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Direct Multipixel Imaging and Spectroscopy of an Exoplanet with a Mission to the Focus of the Solar Gravitational Lens

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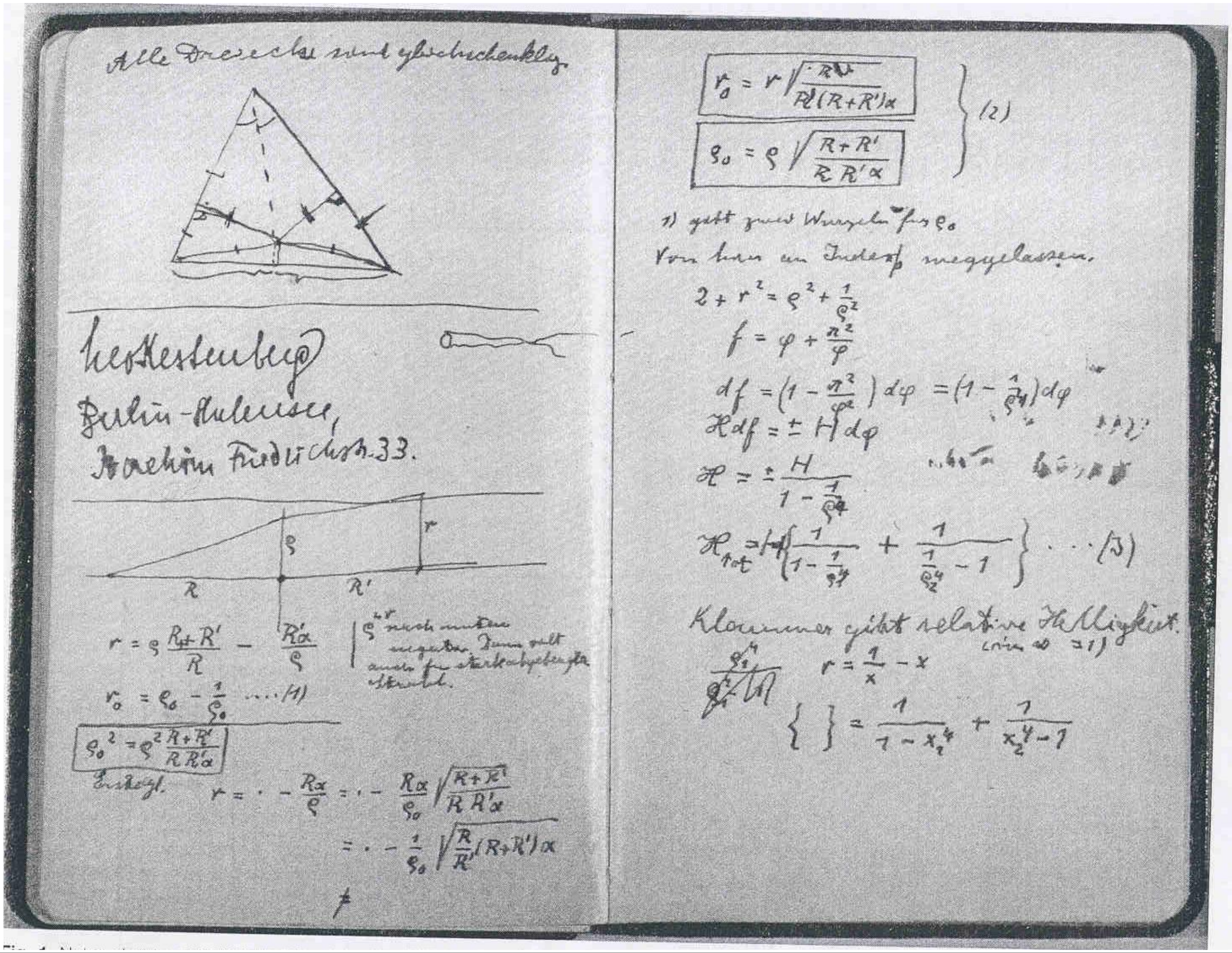
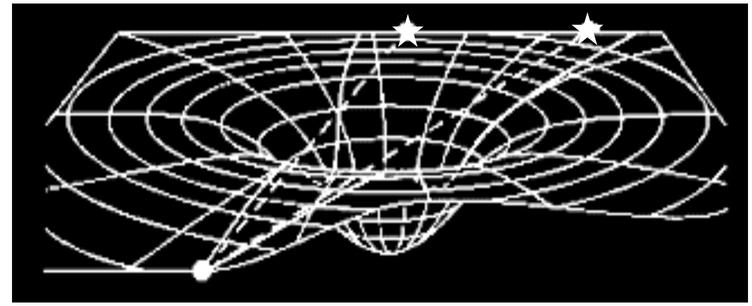
Original gravity lens derivation (Einstein c.1911)



Precision alignment between a Lens and the Earth is very unlikely…

The First Test of General Theory of Relativity



Gravitational Deflection of Light:

$$\alpha_{\rm GR}(b) = \frac{2(1+\gamma)GM_\odot}{c^2 b} \simeq 1.75\left(\frac{1+\gamma}{2}\right)\left(\frac{\mathcal{R}_\odot}{b}\right)\ \text{arcsec}$$

Solar Eclipse 1919: possible outcomes

Deflection = 0;
Newton = 0.87 arcsec;
Einstein = 2 x Newton = 1.75 arcsec

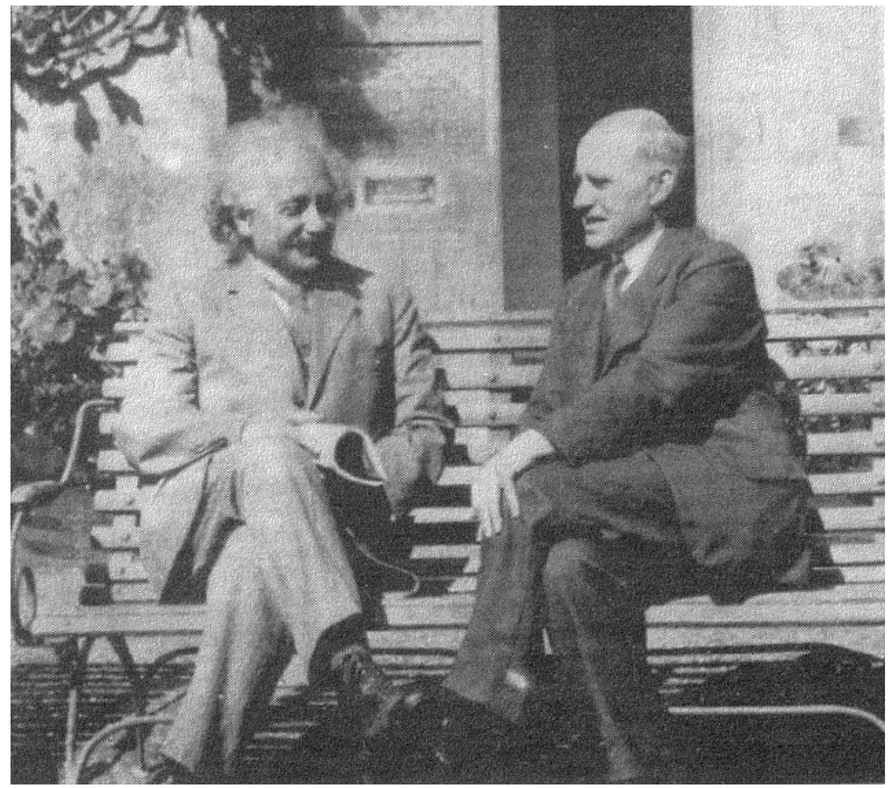
einstein akademie vissenchaften
berlin

Telegraphie des Deutschen Reichs.
[redacted] Antw S
Berlin, Haupt-Telegraphenamt
Antw S

