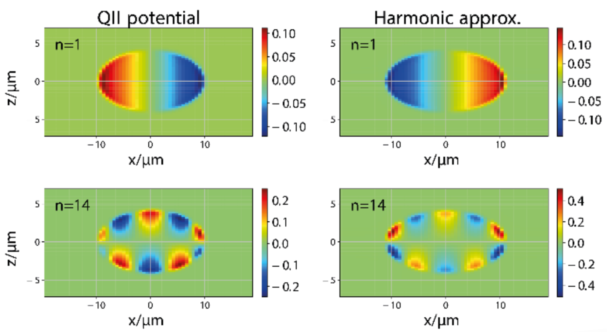
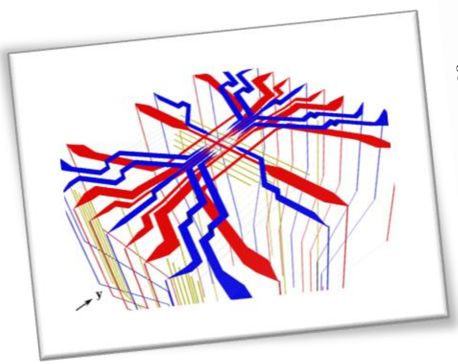
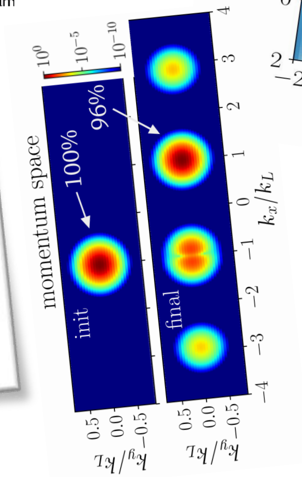
DLR

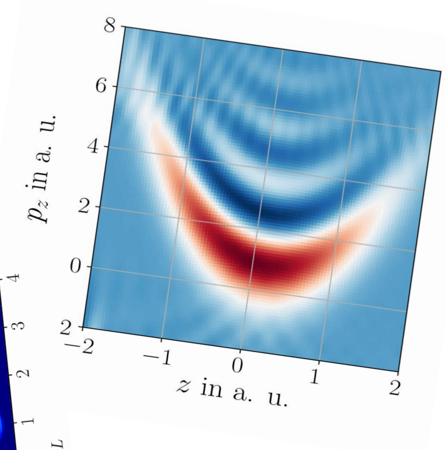
J. Teske
A. Neumann
J. Battenberg
S. Srinivasan
O. Gabel
J. Richhardt
L. Plimak
R. Walser

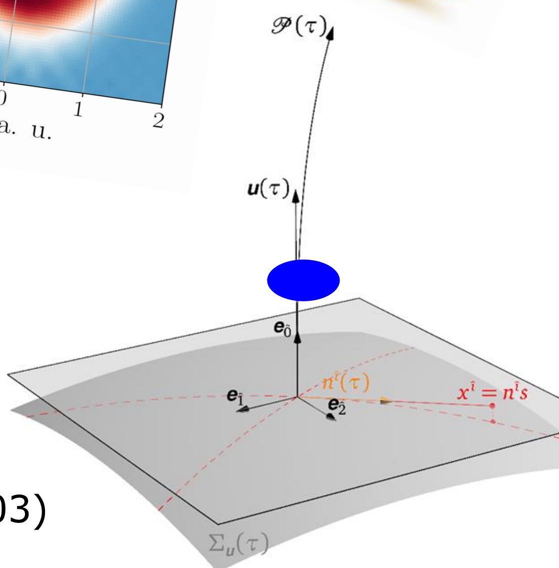
QUANTUS
Collaboration











*Murphy`s Law: The 26th Anniversary Edition, A. Bloch, Penguin (2003)

Aberrations everywhere!

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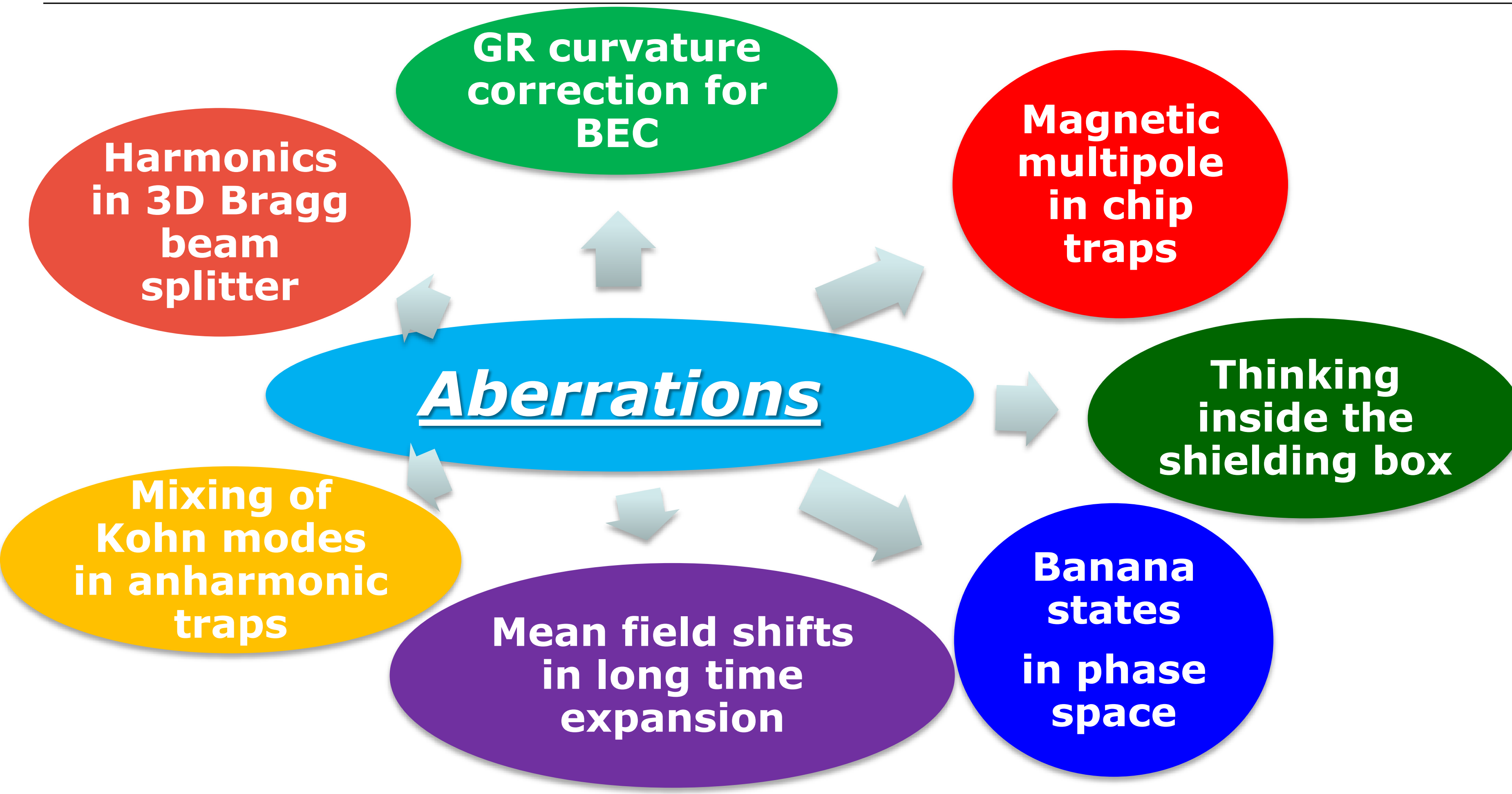
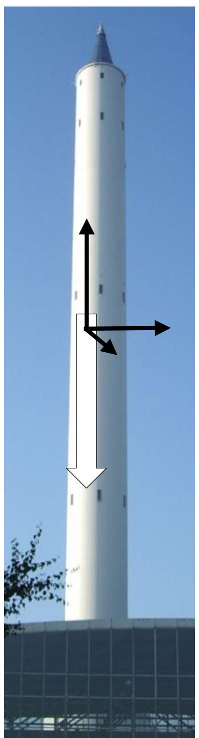


Quantum gases in free fall

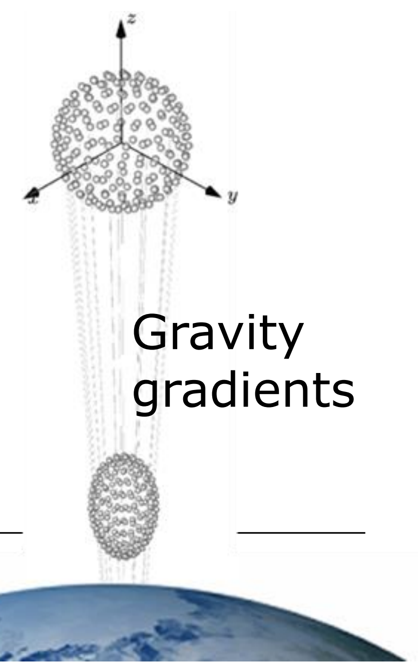
- BEC in curved space-time
- O. Gabel



- **Drop tower experiment:** free fall of atomic gas Bose-Einstein condensates BEC's in Newtonian gravity **NBEC**
- **Measurment of g:** interferometry
- **Classical gravity model EGM** N=2159 multipoles

$$V(r',\theta',\phi') = -\frac{GM_{\oplus}}{r'}\Big[1+\sum_{n=2}^{N}\left(\frac{a}{r'}\right)^{n}\sum_{m=-n}^{n}P_{nm}(\cos\theta')$$
$$\times(C_{nm}\cos m\phi' + S_{nm}\sin m\phi')\Big]. \qquad (3)$$

- **What about GR corrections to BEC's?**



Nandi et. al. PRA **76**, 063617 (2007)
Müntinga et al, PRL 110, 093602 (2013)
Pavlis et al., J. Geophys. Res. 117, B04406 (2012).

BEC in curved space-time

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- **Newtonian physics**: Gross-Pitaevskii mean field BEC's $\psi \in \mathbb{C}$

$$i\hbar\,\partial_t \psi(\boldsymbol{x},t) = \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + V_{\mathrm{trap}}(\boldsymbol{r},t) + \frac{4\pi\hbar^2 a_{\mathrm{s}}}{m}|\psi|^2\right]\psi(\boldsymbol{x},t) \qquad \int \mathrm{d}^3x\,|\psi|^2 = N$$

- **Flat space** relativistic analog: non-linear Klein-Gordon equation

$$\left[\eta^{\alpha\beta}\partial_\alpha\partial_\beta + \left(\frac{mc}{\hbar}\right)^2 + \xi|\phi|^2\right]\phi = 0 \qquad \phi = \psi\,\exp\left(-i\frac{mc^2}{\hbar}\tau\right)$$

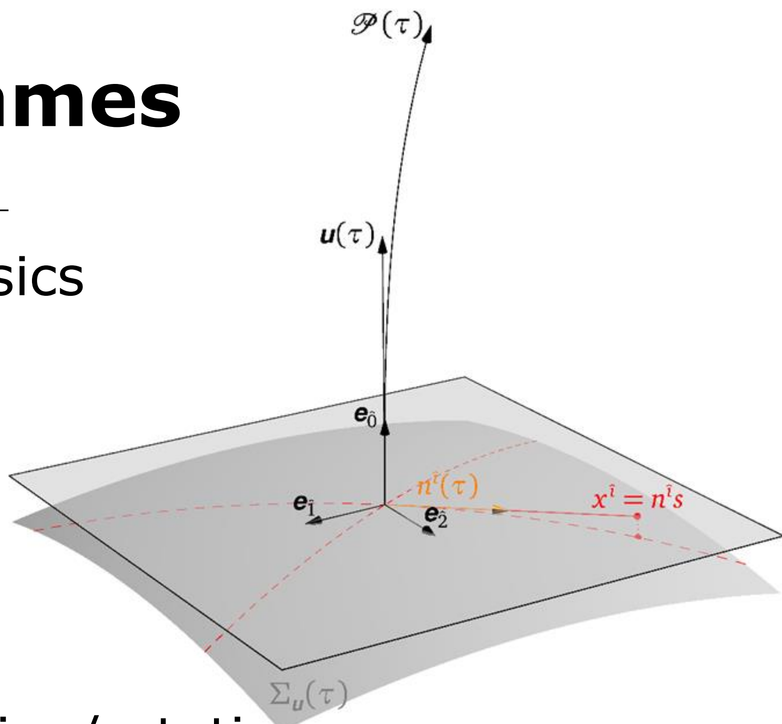
$$\eta^{\alpha\beta} \longrightarrow g^{\alpha\beta}$$
$$\partial_\alpha \longrightarrow \nabla_\alpha$$

- **Curved space-time**: correspondence principle
- **EBEC:** curved space, non-linear Klein-Gordon equation[3]

$$\left[g^{\mu\nu}\nabla_\mu\nabla_\nu + \left(\frac{mc}{\hbar}\right)^2 + \xi|\phi|^2\right]\phi = 0 \qquad \nabla_\mu\nabla^\mu = g^{\mu\nu}\left(\partial_\mu\partial_\nu + \Gamma^\sigma{}_{\mu\sigma}\partial_\nu\right)$$

[3] Anandan, PRL **47**, 463 (1981), Gr. Volovik, Universe in Helium droplet, analog gravity

Observers in GR: local frames



- **Einstein:** Observers fall along geodesics

$$\frac{Du^\sigma}{d\tau} \equiv \frac{du^\sigma}{d\tau} + \Gamma^\sigma{}_{\mu\nu} u^\mu u^\nu = 0$$

- **Inertial frames**

$$e_{\hat{\alpha}}{}^\sigma := \frac{\partial X^\sigma}{\partial x^{\hat{\alpha}}}$$ tetrad of basis vectors $$e_{\hat{0}}{}^\sigma = u^\sigma / c$$

- **Frame evolution** including acceleration/rotation

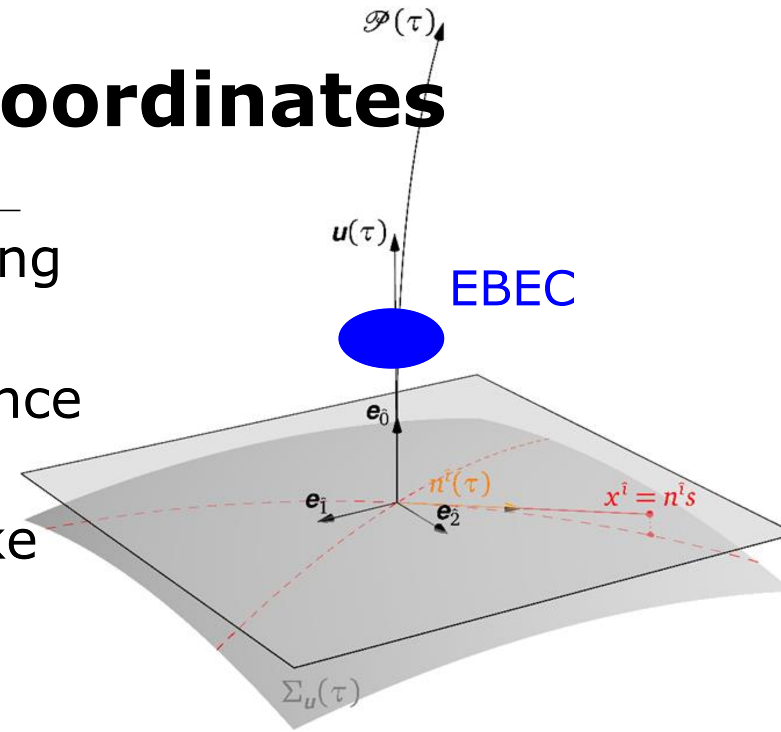
$$\frac{D}{d\tau} e_{\hat{\alpha}}{}^\mu = -\Omega^{\hat{\kappa}}{}_{\hat{\alpha}}\, e_{\hat{\kappa}}{}^\mu \quad \Omega^{\hat{\kappa}}{}_{\hat{\alpha}} = -\frac{1}{c^2}\left[a^{\hat{\kappa}} u_{\hat{\alpha}} - u^{\hat{\kappa}} a_{\hat{\alpha}}\right] - \omega^{\hat{\jmath}} \epsilon_{\hat{\jmath}}{}^{\hat{k}}{}_{\hat{a}}$$

acceleration part — rotation part

- **Observer** measures projection of space-time tensors in frame

  e.g. curvature tensor: $$R_{\hat{\gamma}\hat{\delta}\hat{\alpha}\hat{\beta}} = R_{\sigma\rho\mu\nu} e_{\hat{\gamma}}{}^\sigma e_{\hat{\delta}}{}^\rho e_{\hat{\alpha}}{}^\mu e_{\hat{\beta}}{}^\nu$$

Extended objects: Fermi coordinates



- **Local coordinates:** inertial frame along world-line
- **Extended objects:** „beyond equivalence principle"
- **Taylor expand** metric along space-like geodesics away from world-line $\mathcal{P}(\tau)$
- **Tidal potential:** non-rel. limit

$$V_{\text{tidal}}(\tau, x^{\hat\imath}) = \frac{c^2}{2}\left[g_{00} - \eta_{\hat0\hat0}\right]$$

Order:	0th	1st	2nd	3rd	
$g_{00} =$	$\eta_{\hat0\hat0}$	$-\frac{2}{c}\Omega_{\hat0\hat\imath}x^{\hat\imath}$	$+\left(R_{\hat0\hat\imath_1\hat\imath_2\hat0} + \frac{1}{c^2}\Omega^{\hat\kappa}{}_{\hat\imath_1}\Omega_{\hat\kappa\hat\imath_2}\right)x^{\hat\imath_1}x^{\hat\imath_2}$	$-\frac{1}{3}\left(R_{\hat0(\hat\imath_1\hat\imath_2\vert\hat0;\vert\hat\imath_2)} - \frac{4}{c}\Omega^{\hat\delta}{}_{(\hat\imath_1\vert}R_{\hat\delta\vert\hat\imath_2\hat\imath_3)\hat0}\right)x^{\hat\imath_1}x^{\hat\imath_2}x^{\hat\imath_3}$	$+\mathscr{O}(x^4)$
$g_{0b} =$		$-\frac{1}{c}\Omega_{\hat b\hat\imath}x^{\hat\imath}$	$+\frac{2}{3}R_{\hat0\hat\imath_1\hat\imath_1\hat b}x^{\hat\imath_1}x^{\hat\imath_2}$		$+\mathscr{O}(x^3)$
$g_{ab} =$	$\eta_{\hat a\hat b}$		$+\frac{1}{3}R_{\hat a\hat\imath_1\hat\imath_2\hat b}x^{\hat\imath_1}x^{\hat\imath_2}$		$+\mathscr{O}(x^3)$
	flat space	connection	Riemann/inertial curvature	curvature gradients	

equivalence principle | **beyond equivalence principle**

EBEC's in Fermi coordinates

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- **EBEC:** non-linear KG mean-field
- **Perturbation ansatz** for 2nd-order Fermi metric

$$g_{\alpha\beta}(x) = \eta_{\alpha\beta} + h_{\alpha\beta}(x) \qquad h_{\alpha\beta} \propto R_{\hat{\alpha}\hat{\imath}_1\hat{\imath}_2\hat{\beta}} x^{\hat{\imath}_1} x^{\hat{\imath}_2}$$

- **Low energy limit for EBEC:** curvature corrections to NBEC (Gross-Pitaevskii)

$$i\hbar\,\partial_t\psi = \left[\hat{H}_0 + \hat{H}_1\right]\psi$$

gravity gradient

$$\hat{H}_0 = -\frac{\hbar^2}{2m}\left(\boldsymbol{\nabla}^2 + \xi|\psi|^2\right) + V_{\text{tidal}} \qquad \xi = 8\pi a_{\text{s}}$$

$$\hat{H}_1 = -\frac{\hbar^2}{2m}\left(h^{\hat{\imath}\hat{\jmath}} - h^{\hat{0}\hat{0}}\eta^{\hat{\imath}\hat{\jmath}}\right)\partial_{\hat{\imath}}\partial_{\hat{\jmath}} - i\hbar c\, h^{\hat{0}\hat{\jmath}}\partial_{\hat{\jmath}} + \delta V_{\text{tidal}} + h^{\hat{0}\hat{0}}\frac{\hbar^2\xi}{2m}|\psi|^2$$

- Tidal potential $\frac{1}{2}mc^2\, h^{\hat{0}\hat{0}} = V_{\text{tidal}} + \delta V_{\text{tidal}}$
- Very small residual curvature corrections from $h^{\hat{\imath}\hat{\jmath}}, h^{\hat{0}\hat{\jmath}}, h^{\hat{0}\hat{0}}$

Dominant tidal corrections

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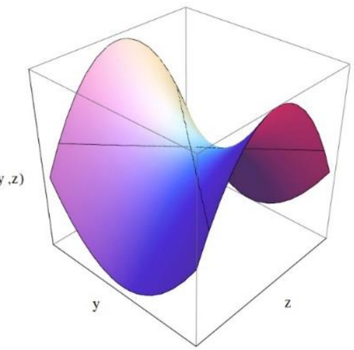
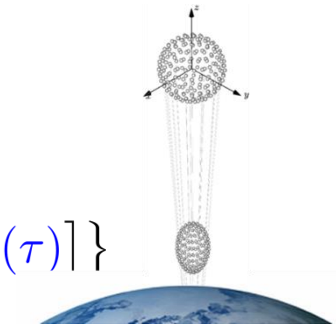
- **Tidal potential** in Fermi coordinates $O(x^2)$

$$V_{\text{tidal}} + \delta V_{\text{tidal}} = \frac{GMm}{4R_0^3}\Gamma^2\{2x^2 - z^2 - y^2 - 3\,(1-\epsilon)\left[(z^2-y^2)\cos 2\Phi(\tau) + yz\sin 2\Phi(\tau)\right]\}$$

$$\approx \frac{m\Omega^2}{2}\left\{x^2 + y^2 - 2z^2 - \tfrac{1}{2}\epsilon(z^2 - 3x^2) + \mathcal{O}(\epsilon^2)\right\}$$

$$\epsilon = \frac{R_S}{R} \approx \frac{9\,\text{mm}}{6378\,\text{km}} \approx 10^{-9} \qquad \Gamma = (1 - \tfrac{3}{2}\epsilon)^{-1/2}$$





- **Newtonian correction:** V_{tidal} normal/inverted harmonic oscillator

$$\nu(0) = 0.197\,\text{mHz}$$

$$\nu(240\,\text{km}) = 0.186\,\text{mHz}$$

$$L = \sqrt{\frac{\hbar}{m\Omega(0)}} = 767.8\,\mu\text{m}$$

- **Einstein GR correction :** δV_{tidal} residual curvature corrections (1 cm BEC on Earth)

$$\frac{\epsilon}{2}\left(\frac{r = 1\,\text{cm}}{R_0 = 6378\,\text{km}}\right)^2 \approx 4 \times 10^{-27}$$

Magnetic traps & lenses on atom chips

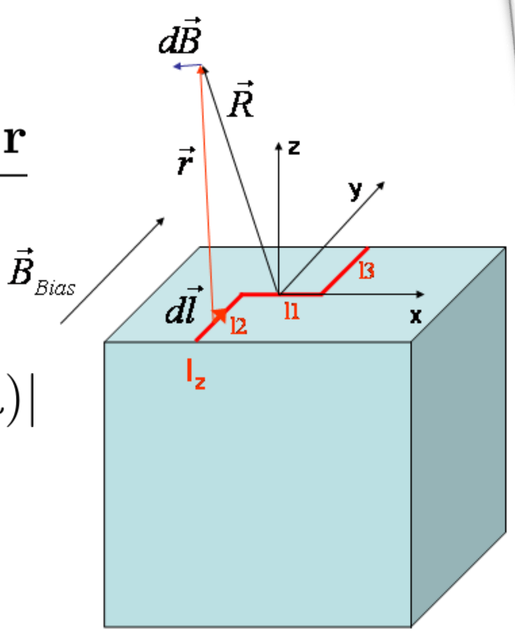
– **Fast multipole expansion**
– **J. Battenberg**

- 2D geometry of conducting strips
- several active layers
- multiple currents
- Biot-Savart law

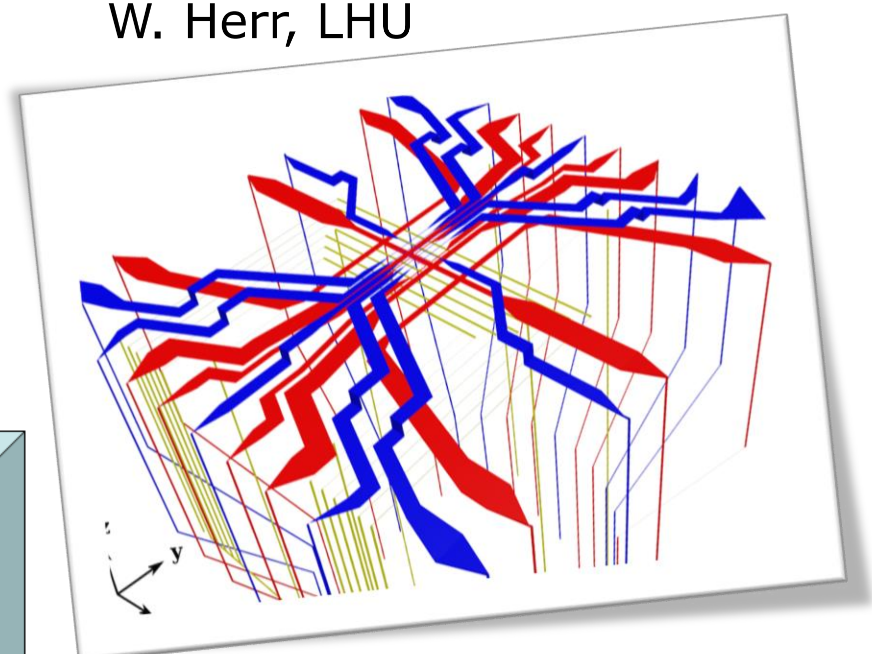
$$\mathbf{B}(\mathbf{R}) = \frac{\mu_0 I}{4\pi} \int_C \frac{d\mathbf{l} \times \mathbf{r}}{r^3}$$

- Zeeman potential

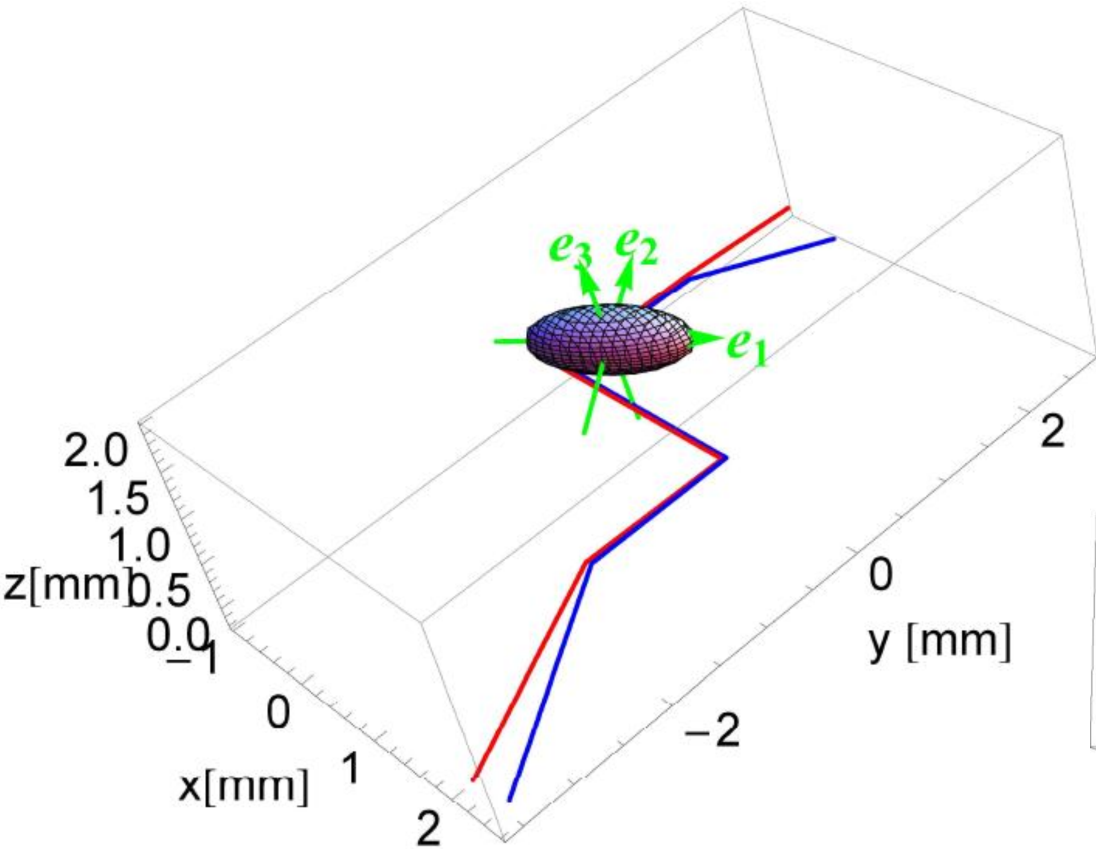
$$V(\mathbf{R}) = m\mu_B g_F |\mathbf{B}(\mathbf{R})|$$