lickobservaory calif 129/12 50 lr 19/16.- vesternunion

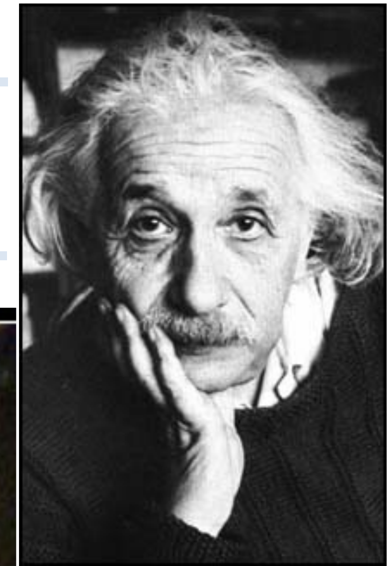
three pairs australia tahiti eclipse plates measured by camp[b]
trumpler sixty two to eighty four stars each five of six
measurements completely calculated give einstein deflection
between one point fifty nine and one point eighty six seconds
arc mean value one point seventy four seconds = campbell .+

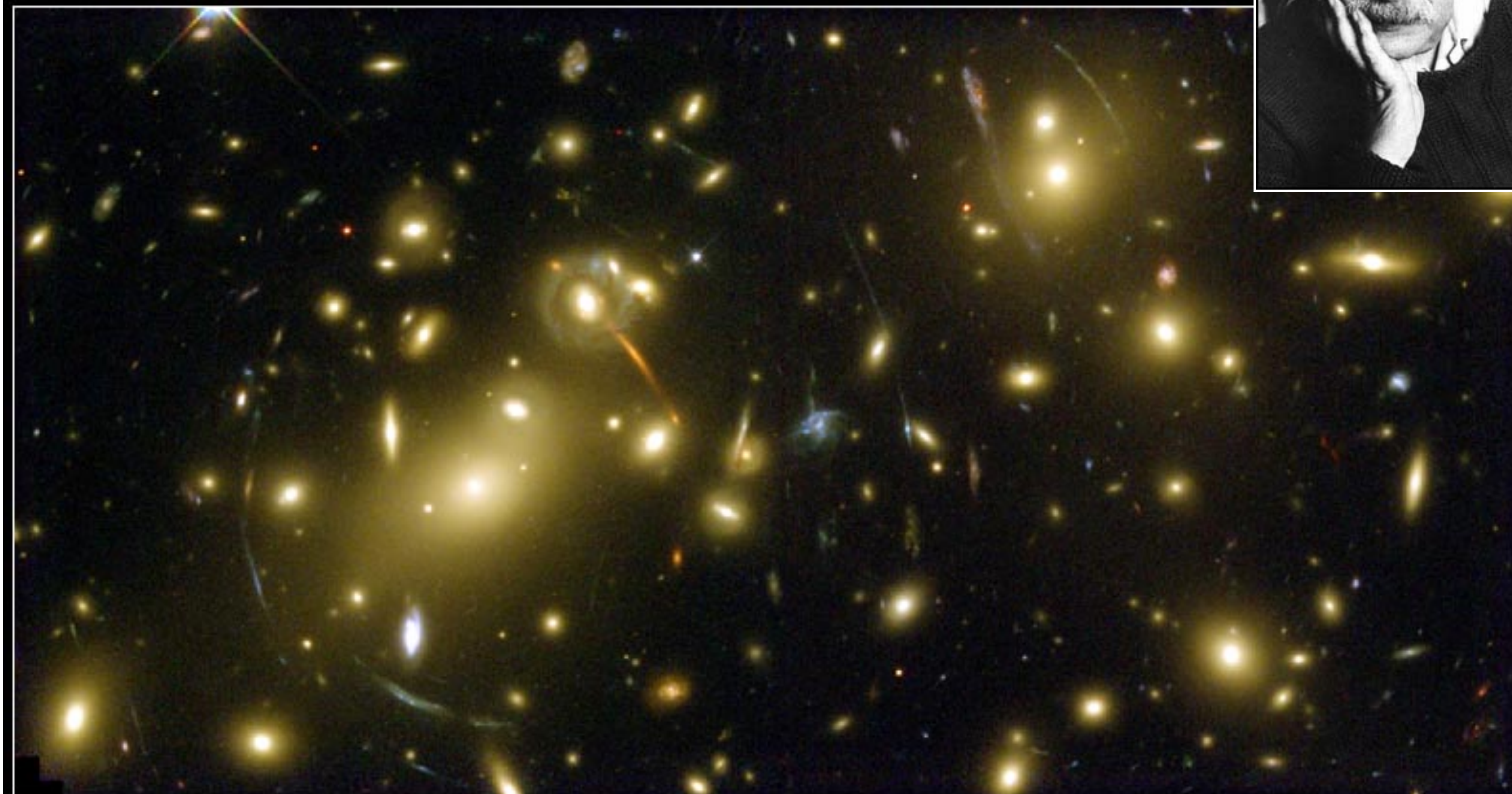
Campbell's telegram to Einstein, 1923



Einstein and Eddington, Cambridge, 1930

Gravitational Deflection of Light is a Well-Known Effect Today

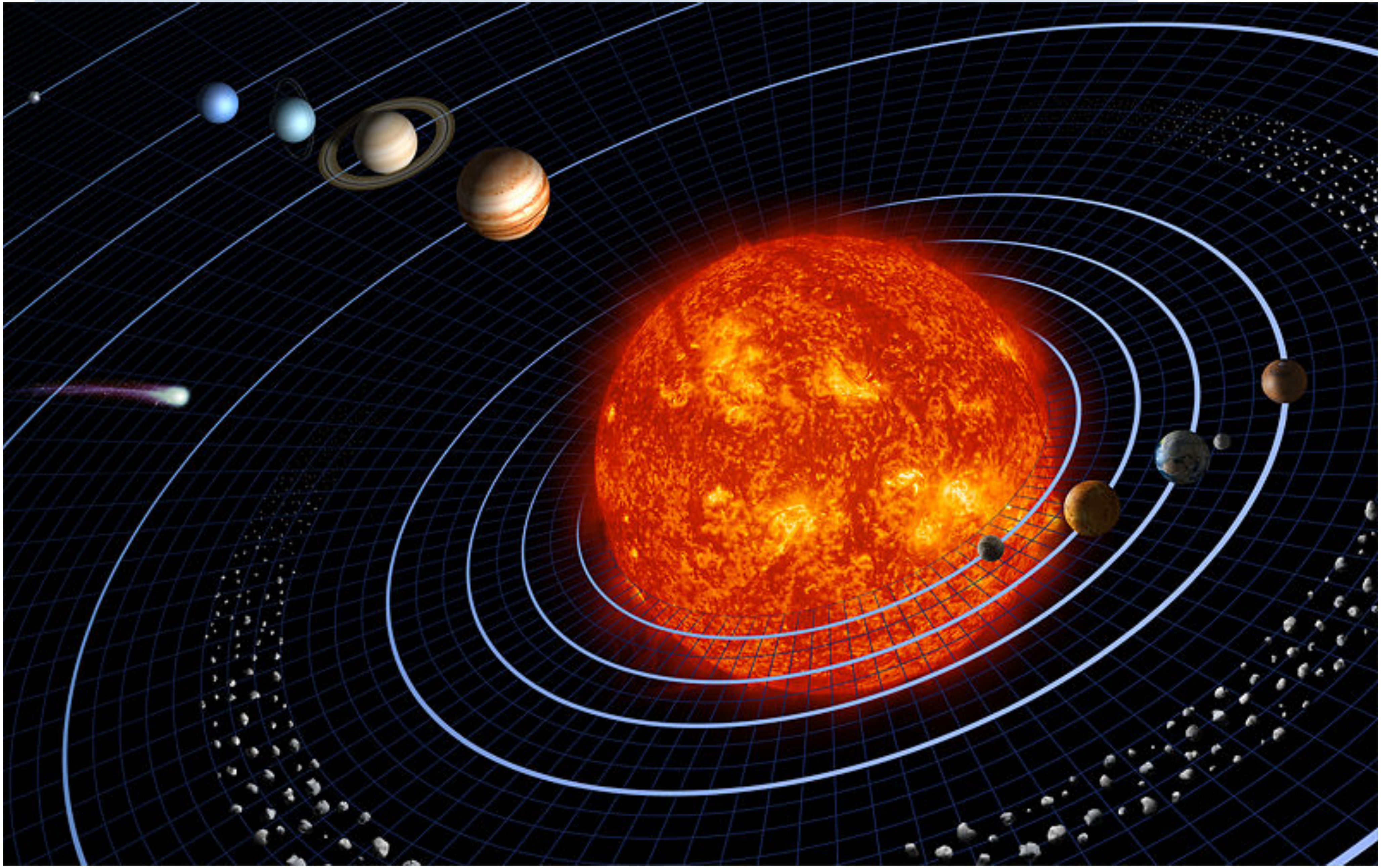




Galaxy Cluster Abell 2218 **HST • WFPC2**

NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08

Our solar system and tests of gravity



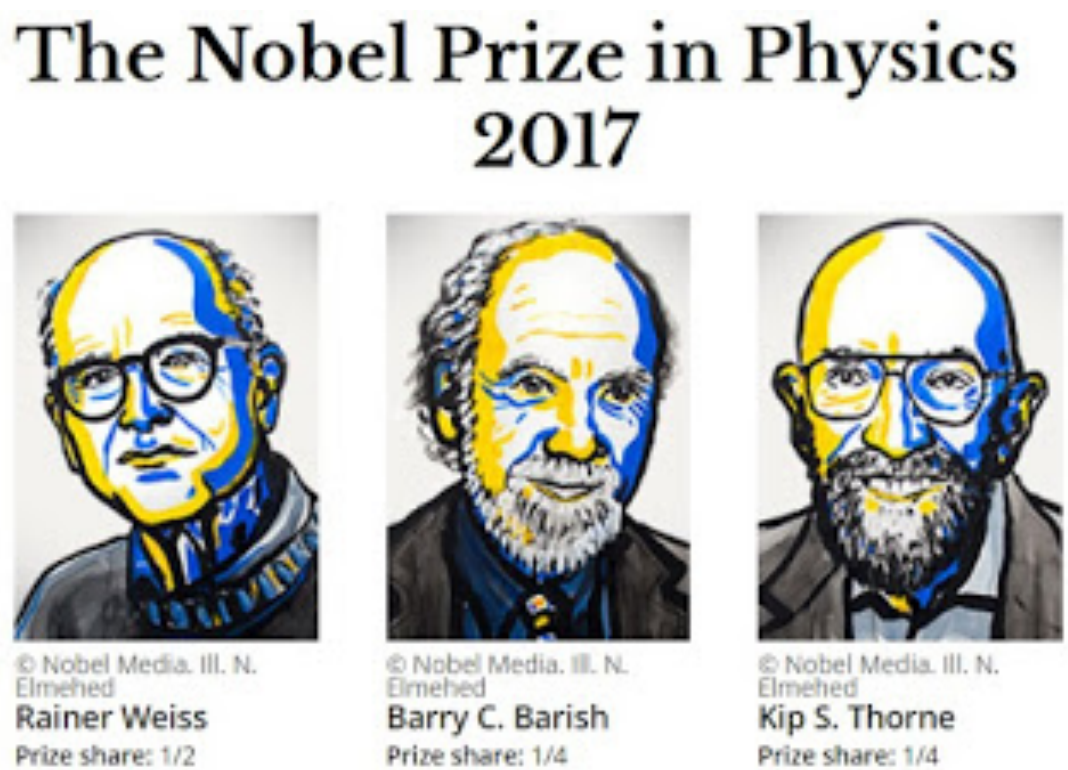
40+ Years of Solar System Gravity Tests

Techniques for Gravity Tests:

Radar Ranging:
- Planets: Mercury, Venus, Mars
- s/c: Mariners, Vikings, Pioneers, Cassini, Mars Global Surveyor, Mars Orbiter, etc.
- VLBI, GPS, etc.

Laser:
- SLR, LLR, interplanetary, etc.

Dedicated Gravity Missions:
- **LLR (1969 - on-going!!)**
- GP-A,’76; LAGEOS,’76,’92; GP-B,’04; LARES,’12; MicroSCOPE,’16, ACES, ‘18; LIGO,’16; eLISA, 2030+(?)



"for decisive contributions to the LIGO detector and the observation of gravitational waves"

New Engineering Discipline – Applied General Relativity:
- Daily life: GPS, geodesy, time transfer;
- Precision measurements, deep-space navigation & μas-astrometry (Gaia)

General relativity is now well tested. Can we use it to build something?

Mission to the Gravity Lens of the Sun

Eshleman V.R., Science **205**, 1133 (1979)

Gravitational Lens of the Sun: Its Potential for Observations and Communications over Interstellar Distances

Abstract. *The gravitational field of the sun acts as a spherical lens to magnify the intensity of radiation from a distant source along a semi-infinite focal line. A spacecraft anywhere on that line in principle could observe, eavesdrop, and communicate over interstellar distances, using equipment comparable in size and power with what is now used for interplanetary distances. If one neglects coronal effects, the maximum magnification factor for coherent radiation is inversely proportional to the wavelength, being 100 million at 1 millimeter. The principal difficulties are that the nearest point on the focal half-line is about 550 times the sun-earth distance, separate spacecraft would be needed to work with each stellar system of interest, and the solar corona would severely limit the intensity of coherent radiation while also restricting operations to relatively short wavelengths.*

About 40 years ago, Einstein (*1*) published a short note in *Science* on the focusing of starlight by the gravitational field of another star. He emphasized the improbability of observing this phenomenon by the chance alignment of two stars and the earth. From concepts based on current technology and trends, however, it appears that gravitational focusing of electromagnetic radiation might be employed, by design, for highly directional observations and communications over interstellar distances.