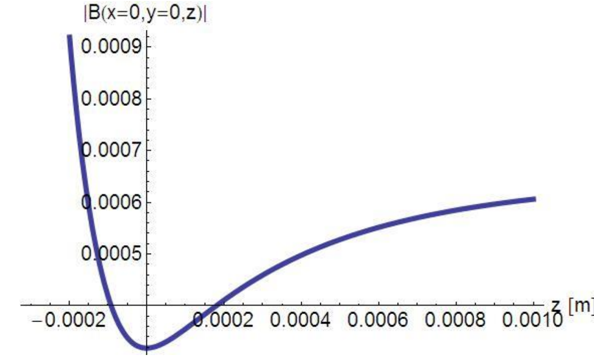


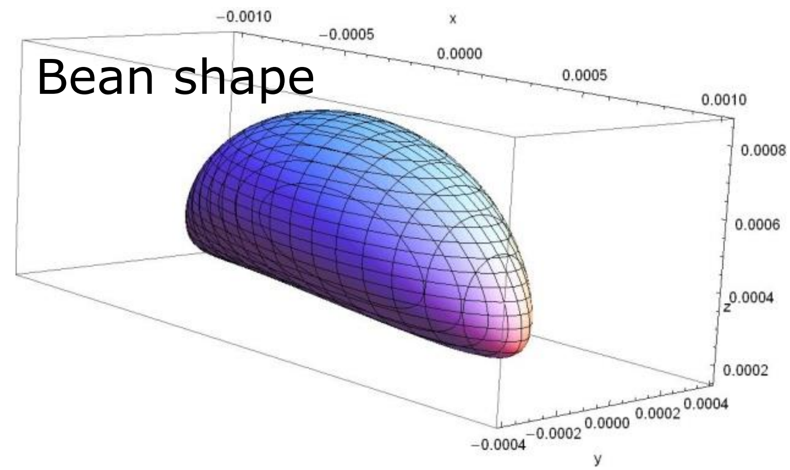
exp. chip design
W. Herr, LHU



Shape of Zeeman potential







QII frequency manifold

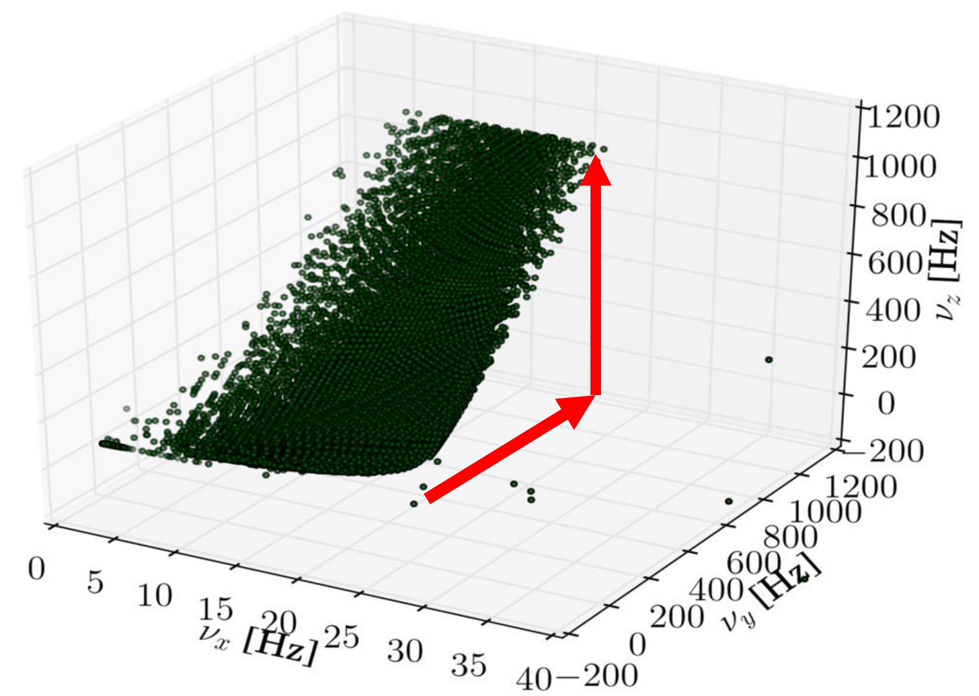
$$\{(\nu_1,\nu_2,\nu_3) \in R^3 | (I_x, I_y, I_b, I_s) \in R^4 \text{ with feasable solution}\}$$

Eigen-frequencies of potential Hess-matrix at potential minimum

$$V(\mathbf{x}) = \frac{1}{2} m \mathbf{x} \Omega^2 \mathbf{x} + \dots$$

Feasible frequencies
2D planar manifold
Earnshaw theorem
magnetic shield

Map of possible frequencies is current QII setup



Magnetic multipoles

Scalar potential in magneto-statics

$$\nabla \times \mathbf{B} = 0, \quad \nabla \cdot \mathbf{B} = 0$$

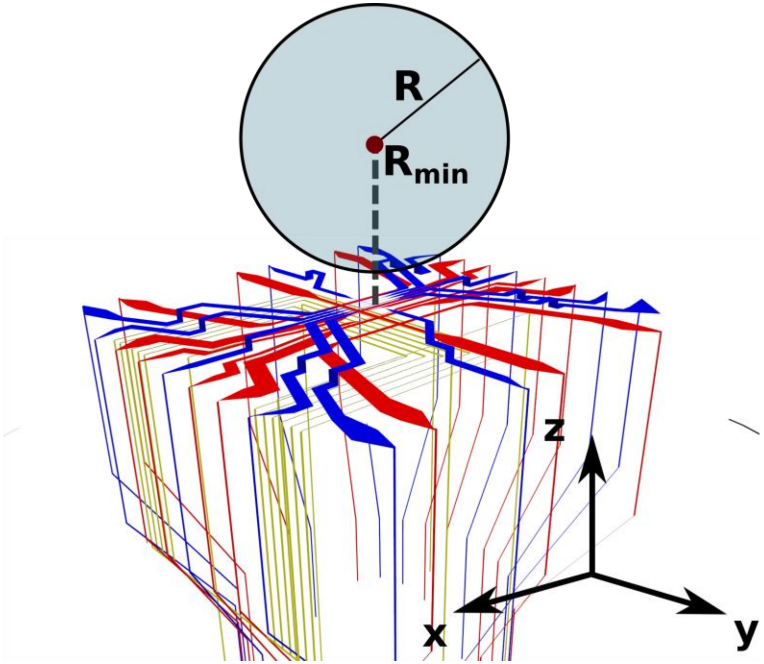
$$\mathbf{B} = -\nabla\Phi, \quad \Delta\Phi = 0$$

Regular harmonic functions

$$R^{lm}(\mathbf{r}) = r^l Y^{lm}(\frac{\mathbf{r}}{r})$$

$$\sim \{1, x \pm iy, z, x^2 + y^2 - 2z^2, \ldots\}$$

$$\Phi(\mathbf{r}) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} \Phi_{lm} R^{lm}\left(\frac{\mathbf{r}}{R}\right) \; [Gm]$$



Multipole coefficients from mag. field $\mathbf{B}(x \in S^2)$

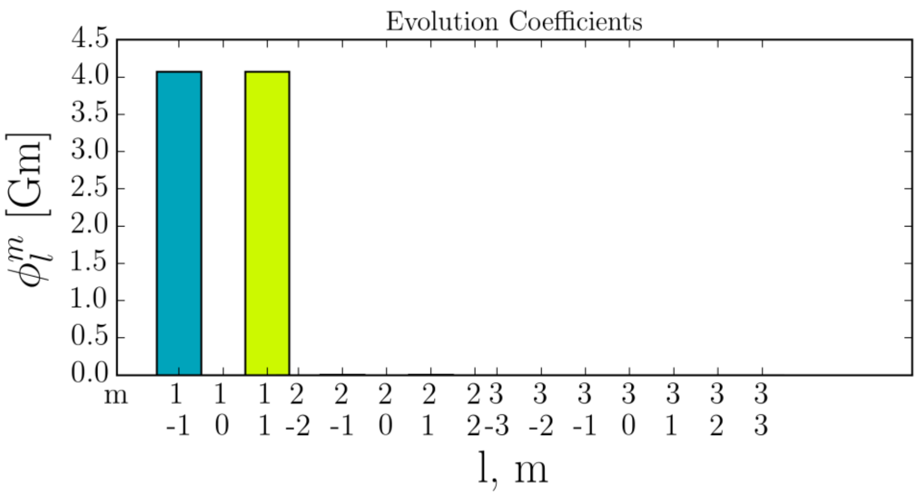
$$\Phi_{lm} = -\sqrt{\frac{2l+1}{4\pi}} \frac{R}{l} \int d^2\,\Omega Y_l^{m*}(\theta,\phi) B_r(R,\theta,\phi) \text{ [Gm]}$$

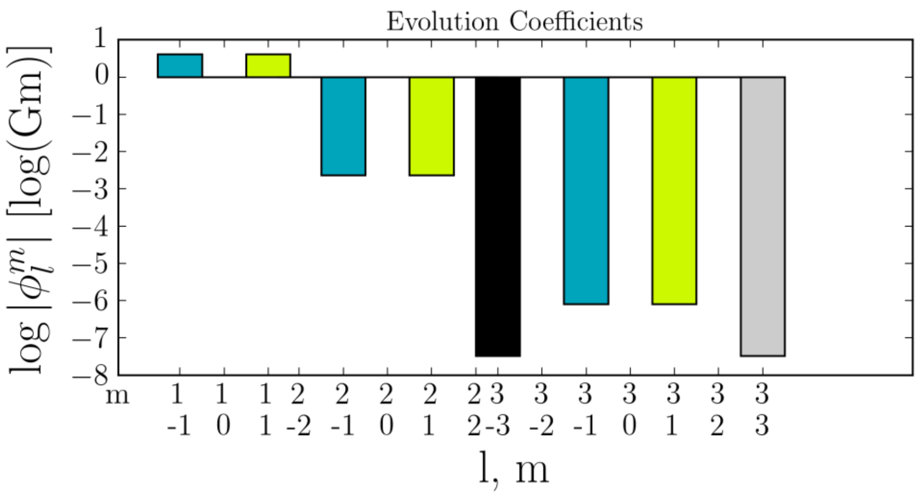
Multipoles of QII release trap

- Multipole coefficients: l>2 very small
- C2 symmetry $\phi_l^{2m} = 0$

QII Parameters:

$$I_{science} = 2.0A, I_{base} = 6.0A, I_{xcoils} = 0.1A,$$
$$I_{ycoils} = -0.37431A,$$
$$R_{min} = (0,0,1461)\mu m,$$ R = 40 µm ≈ $RTF_{,x}$
$$(\nu_1, \nu_2, \nu_3) = (9.1, 27.9, 24.6)Hz$$
$$a_x = 3.574\mu m; a_y = 2.042\mu m; a_z = 2.173\mu m$$





Zeeman potential from mag. potential

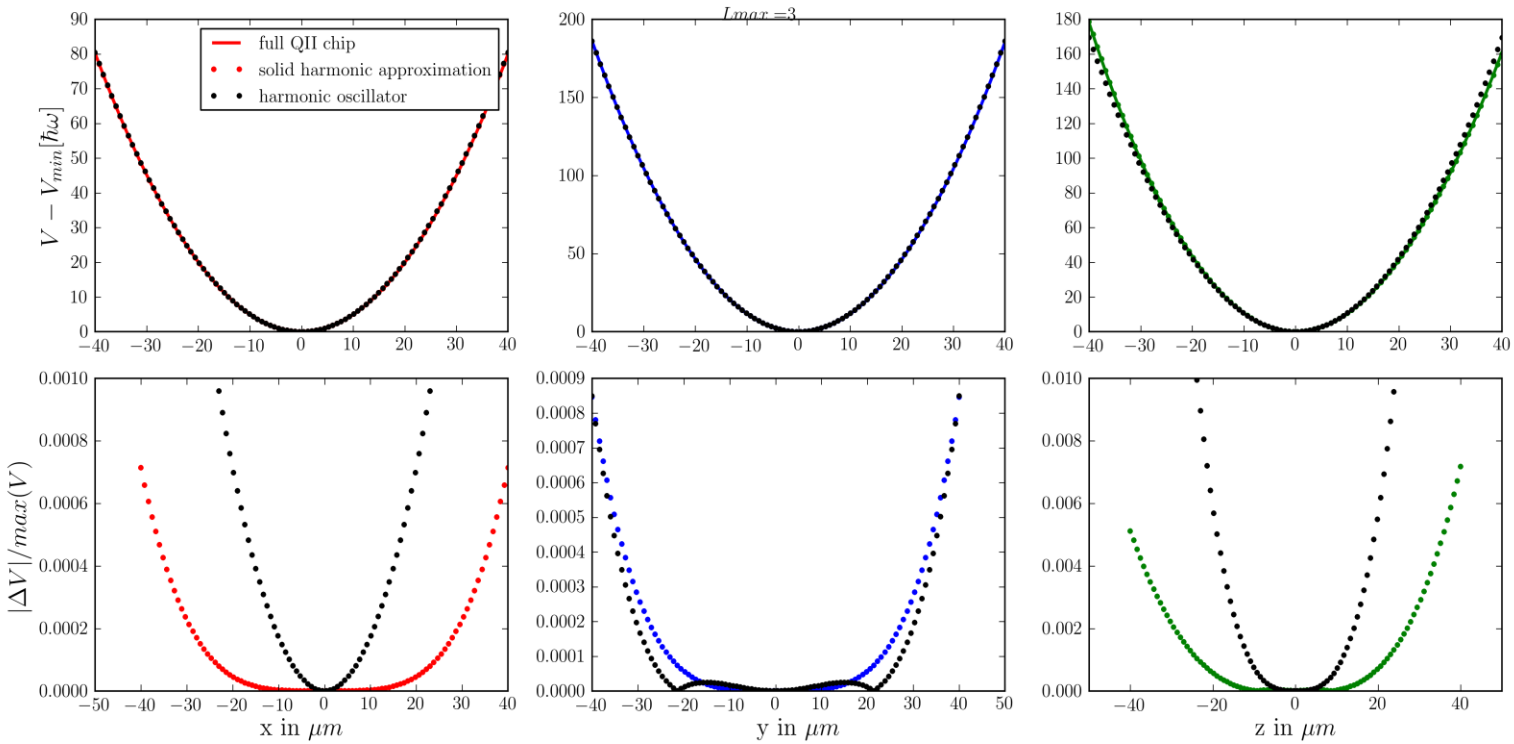
- **Zeeman potential:** $V(\mathbf{x}) = m\mu_b g_J |\nabla\Phi| = m\mu_b g_J \sqrt{\frac{1}{2}\Delta\Phi^2}$
  using Green's identies
- **Squaring harmonic polynomials**:
  angular momentum addition

$$\Delta\Phi^2(\mathbf{x}) = \sum_{l_1,l_2=1}^{\infty} r^{l_1+l_2-2} \sum_{l=|l_1-l_2|}^{l_1+l_2} C_{000}^{l_1 l_2 l} \sum_{m=-l}^{l} Y_m^l \Phi_m^{l(l_1 l_2)}$$
$$\times\sqrt{\frac{(2l_1+1)(2l_2+1)}{4\pi(2l+1)}}(l_1+l_2-l)(l_1+l_2+l+1)$$

- **Gradient evaluation**

$$\nabla\phi_{mag} = \sum_{l=1}^{\infty}\sum_{m=-l}^{l} \phi_l^m r^{l-1} \kappa \mathrm{e}^{im\phi} \Big(l P_l^m(\cos\theta)\mathbf{e}_r$$
$$- \frac{i}{2}\left[P_{l+1}^{m+1}(\cos\theta) + (l-m+1)(l-m+2)P_{l+1}^{m-1}(\cos\theta)\right]\mathbf{e}_\phi$$
$$- \frac{1}{2}\left[(l+m)(l-m+1)P_l^{m-1}(\cos\theta) - P_l^{m+1}(\cos\theta)\right]\mathbf{e}_\theta \Big)$$

Zeeman potential for release trap



$40\mu m = 11.2\, a_x = 19.6\, a_y = 18.4\, a_z$

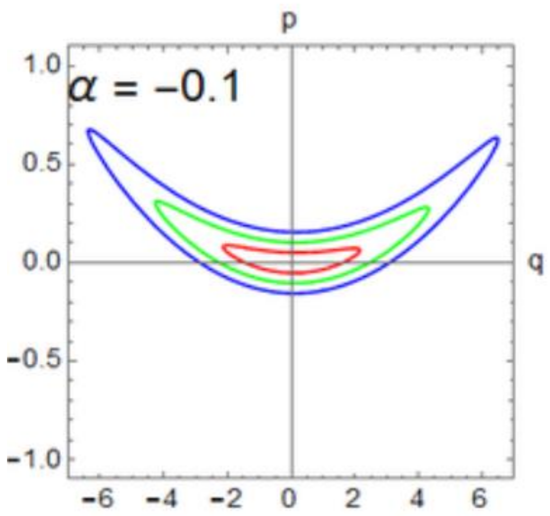
Relative error small even at the edges small; fit near the center very good

Classical wave front distortion by a anharmonic lens

- **straigthen banana states**
- **J. Richhardt**

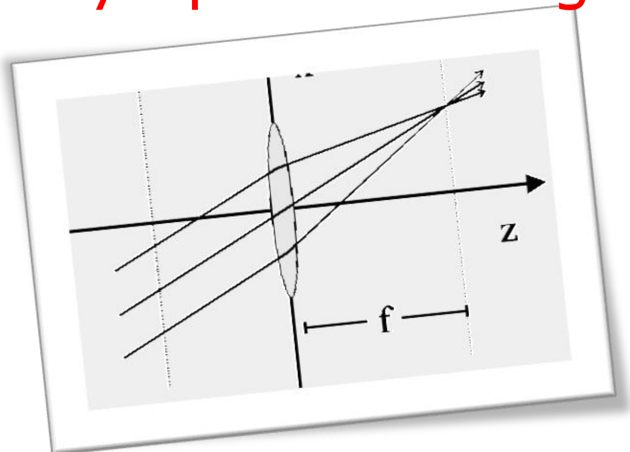
Distorted Gaussian Wigner-function after anharmonic lens

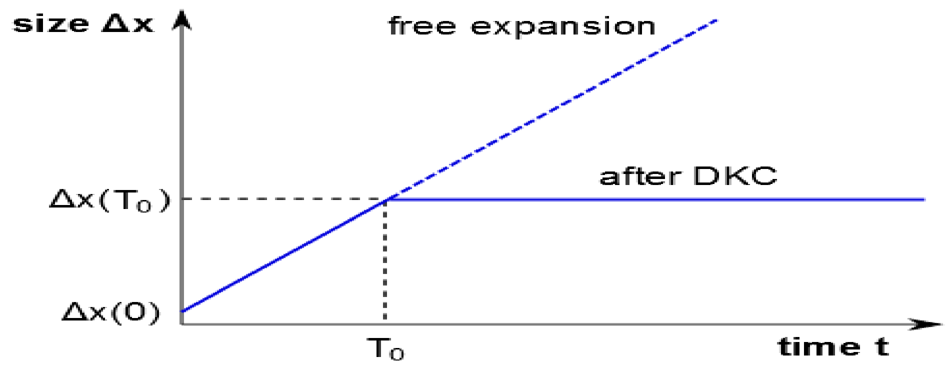
How to describe and correct an aspherical lens

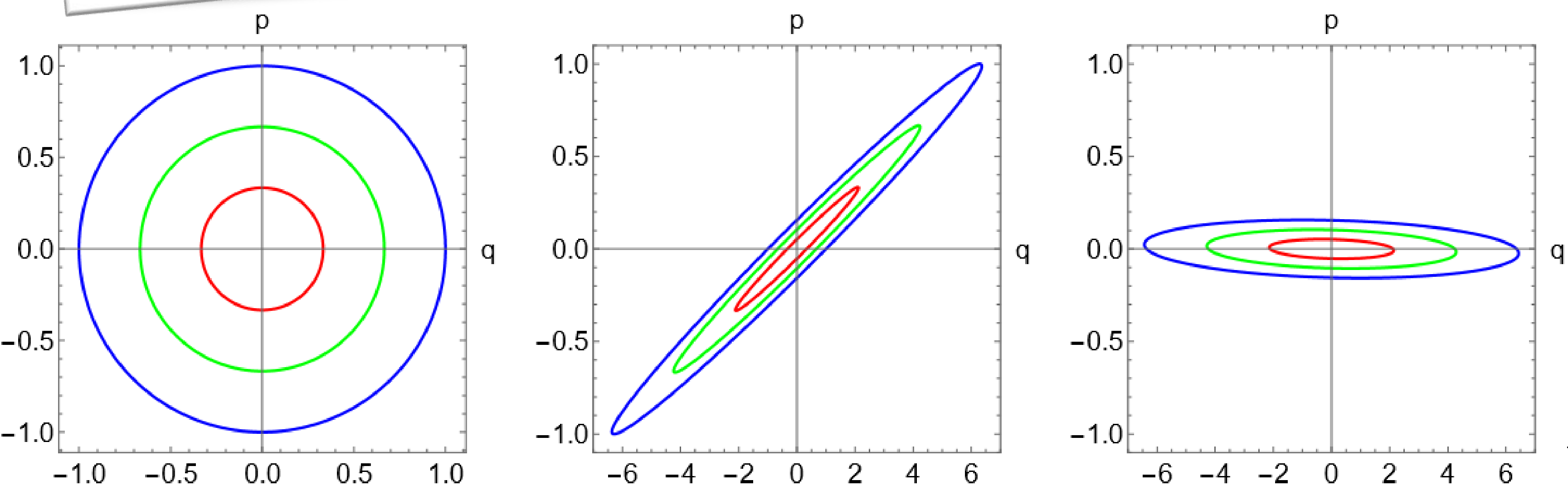


Phase space: delta kick collimation

Ray optics with light







Perturbed harmonic oscillator

- 1D harmonic oscillator with a perturbation

$$H = H_0 + H_1 = \frac{p^2}{2} + \frac{q^2}{2} + \epsilon W(q).$$

(dimensionless)

Harmonic oscillator:
Kinetic energy
+
quadratic potential

Perturbation:
Potential beyond the
harmonic approximation.

Phase space flow maps

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- Interaction picture Hamilton function

$$H'(Q,P) = \epsilon W(Q\cos(t) + P\sin(t)).$$

(Explicit time dependent!)

- Dynamics is govern by Hamiltonian flow:

$$\dot{Q} = \epsilon W'(Q\cos(t) + P\sin(t))\sin(t)$$

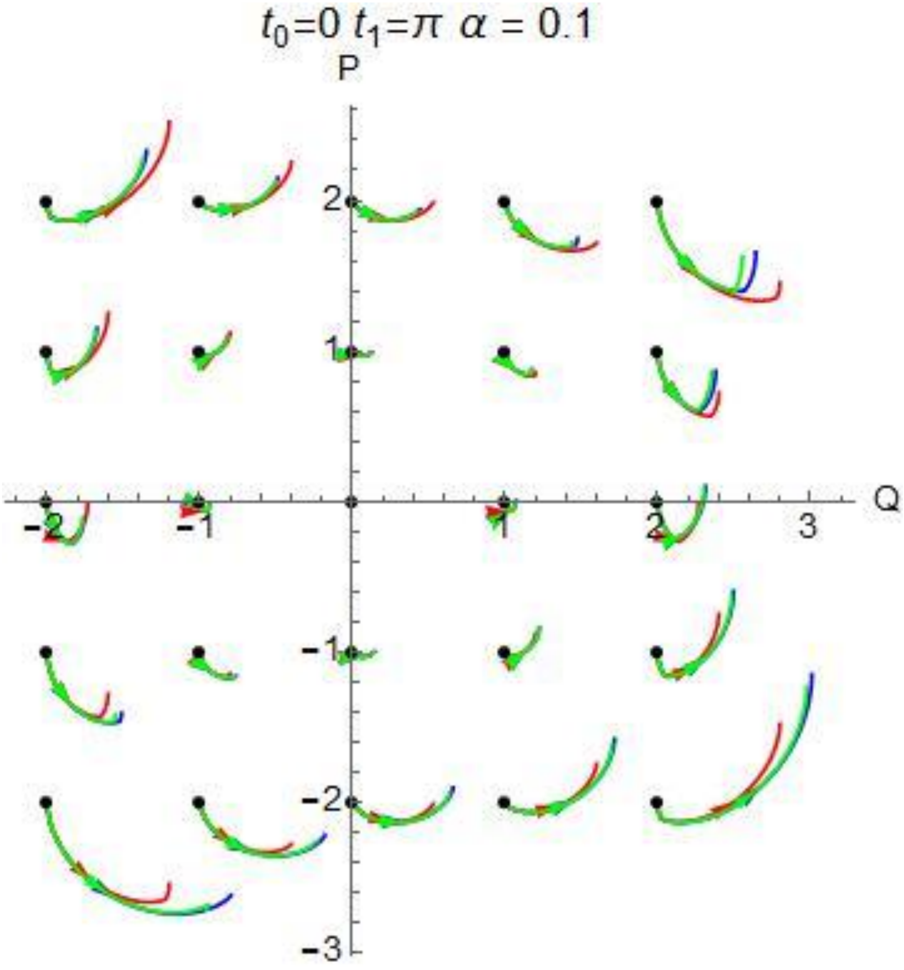
$$\dot{P} = -\epsilon W'(Q\cos(t) + P\sin(t))\cos(t)$$

- Phase space map: Picard iteration generates power series in ε

$$\Phi(\tau,(q_0,p_0)) = e^{\tau\mathcal{L}_f}(q_0,p_0) = \sum_{n=0} \varepsilon^n \Phi^{(n)}(\tau,(q_0,p_0))$$

Phase space flow: cubic potential

Sample trajectories
First order flow map
Up to second order flow map
Full numerical solution