In such use, the gravitational field of the sun could play several roles. First, it might be used to reduce fuel and time re- [...] $1 + \nu$, where the refractivity $\nu = g/r$ at radius r. A ray is deflected through the angle $\alpha = 2g/a$, where a is the ray impact parameter and g is the gravitational radius ($g = 2Gm/c^2$, where G is the gravitational constant, m is the mass of the central body, and c is the speed of light). It is assumed throughout that $\alpha << 1$. An observer at position z behind the lens and x from the center line, as illustrated, would see an energy density lessened by defocusing in the plane of propagation, but increased by focusing due to the curved limb normal to this plane. The relative single-ray intensity $I = F_h^2 F_v^2$, where in ray optics $F_h^2 =$ [...] nel scales along the circumference of a circle at the ray-impact radius. Using also the wave number $k = 2\pi/\lambda$, the maximum intensification of the coherent signal is simply

$$I_{max} = 2\pi kg \qquad (2)$$

As an approximation, let the focal "spot" radius x_s be the value of x where I falls to $I_{max}/4$, so that $x_s = (2/\pi k)(z/2g)^{1/2}$. Thus the angular resolution for distinguishing two adjacent coherent sources by a corresponding change in intensity is x_s/z radians. (The first null off the center line is at $x = \pi^2 x_s/2$, and the first sidelobe is twice this distance with intensity I_{max}/π^2.) The periapsis or minimum radius of the ray relative to the center of mass is $a - g$, or essentially a, and this must be greater than r_0, the physical radius of the spherical mass. Thus $\alpha_{max} = 2g/r_0$ and the focal line begins at $z_{min} = r_0^2/2g$.

Now consider the focusing at $z > z_{min}$ of incoherent radiation from a uniformly bright, circular, extended source of radius r_p and distance $z_p >> z$. This is the problem considered by Einstein (*1*) and more completely by others, notably Liebes (*4*). The gain factor A of the gravitational lens for the intensity observed from the two individual images com- [...]

Kraus J.D., Radio Astronomy, Cygnus-Quasar Books, Powell, Ohio, 6-115 (1986)

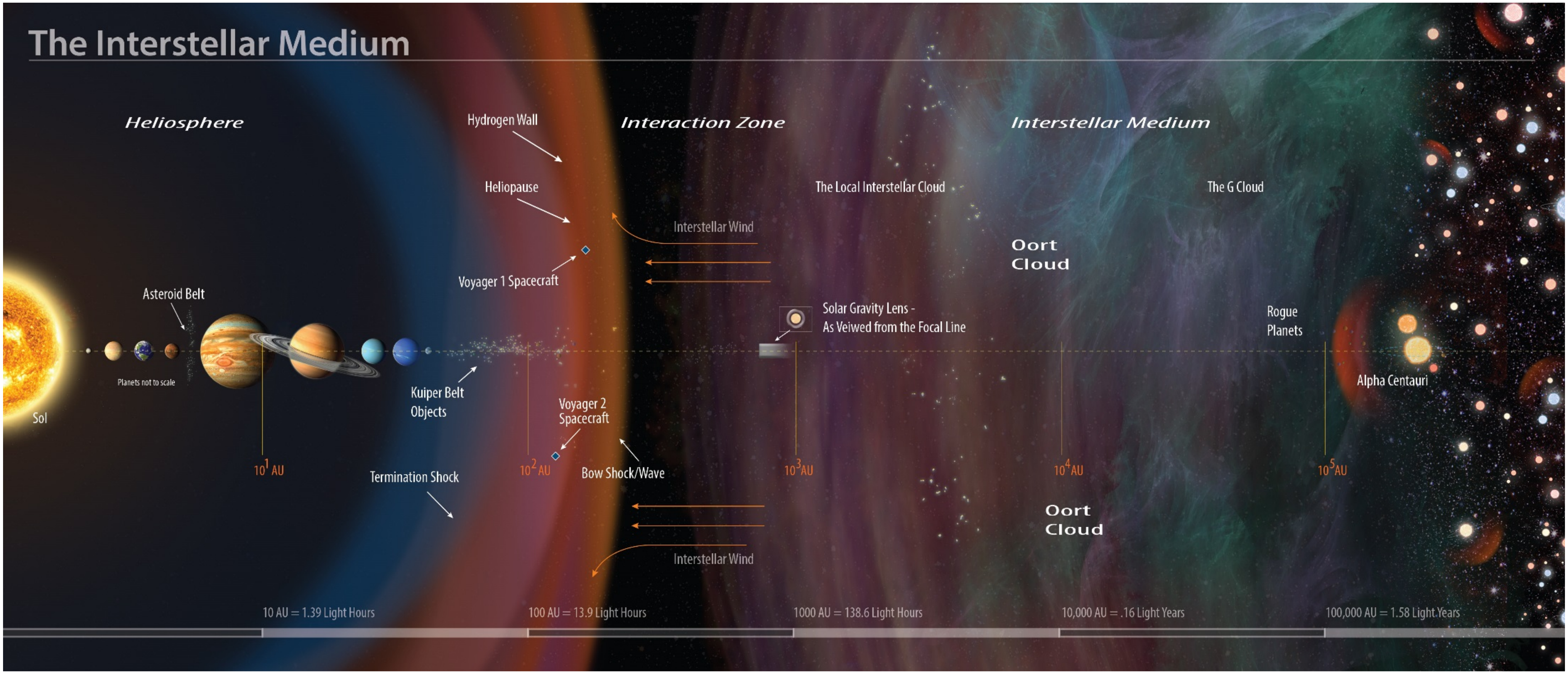
Maccone C., many papers, 1999-present

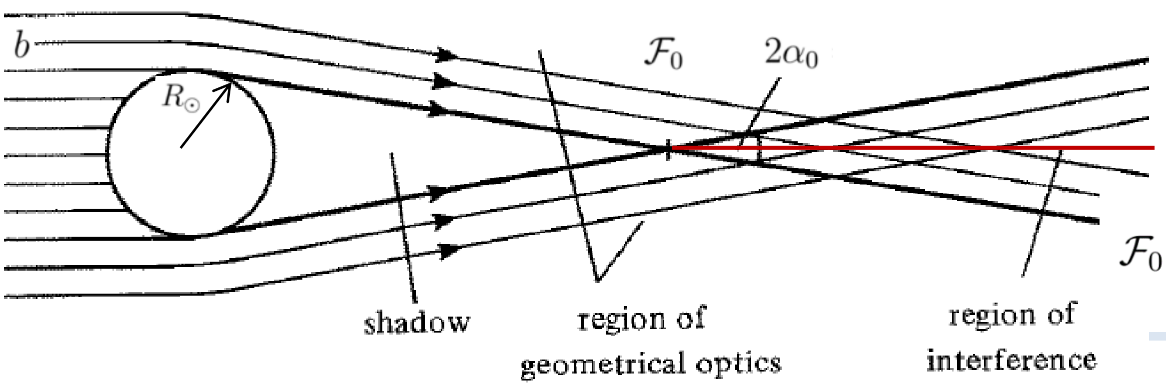
Turyshev & Andersson, MNRAS **341**, 577 (2003)