Banana states: $W(q) = \alpha q^3/3$

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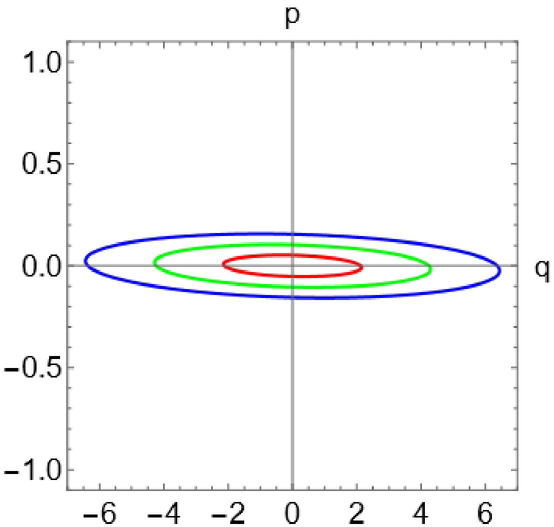
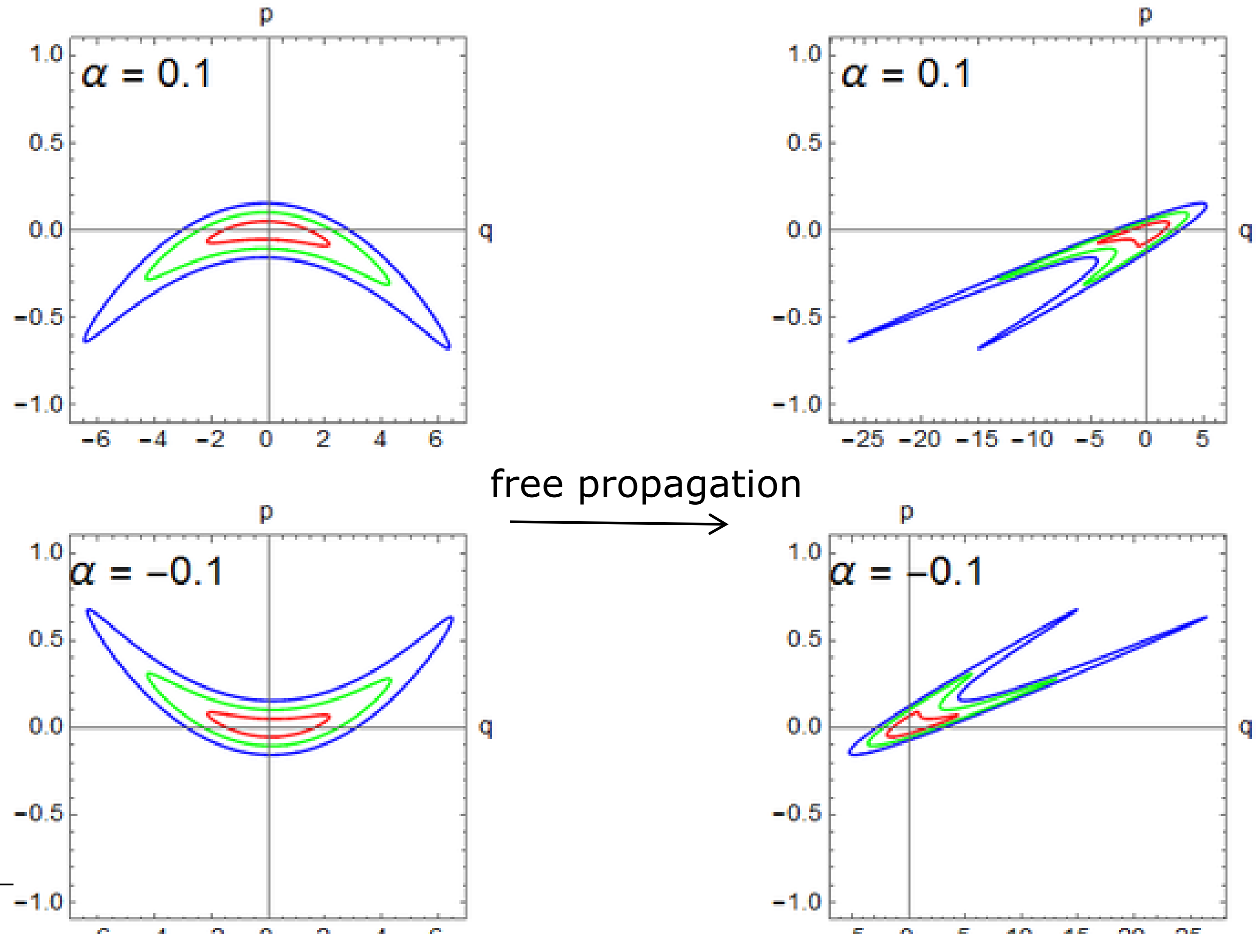
Ideal collimation with harmonic lens after lens



Cubic lens aberration



Delta kick collimation of coherent matter-waves

- 3D stationary and time-dependent simulation
- J. Teske, S. Srinivasan

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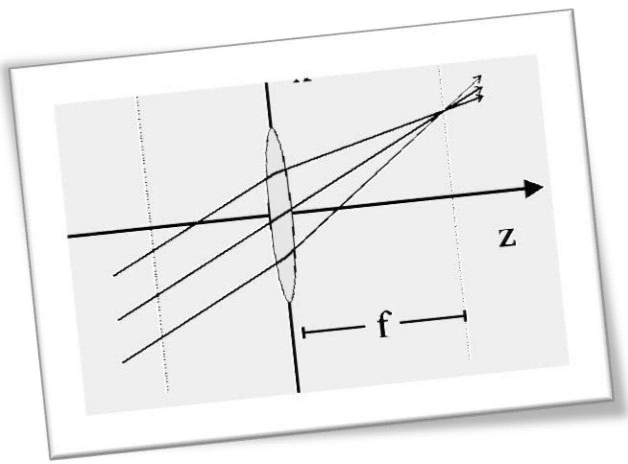
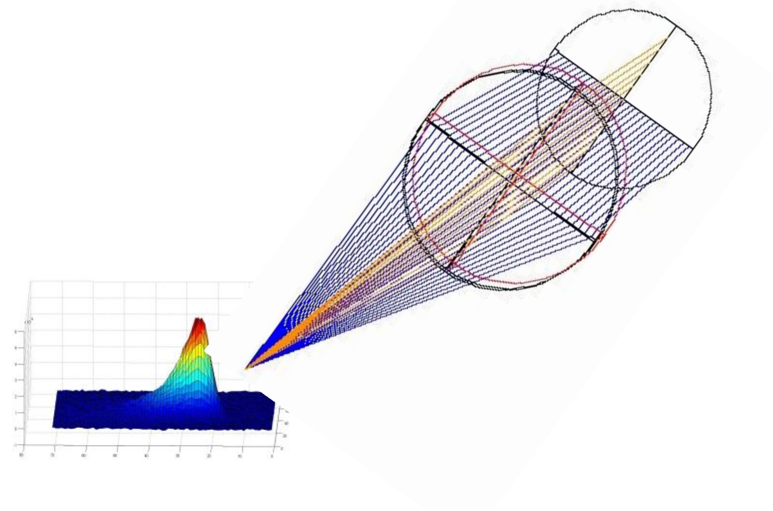
Ray optics with light (2+1D) Matter wave optics (3+1D)





AMMANN, Hubert ; CHRISTENSEN, Nelson: Delta Kick Cooling: A New Method for Cooling Atoms, *Phys. Rev. Lett.* 78, 2088 (1997)

Matter wave optics DKC-sequence:

$$t_{lens} = \frac{1}{\omega} \tan\left(\frac{1}{\omega t_1}\right)$$

Methods

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- **Dynamics**
  GP mean-field

$$i\hbar\partial_t\psi(\boldsymbol{x},t) = \left[-\frac{\hbar^2\boldsymbol{\nabla}^2}{2m} + V(\boldsymbol{x},t) + g|\psi(\boldsymbol{x},t)|^2\right]\psi(\boldsymbol{x},t)$$

- **Adaptive frames**
  Classical center-of-mass and scale parameters (eliminates COM, ballistic expansion)

$$\begin{pmatrix}\boldsymbol{\xi}\\ \boldsymbol{\kappa}\end{pmatrix} = \begin{pmatrix}\Lambda^{-1} & 0\\ -m\dot{\Lambda}^T & \Lambda^T\end{pmatrix}\begin{pmatrix}\boldsymbol{x}-\boldsymbol{\eta}(t)\\ \boldsymbol{p}-m\dot{\boldsymbol{\eta}}(t)\end{pmatrix}$$

$$\Lambda^T(t)\left(\frac{d^2\Lambda}{dt^2} + \Omega^2(t)\Lambda(t)\right) = \frac{\Omega^2(0)}{\det\Lambda(t)}$$

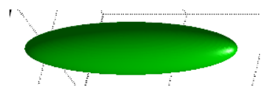
- **Linear response**
  Bogoliubov-de-Gennes Equation for collective modes

$$\begin{bmatrix}\Sigma_A & \Sigma_B\\ -\Sigma_B^* & -\Sigma_A\end{bmatrix}\begin{bmatrix}u(\mathbf{x})\\ v(\mathbf{x})\end{bmatrix} = \omega\begin{bmatrix}u(\mathbf{x})\\ v(\mathbf{x})\end{bmatrix}$$

M. Meister et al. Advances in Atomic, Molecular, and Optical Physics **66**, 375, (2017)

Initial state in release trap

Cigar-shaped trap



$$\boldsymbol{\nu} = (9.09, 27.89, 24.61)\text{Hz}$$

$$\boldsymbol{x}_{\text{TF}} = (25.93, 8.47, 9.60)\mu\text{m}$$

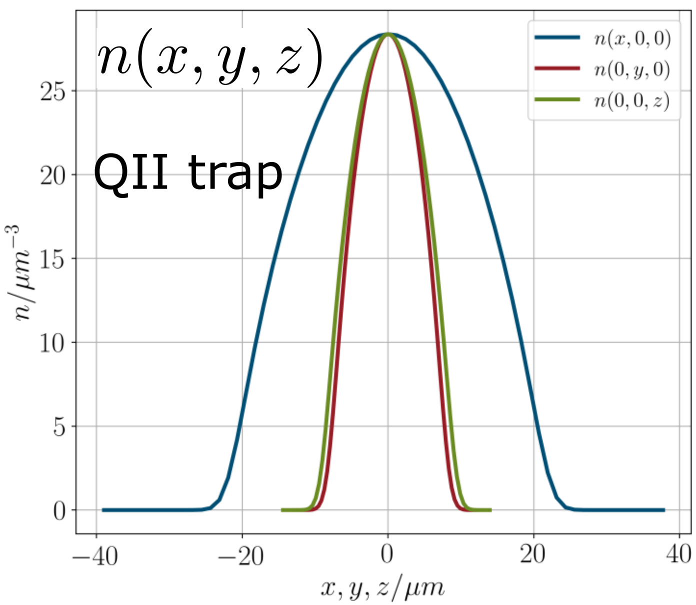
$$N_c = 10^5 \quad \text{Rb}^{87} \quad a_s = 5.8 \text{ nm}$$

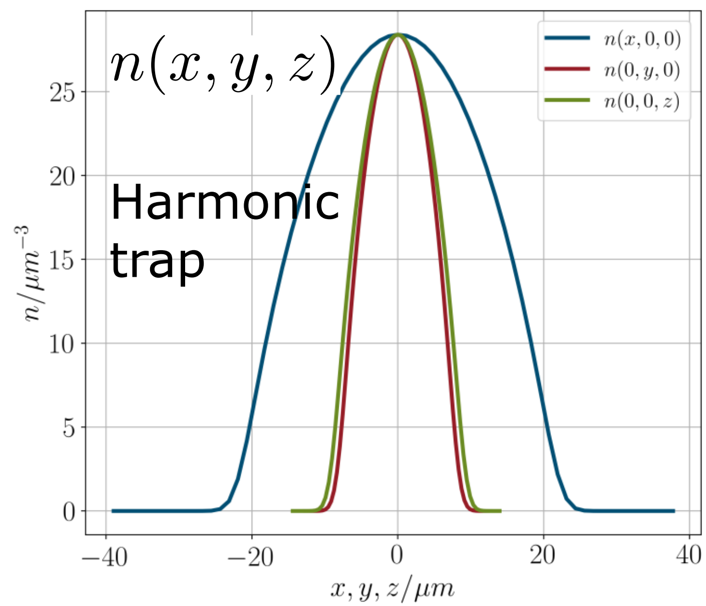
$$\left[-\frac{\hbar^2 \boldsymbol{\nabla}^2}{2m} + V(\boldsymbol{x}) + gn(\boldsymbol{x})\right] \psi(\boldsymbol{x}) = \mu\psi(\boldsymbol{x})$$

$$R_{\min} = (-0.143, -4.8 \cdot 10^{-2}, 1460)\,\mu\text{m}$$

$$I_{sc} = 2 \text{ A} \qquad I_{ycoil} = -0.37431 \text{ A}$$

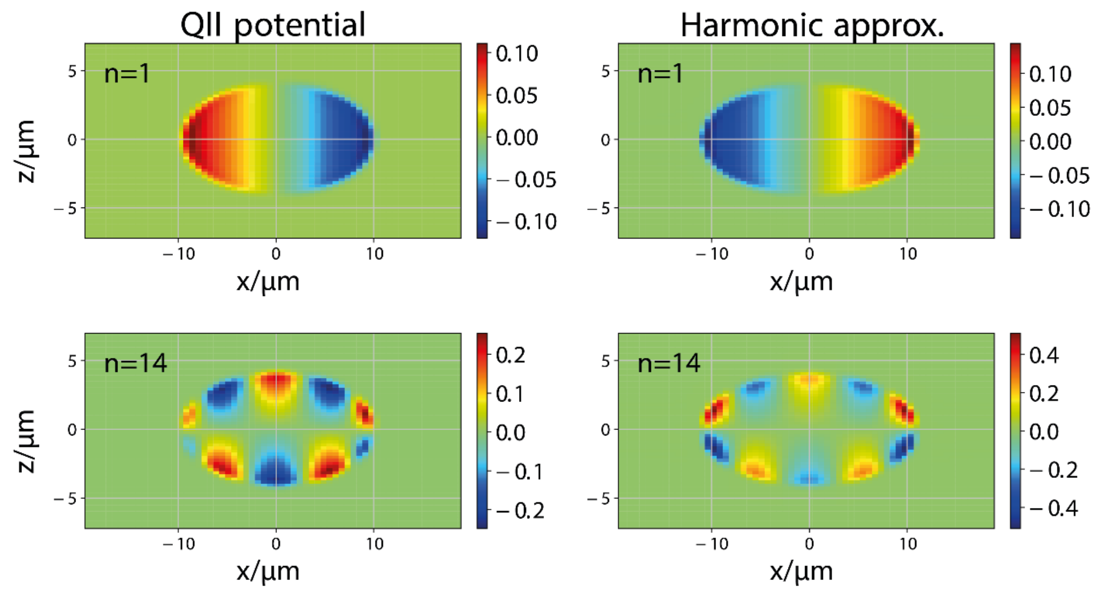
$$I_{bc} = 6.0 \text{ A} \qquad I_{xcoil} = 0.1 \text{ A}$$