SGL enables direct multipixel imaging

- Overcoming the issue of a small target size:
  - Consider an exo-Earth @ 30pc (100 l.y.) is $\sim 1.4\times 10^{-11}$ rad;
  - A diffraction-limited telescope needed to resolve an object with this size at such distance must have a diameter of ~76 km;
    - But, even this telescope would barely resolve the disk of the planet.
  - To resolve the planet with 1,000 pixels one needs a telescope with a diameter of 7.6×10^4 km (or $\sim 12\ R_\oplus$), which is impractical...
    - An imaging interferometer with a set of such baselines - not feasible.
  - Even more challenging is the integration time needed to reach SNR=10:
    - a 50m telescope would need an integration time of $t \sim 10^6$ years (zodi);
    - with SGL's light amplification ($\sim 2\times 10^{11}$) we could do the job in ~5 weeks.
- Solving the parent start light contamination issue:
  - Current exoplanet-imaging concepts detect light of a planet as a single pixel. Contamination from the parent star (~0.1" off the planet) is a major problem;
    - Due to the high angular resolution of the SGL (~0.5 nas), the parent star is resolved from the planet with its light amplified 0.01 AU away from the optical axis, making the parent star contamination issue negligible.

The Solar Gravitational Lens (KISS study, 2015)





$$\alpha_0 = \frac{2r_g}{R_\odot} \approx 8.5\ \mu\text{rad} \quad \rightarrow \quad \alpha(b) = \alpha_0 \frac{R_\odot}{b}$$

$$\mathcal{F}_0 = \frac{R_\odot}{\alpha_0} = \frac{R_\odot^2}{2r_g} \approx 547\ \text{AU} \quad \rightarrow \quad \mathcal{F}(b) = \mathcal{F}_0 \frac{b^2}{R_\odot^2}$$

Focal beam of extreme intensity

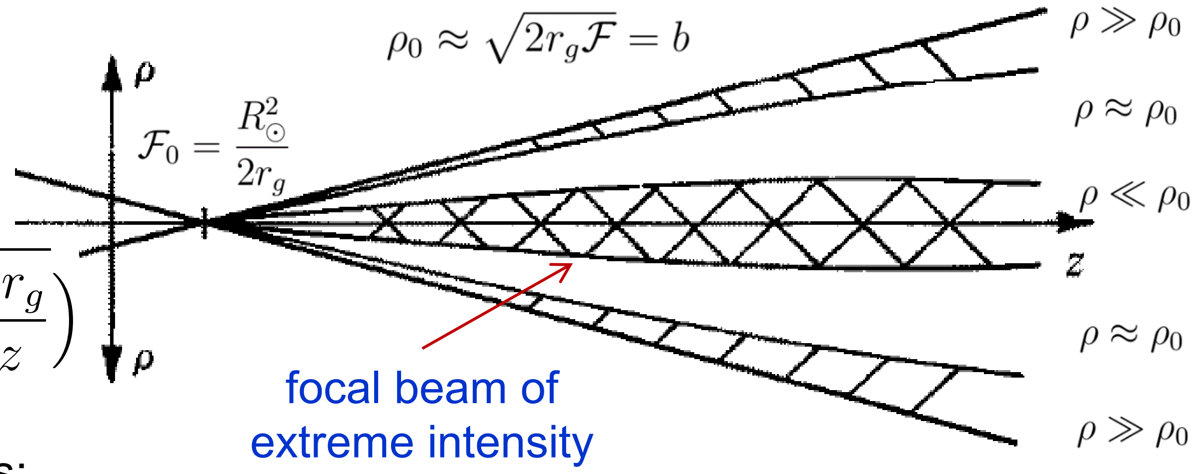
Herlt & Stephani, IJMP 15, 45 (1976)

- Major brightness increase:
  - For small departures from the optical axis, ρ, magnification of the SGL is:

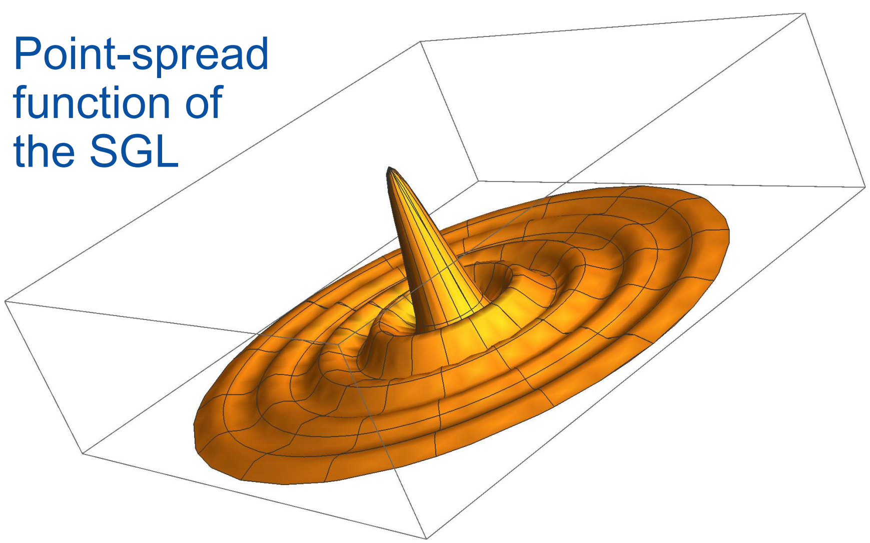
$$\mu_z(\rho, z, \lambda) \;\cong\; 4\pi^2 \frac{r_g}{\lambda}\, J_0^2\Big(2\pi \frac{\rho}{\lambda}\sqrt{\frac{2r_g}{z}}\Big)$$

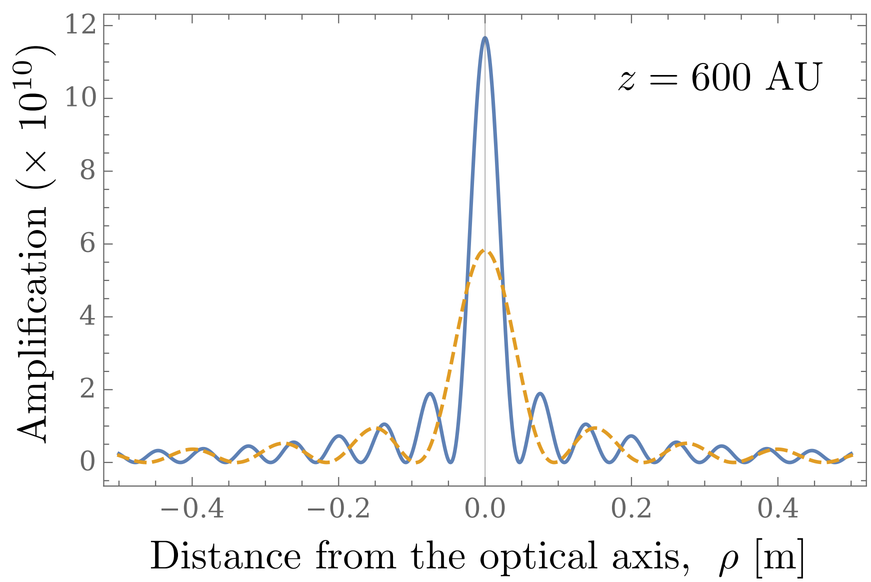
  - Max value of $G(\rho, \lambda)$ is on axis:

$$\mu_z(0, z, \lambda) \;\cong\; 4\pi^2 \frac{r_g}{\lambda}$$



Point-spread function of the SGL

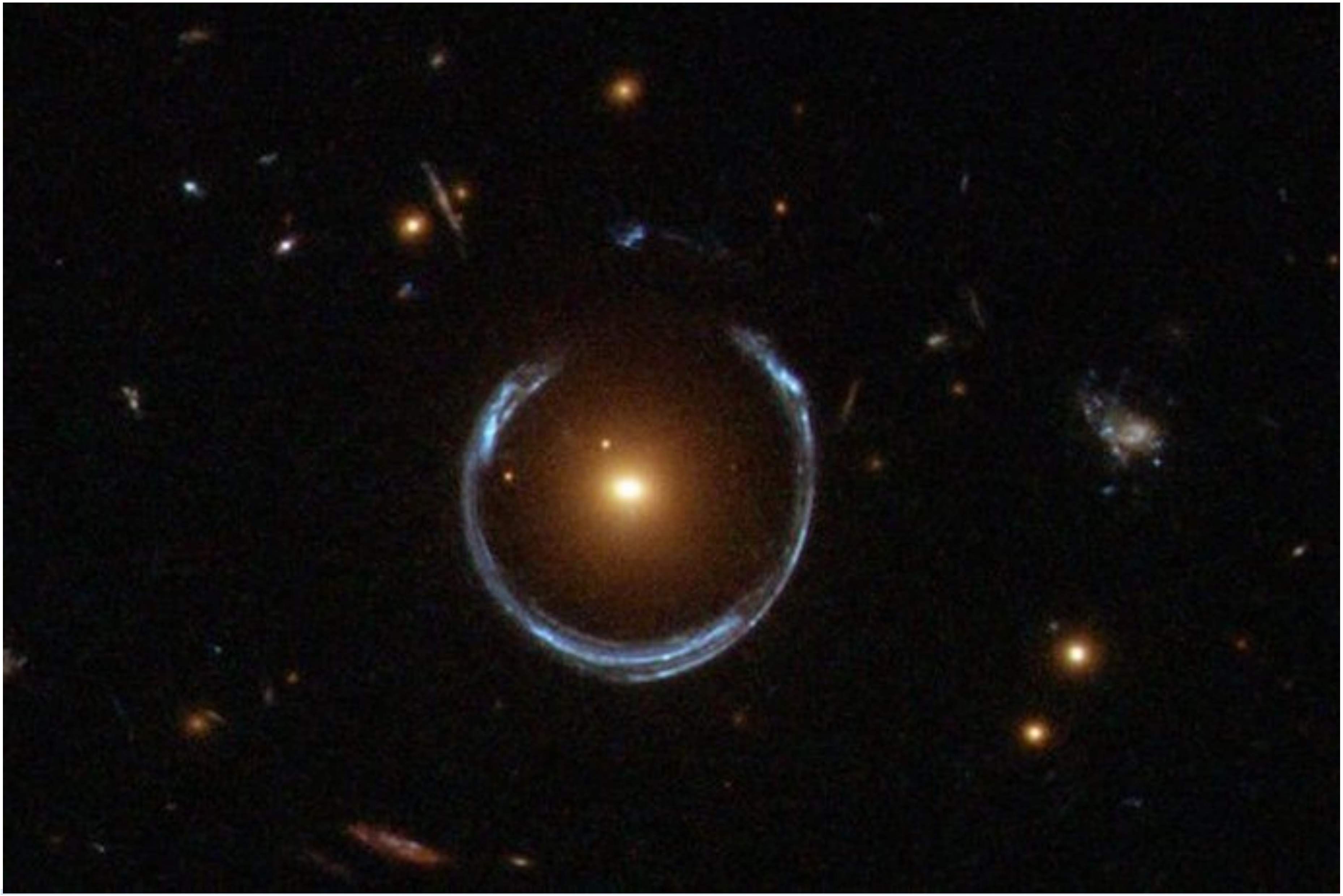




Turyshev & Toth, Phys. Rev. D 96, 024008 (2017)

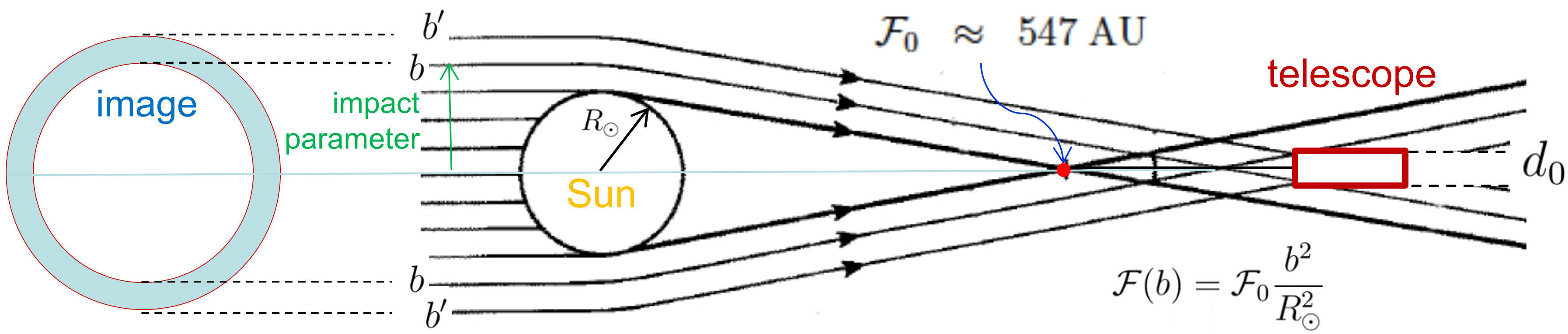
Gain of the SGL as seen in the image plane as a function of possible observational wavelength

The image in a form of the Einstein ring



Credit: ESA, Hubble & NASA Wikimedia

Properties of the Solar Gravity Lens



- Important features of the SGL (for $\lambda = 1\ \mu$m):
  - Major brightness magnification: a factor of 10^{11} (on the optical axis);
  - High angular resolution: ~0.5 nano-arcsec. A 1-m telescope at the SGL collects light from a ~(10km × 10km) spot on the surface of the planet, bringing this light to one 1-m size pixel in the image plane of the SGL;
  - Extremely narrow "pencil" beam: entire image of an exo-Earth (~13,000 km) at 100 l.y. is included within a cylinder with a diameter of ~1.3 km.
- Collecting area of a 1-m telescope at the SGL's focus:
  - Telescope with diameter d_0 collects light with impact parameters $\delta b \simeq d_0$;
  - For a 1-m telescope at 750AU, the total collecting area is: 4.37×10^9 m^2, which is equivalent to a telescope with a diameter of ~80 km…

Imaging Exoplanets with the Solar Gravitational Lens

Please watch the movie at:

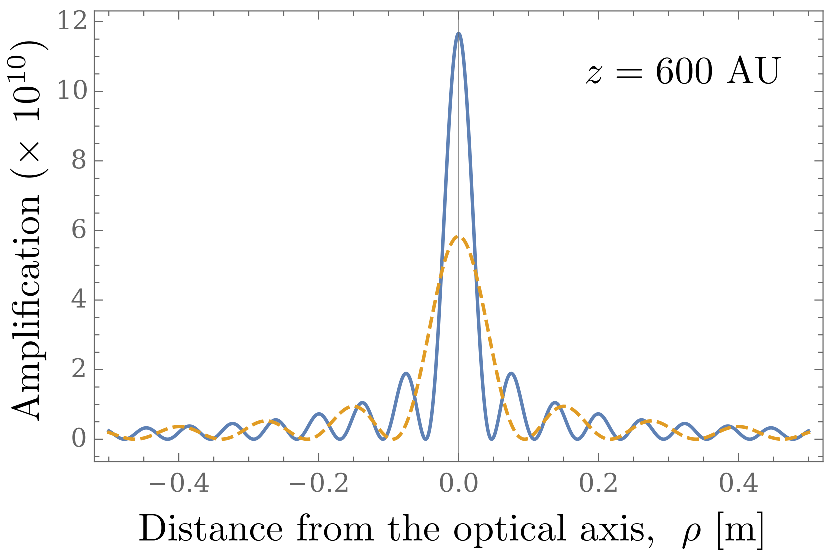
https://www.youtube.com/watch?v=Hjaj-Ig9jBs

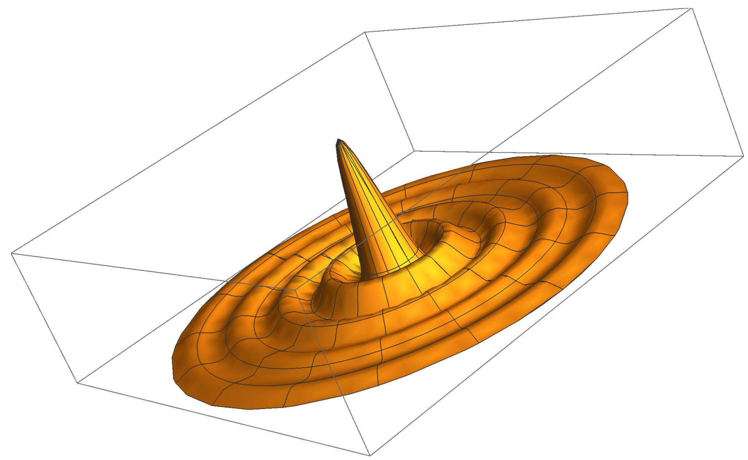
Credit: J. DeLuca

Exploring optical properties of the SGL

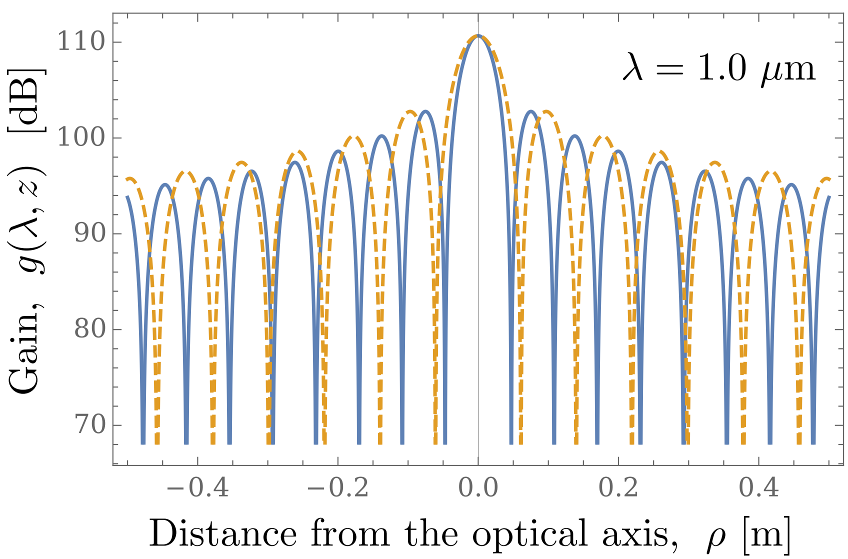
- Significant potential for high-resolution spectroscopy:
  - Spectroscopic signal is high: SNR~10^6 in 1 sec (broadband);
  - Splitting this signal in ~10^6 bands would yield high-resolution spectra.
- Although very powerful, the Sun is not a very good lens:
  - Magnified images will be highly blurred, with any given pixel containing light reflected from adjacent regions on the surface of the exoplanet.
  - Would require correction with modern image reconstruction techniques:
    - Planetary rotation would provide periodic changes. Rotational deconvolution (aka tomography): ~250-300 pixel images in ~1 year;
    - Direct deconvolution: SNR reduction because of blurring, leading to longer integration times to reach ~800 pixel (study is ongoing).
- Observing on the background of the solar corona:
  - Corona pushes us to higher heliocentric distances ~650-850 AU;
  - Required chronographic performance 10^{-6} (WFIRST needs 10^{-9});
    - Internal coronagraph that has ~2×10^{-7} attenuation, 15% throughput, but pushes to larger apertures 1.5-2.2m;
    - External coronagraph (i.e., starshade) allows for smaller aperture(s).

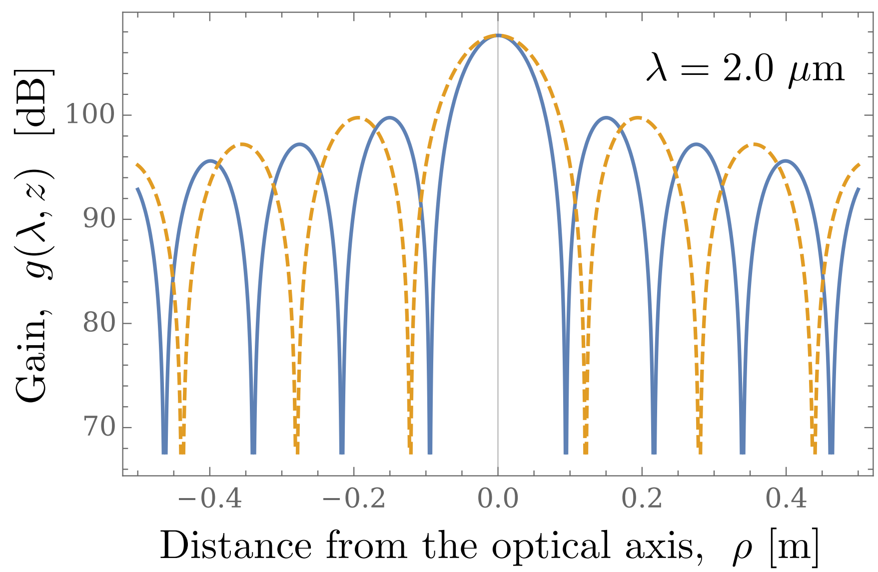
Point spread function & gain of the SGL





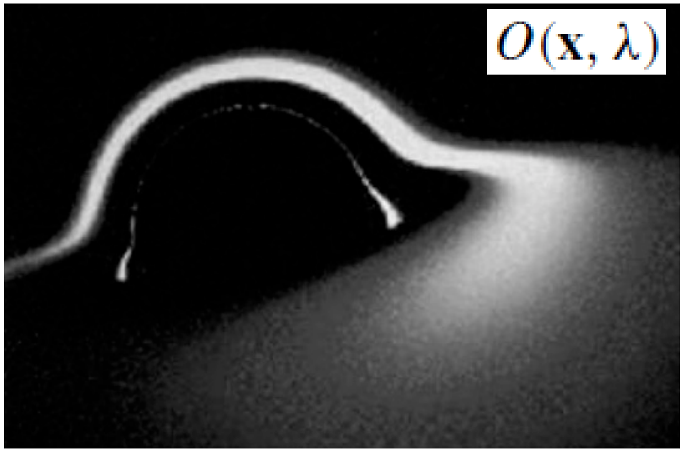
3-D Airy pattern of the SGL





Turyshev & Toth, Phys. Rev. D 96, 024008 (2017)