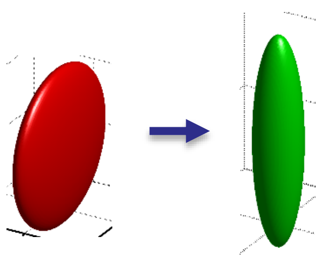


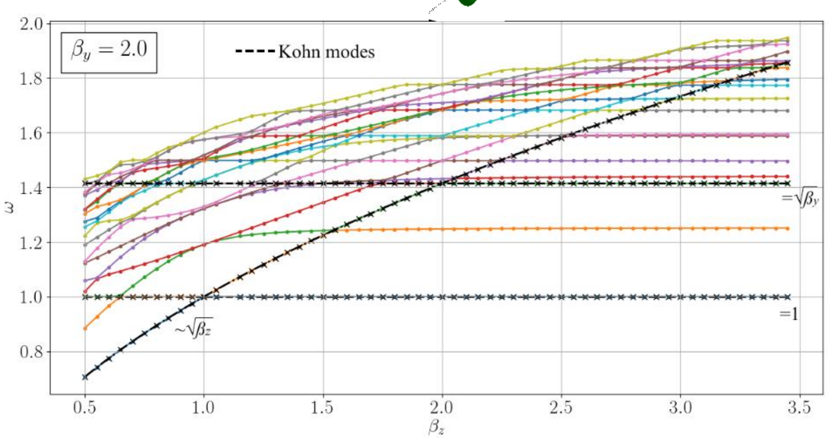
Collective excitations

Bogoliubov modes:
linear response to any form of weak time-dependent perturbation

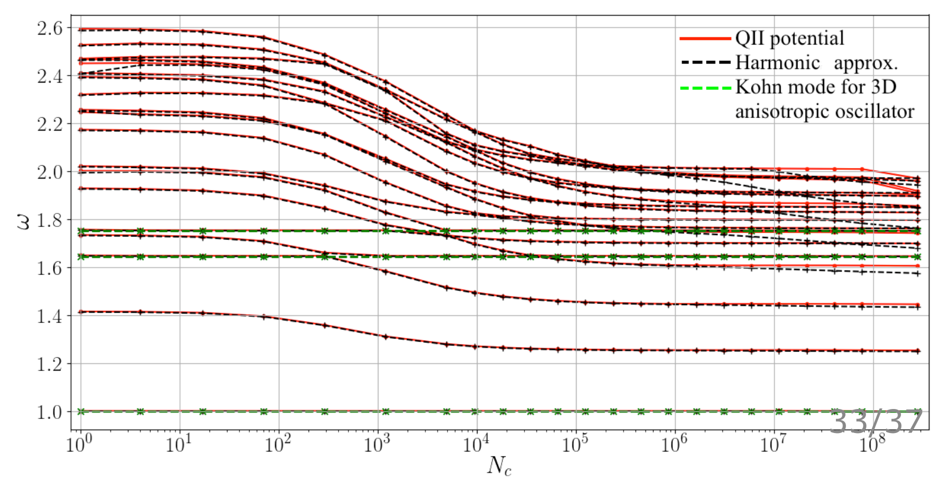


Geometric deformation





Increasing particle number



Bre

Evolving momentum distribution

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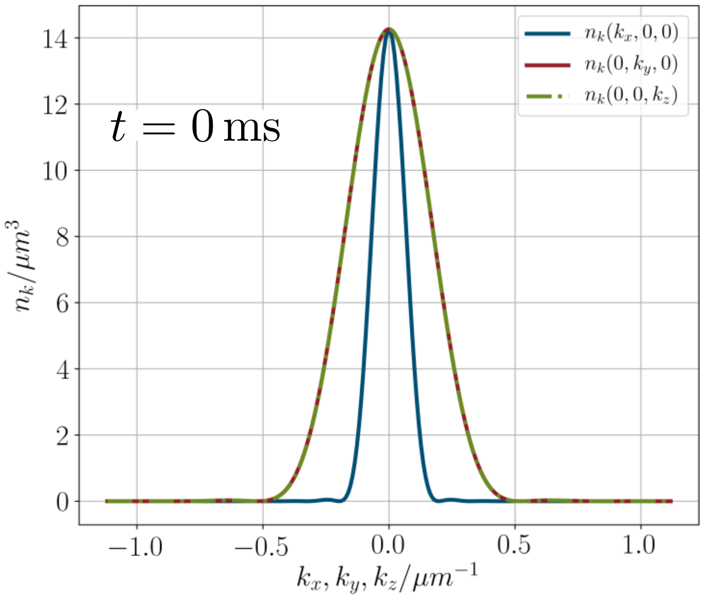
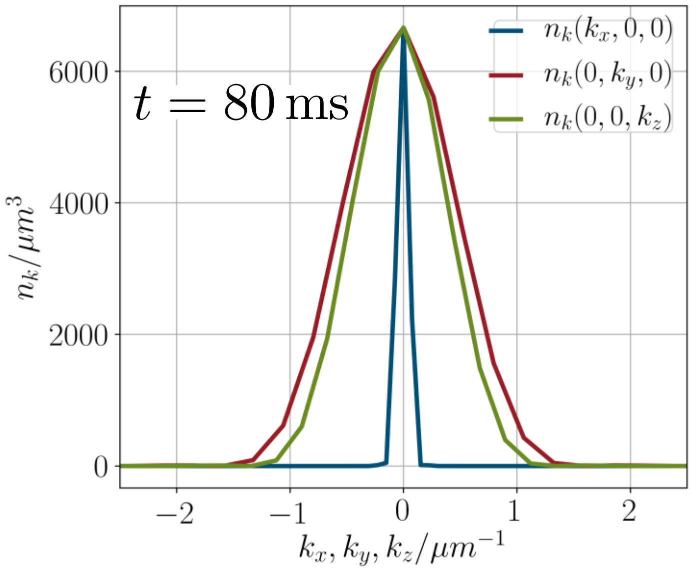
Thomas-Fermi approximation
for the momentum distributions

$$n(\boldsymbol{k}) \sim J_2^2(\tilde{k})/\tilde{k}^2 \qquad \overline{\alpha}_{\mathrm{TF}}(\tilde{k}) = R_{\mathrm{TF}}^n \sqrt{\frac{\pi\mu_{\mathrm{TF}}}{2g}} \frac{J_{\frac{n+1}{2}}(\tilde{k})}{\tilde{k}^{\frac{n+1}{2}}}.$$

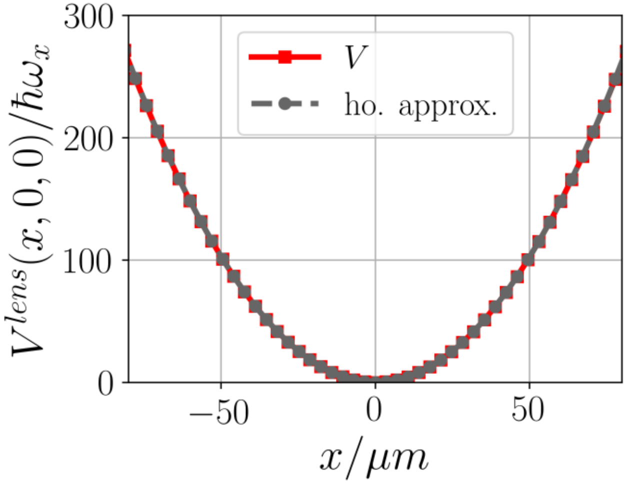
Initial momentum distribution



Momentum distribution after free propagation



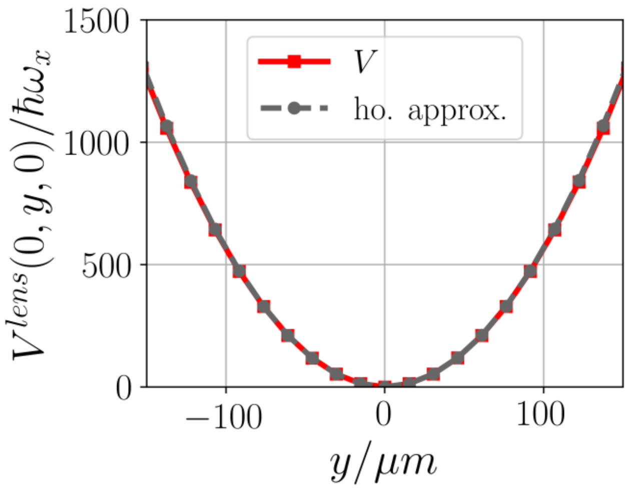
Magnetic Lens

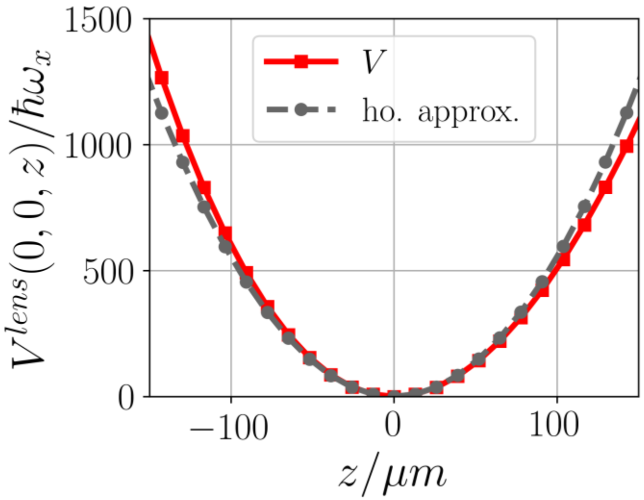


$I_{sc} = 0$ A $I_{ycoil} = -0.0754283$ A
$I_{bc} = 1.8$ A $I_{xcoil} = 0.1$ A

$$\boldsymbol{\nu} = (2.98, 10.82, 10.82)\,\text{Hz}$$

$$\boldsymbol{x}_{\text{TF}} = (72.39, 118.39, 110.72)\mu\text{m}$$



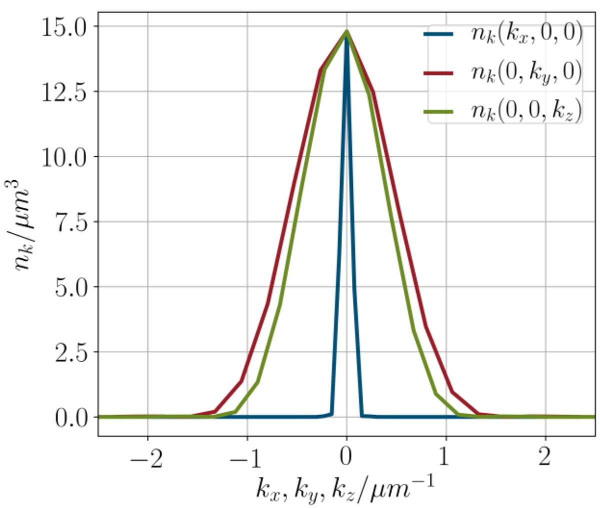


Lens action in k-space

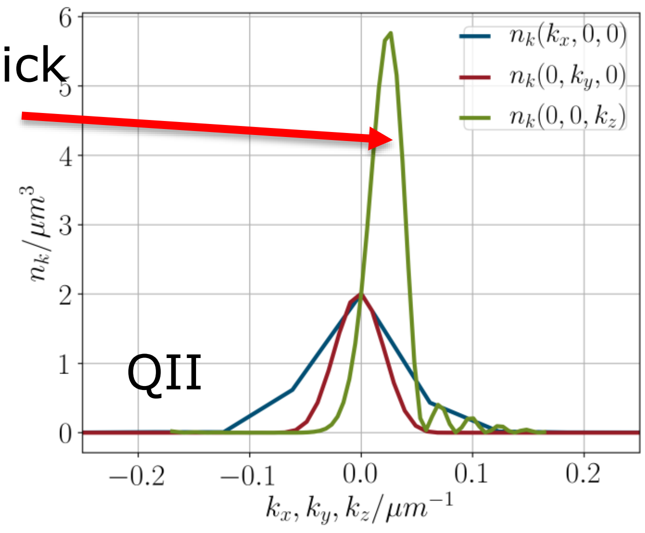
Momentum distribution $n(k_x, k_y, k_z)$

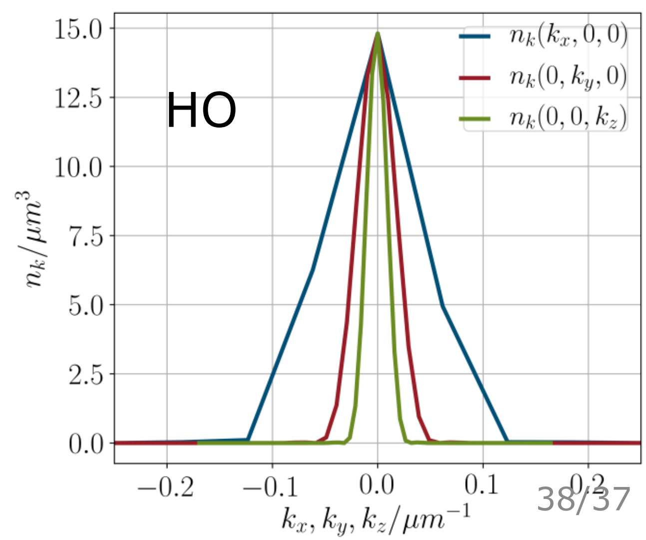
momentum kick in z-direction

before lens



after lens





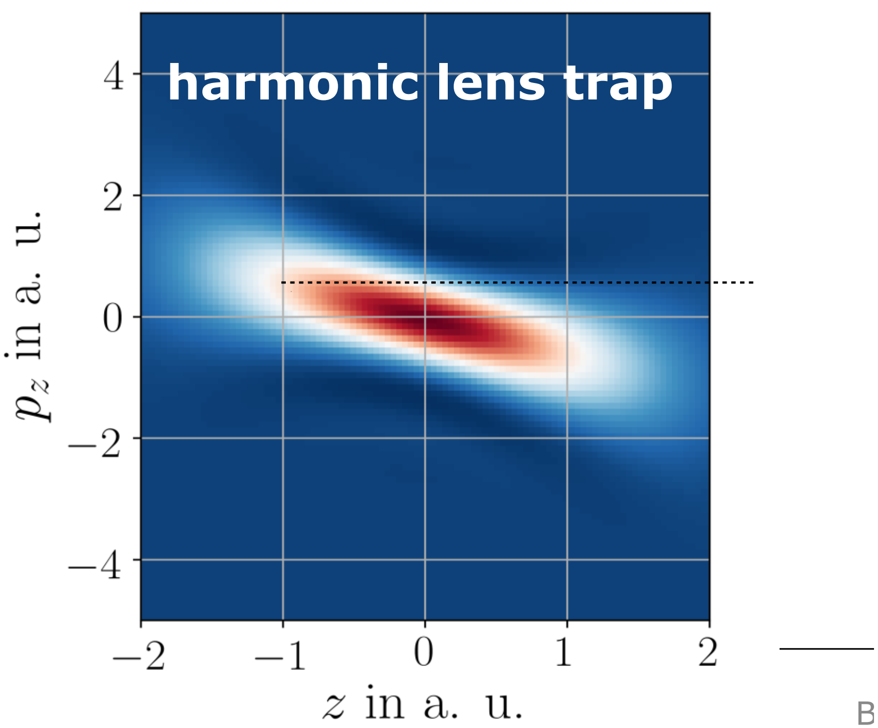
Phase space (z, k_z)

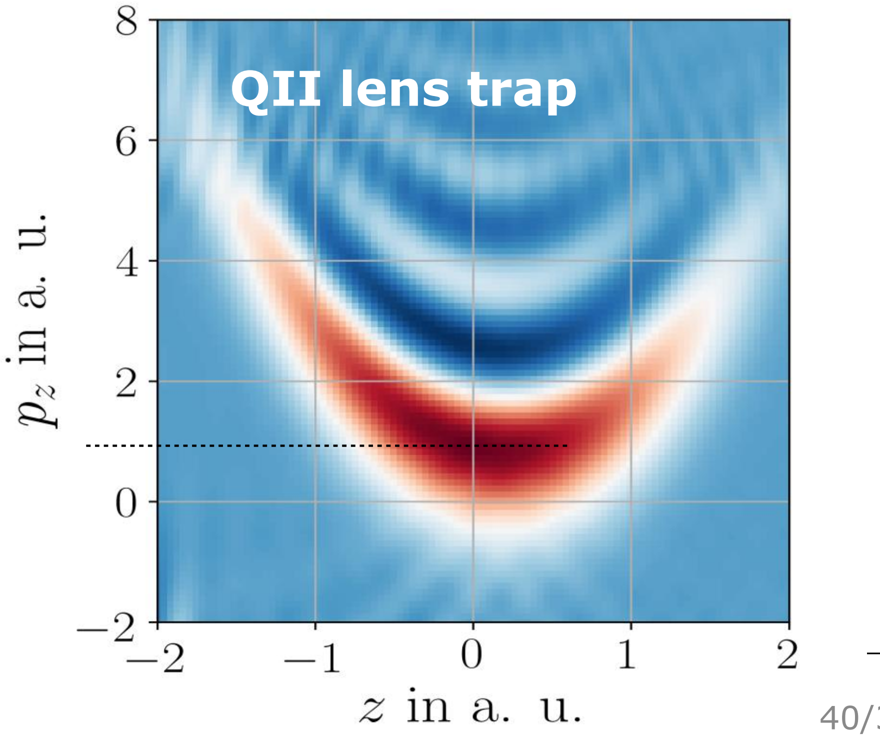
Cut through the six dimensional Wigner function $W(x, y, z, k_x, k_y, k_z)$

Coordinates with respect to atom chip!

$W(x = y = 0, z, k_x = k_y = 0, k_z)$

$t = t_1 + \tau$





B

Long time expansion dynamics

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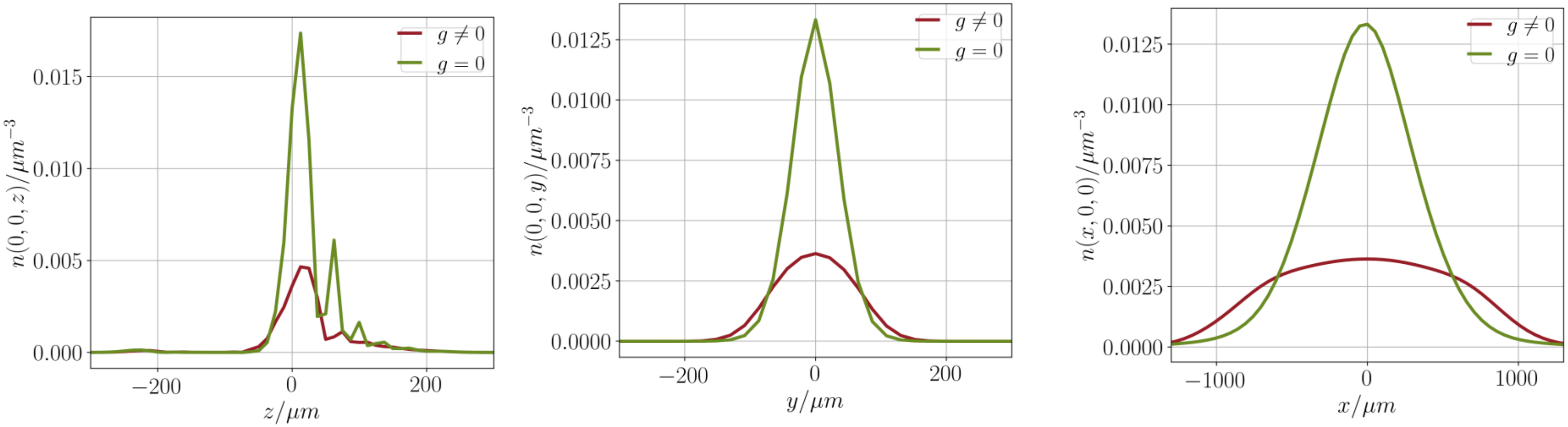
- **After DKC** evelove BEC for 2 sec
- **Size** 2mm
- **Significant:** residual mean field interaction for x,y,z, despite strong drop in density



Matter wave interferometry

– **Aberrations of Bragg beamspitter losses, dissipation, time dependence & spatial effect**
– **A. Neumann**

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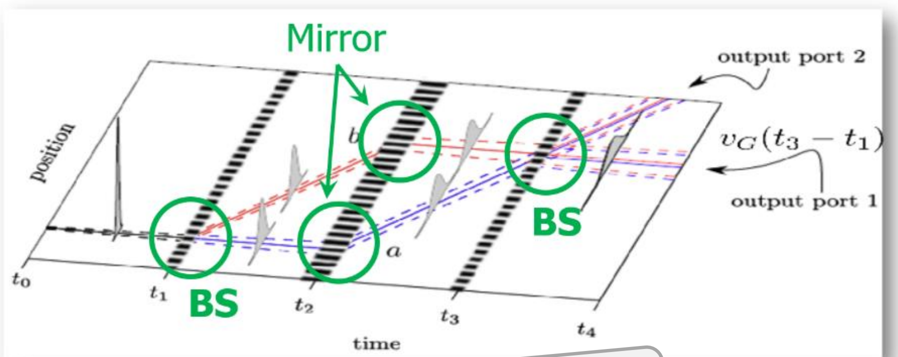
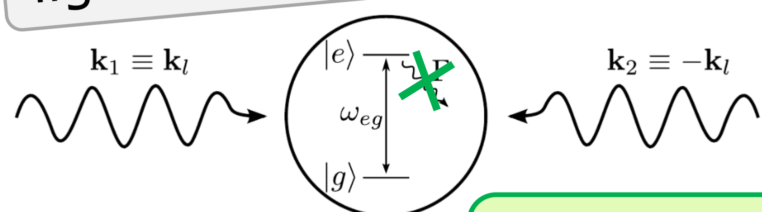


BS goal:
coherently splitting motion of atoms with unit response and wide momentum range

light-matter interaction



$$\omega_1 = \omega_2 \equiv \omega_l$$
$$\Delta = \omega_{eg} - \omega_l$$

BS: Bragg diffraction of atoms by periodic grating (optical standing wave)

$$\longrightarrow \; i\hbar\partial_t|\Psi\rangle = \hat{H}|\Psi\rangle$$

large detuning:
$$\Delta = \omega_{eg} - \omega_l, \;\; |\Delta| \gg \Gamma$$

solve with split operator method

Bragg diffraction

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$$\hat{H} = \frac{\hat{p}^2}{2M} \otimes \mathbb{1} + \frac{\hbar\Delta}{2}\hat{\sigma}_z + \frac{\hbar\Omega_0}{2}$$

$$i\hbar\partial_t|\Psi\rangle = \hat{H}|\Psi\rangle$$

$$\Psi(t) = e^{-i\hat{H}t}|\Psi(0)\rangle = \hat{U}|\Psi(0)\rangle$$

absorb / emit 1 photon:

$$e^{\pm i k_l \hat{x}} = \int dp |p \pm \hbar k_l\rangle\langle p|$$

→ coupling:

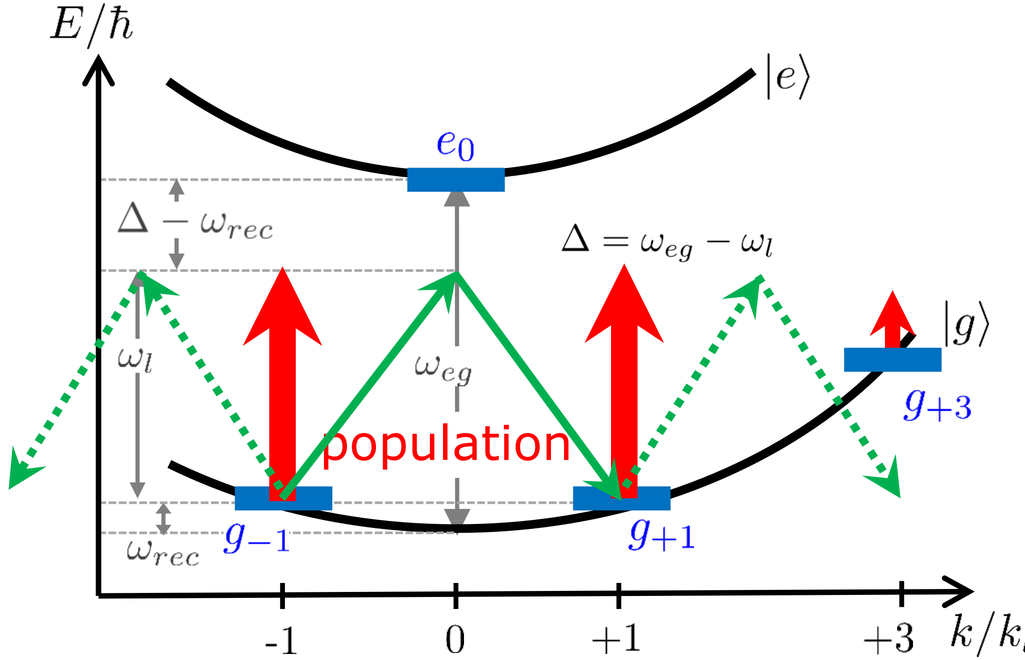
$$|g, k\rangle \leftrightarrow |g, k \pm 2Nk_l\rangle$$

(Bragg order N)

Loss into off resonant higher diffraction orders:

$$|\Psi(t)\rangle = \sum_{m=-N}^{N} g_m |g, m\cdot k_l\rangle + e_{m'}|e, m'\cdot k_l\rangle$$

m odd, m' even



Velocity dispersion

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$$|\Psi(t)\rangle = \sum_{\delta k=-1}^{1} \sum_{m=-N}^{N} g_{m,\delta k}|g,(m+\delta k)k_l\rangle + e_{m',\delta k}|e,(m'+\delta k)k_l\rangle$$

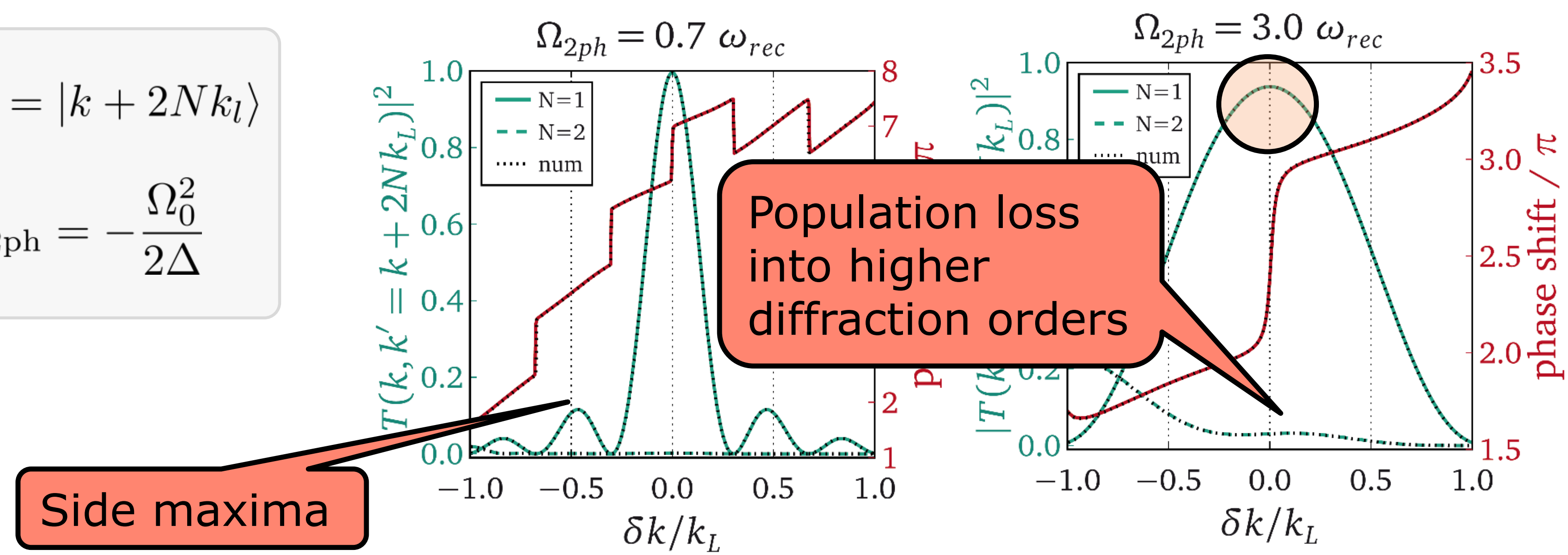
transition amplitude between initial k and final k'

$$T_{kk'} = \langle k'|\,\hat{U}\,|k\rangle = |T_{kk'}|e^{i\phi_{kk'}}$$

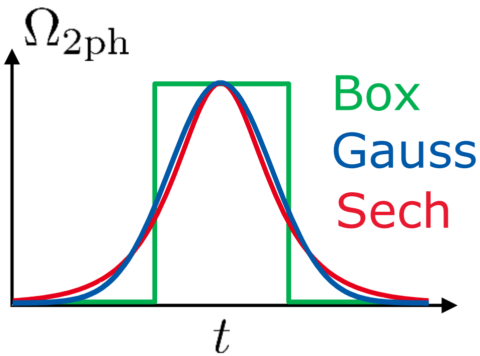
phase shift $\Delta\phi = \phi_{kk} - \phi_{kk'}$

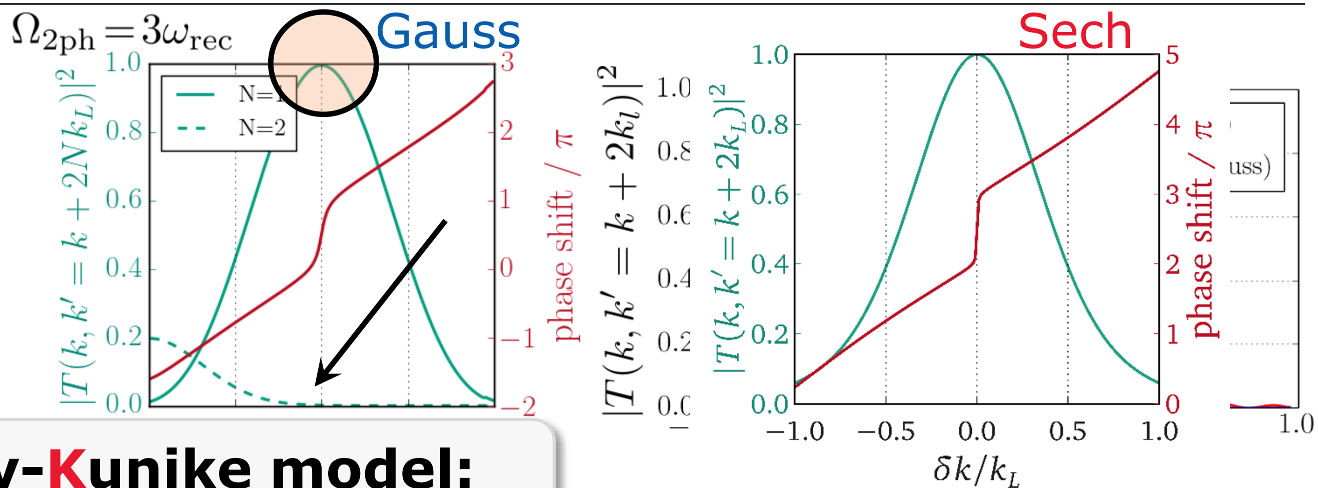
$$|k'\rangle = |k + 2Nk_l\rangle$$

$$\Omega_{\mathrm{2ph}} = -\frac{\Omega_0^2}{2\Delta}$$



Temporal envelopes





Analytic Demkov-Kunike model:
1D 1st order Bragg diffraction

$$\Omega_{2\mathrm{ph}}(t) = \Omega_{2\mathrm{ph}} \cdot \mathrm{sech}\left(\sqrt{\frac{\pi}{2}}\frac{t}{\sigma}\right)$$

$$T_{kk'}^{\mathrm{N}=1} = \frac{\sigma\Omega_{2\mathrm{ph}}\sqrt{\mathrm{sech}\left(\sqrt{\frac{\pi}{2}}\frac{t}{\sigma}\right)}}{i\sqrt{2\pi} - 8\omega_{\mathrm{rec}}\delta k\sigma} \cdot {}_2F_1[a, b; c, z(t)]$$

$$a/b = 1 -/+ \frac{\sigma\Omega_{2\mathrm{ph}}}{\sqrt{2\pi}}, \quad c = \frac{3}{2} + 2i\sqrt{\frac{2}{\pi}}\omega_{\mathrm{rec}}\delta k\sigma, \quad z(t) = \frac{1}{2} + \frac{\tanh\left(\sqrt{\frac{\pi}{2}}\frac{t}{\sigma}\right)}{2}$$

- ✓ suppressed side maxima
- ✓ insignificant population loss into higher diffraction orders
- ✓ Analytic model (DK) for 'sech'- pulses

Comparison of 1D simulation with experimental data*

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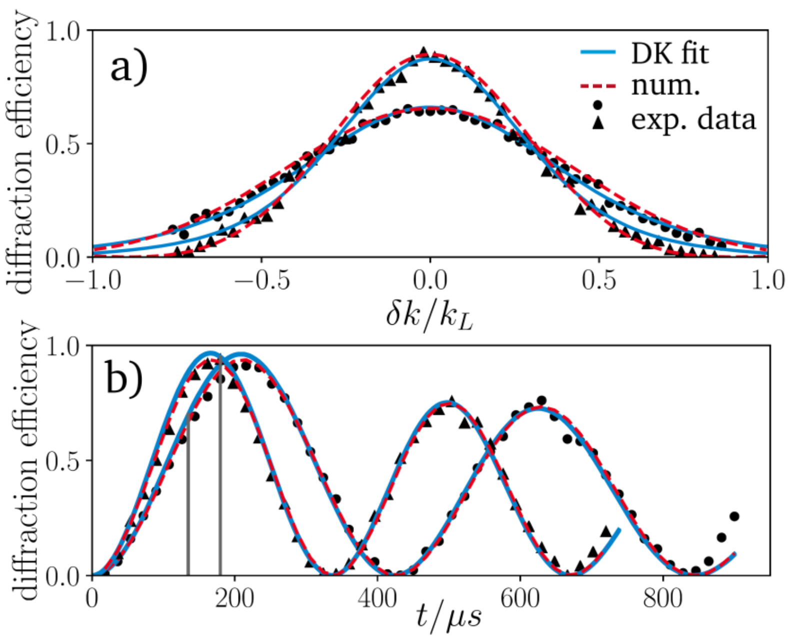
* M.Gebbe (Universität Bremen, priv. com.)

Laser:
- spatial dependence: ~ plane waves
- temporal: Gaussian (+ fit with **D**emkov **K**unike)
- Laser frequency detuned to resonance δ_f

Atoms: BEC @ 50 nK ~ Thomas Fermi (width in momentum space $\ll k_l$)

	$P_\bullet = 20\,\mathrm{mW}$	$P_\blacktriangle = 30\,\mathrm{mW}$	
$\Omega_{2\mathrm{ph}}^{\mathrm{exp}}$	$\lesssim 2.27\,\omega_{\mathrm{rec}}$	$\lesssim 3.40\,\omega_{\mathrm{rec}}$	
t	$135\,\mu s$	$180\,\mu s$	a)
pulse area	$0.61\,\pi$	$1.24\,\pi$	
$\Omega_{2\mathrm{ph}}^{\mathrm{DK,\ num}}$	$1.90\,\omega_{\mathrm{rec}}$	$2.91\,\omega_{\mathrm{rec}}$	
δ_f^{exp}	$\approx 0\,\mathrm{kHz}$	$\approx 0.5\,\mathrm{kHz}$	
$\Omega_{2\mathrm{ph}}^{\mathrm{DK}}$	$2.00\,\omega_{\mathrm{rec}}$	$2.52\,\omega_{\mathrm{rec}}$	b)
δ_f^{DK}	$0.97\,\mathrm{kHz}$	$1.15\,\mathrm{kHz}$	
$\Omega_{2\mathrm{ph}}^{\mathrm{num}}$	$1.97\,\omega_{\mathrm{rec}}$	$2.50\,\omega_{\mathrm{rec}}$	
δ_f^{num}	$1.39\,\mathrm{kHz}$	$1.68\,\mathrm{kHz}$	



qualitative ✓

<u>small</u> quantitative deviations due to neglected 3D dependencies

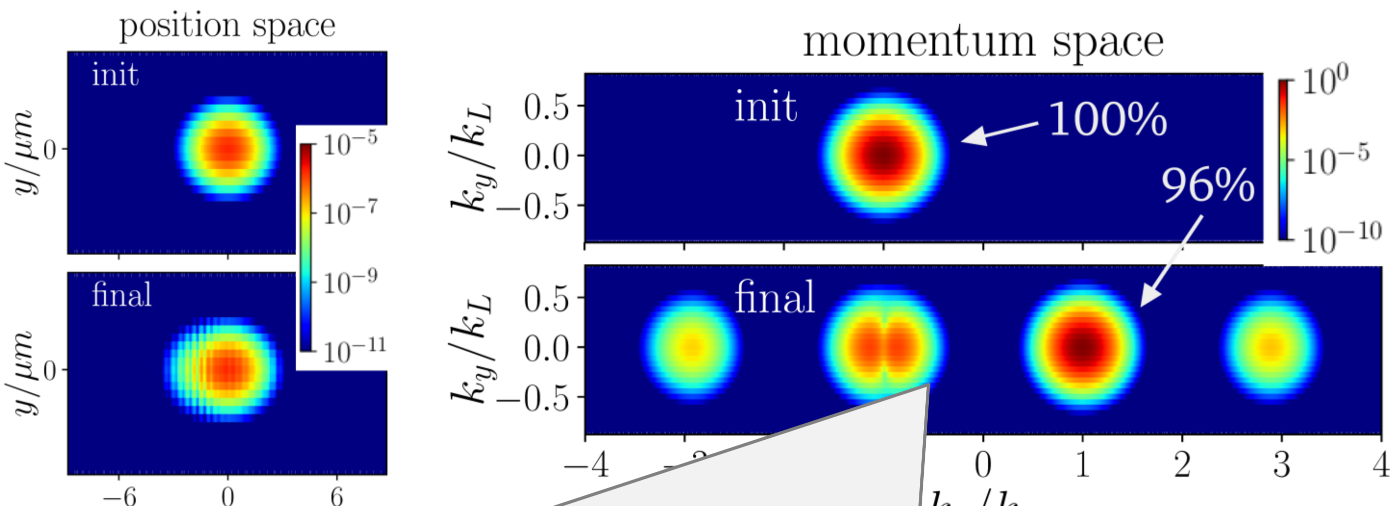
3D simulation:

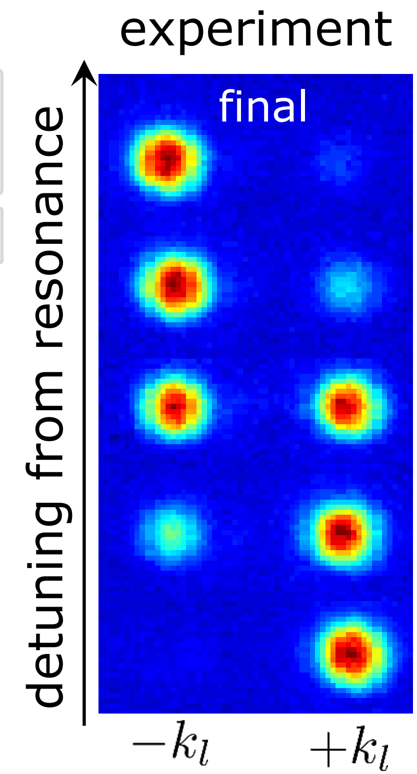
Column integrated 2D density after mirror-pulse

Laser: • spatial: Gaussian beams
• temporal: Gaussian pulse

$\Omega_{2\text{ph}} = 3\ \omega_{\text{rec}}$
$w_0 = 10\ \mu\text{m} \rightarrow x_{\text{ray}} \approx 400\ \mu\text{m}$

Initial state: Gaussian wave packet $\sigma_k = 0.1 k_l,\ \sigma_x \approx 1\ \mu\text{m}$





Fidelity $F = 0.95$ with Gaussian laser beams, $F = 0.96$ with plane waves

Realistic Gaussian laser beams decreases diffraction efficiency!

Summary: technical mw-optics

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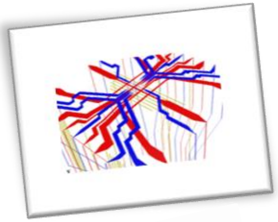
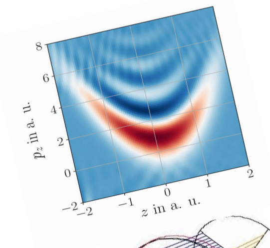
QUANTUS

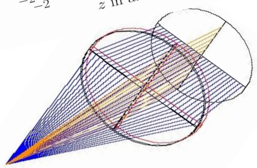


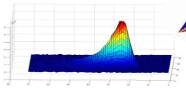
Toolbox mw-optics

- EBEC GR corrections to GP meanfield
- Magnetic traps & lenses
- Classical wavefront aberration banana state
- 3D matter wave optic optics

a. Bragg beam-splitters

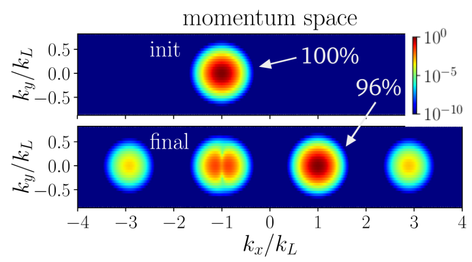






Methods & applications

a. geometical mw-optics: raytracing, aberrations
b. thermal mw-optics: 3D interferometry @ finite T
c. coherent mw-optics: 2 s, delta-kick-collimation
d. quantum mw-optics: JJ's manybody resonances



Thank you for the attention!