Image formation by the SGL



Accretion disk around a black hole as a test object for convolution by the PSF of the SGL.

$$I(\mathbf{x}_2, \lambda) = O(\mathbf{x}, \lambda) \otimes PSF_{\rm diff}(\mathbf{x}_2, \lambda)$$

$$PSF_{\rm diff}(\mathbf{x}_2, \lambda) \simeq J_0\left(\frac{\pi |\mathbf{x}| r_0}{\lambda f}\right) \otimes \frac{r_0}{|\mathbf{x}_2|}$$

- r_0 – impact parameter,
- $|\mathbf{x}_2|$ – distance in the image plane,
- $\otimes$ – 2D convolution operator.

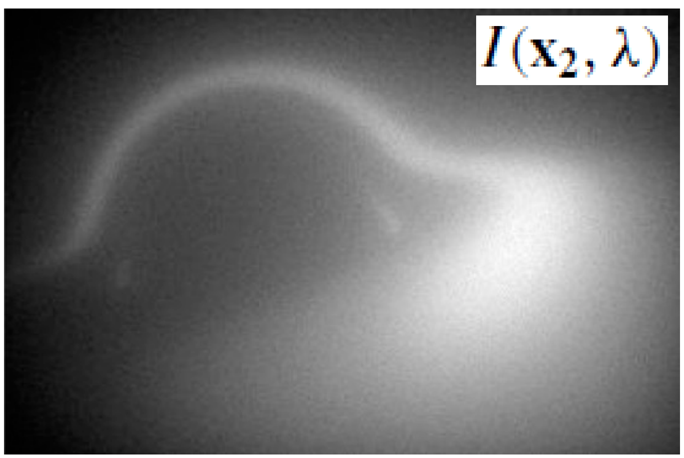
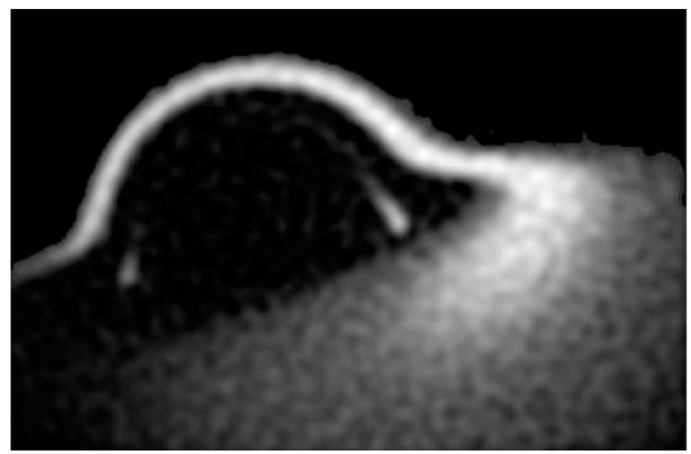


Image obtained after convolution. Photon noise is added, corresponding to 100 ph/pixel



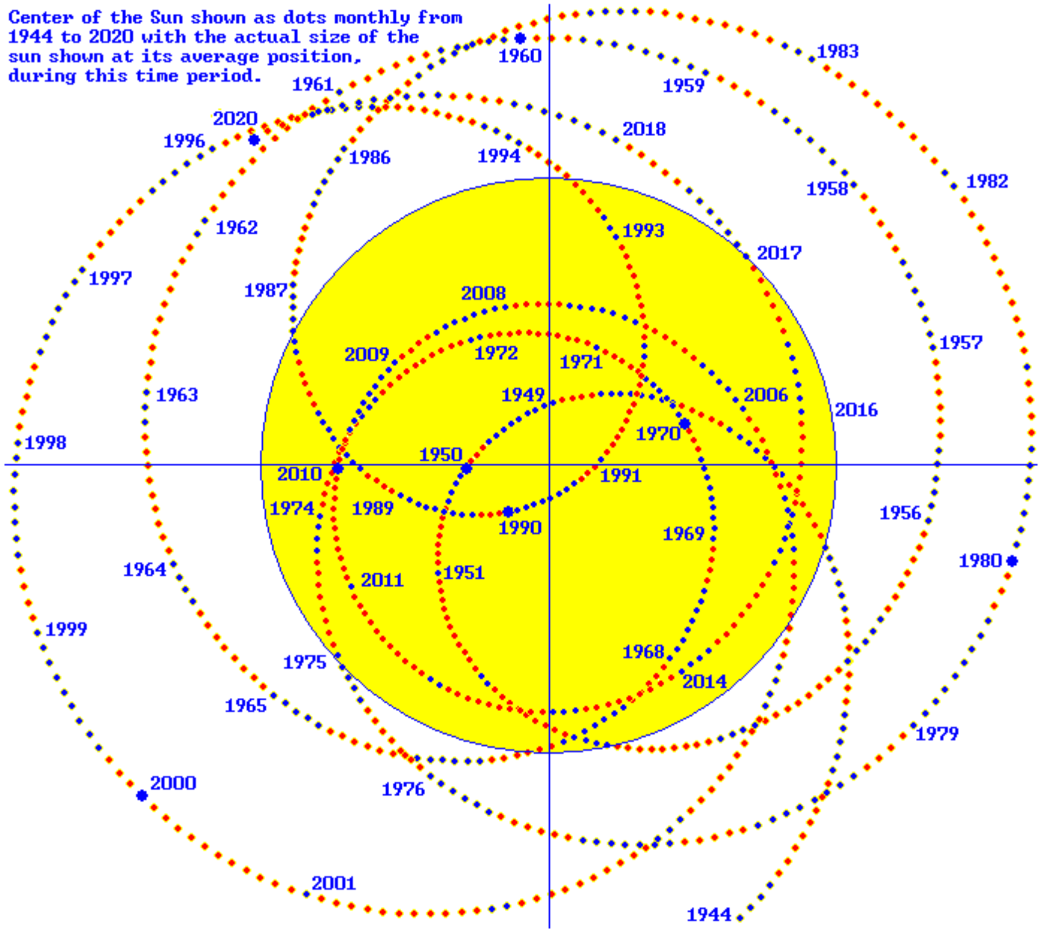
De-convolved image using the SGL’ PSF. Low-pass filtering in spatial frequencies is applied

L. Koechlin et al., Exp Astron (2005) 20:307–315

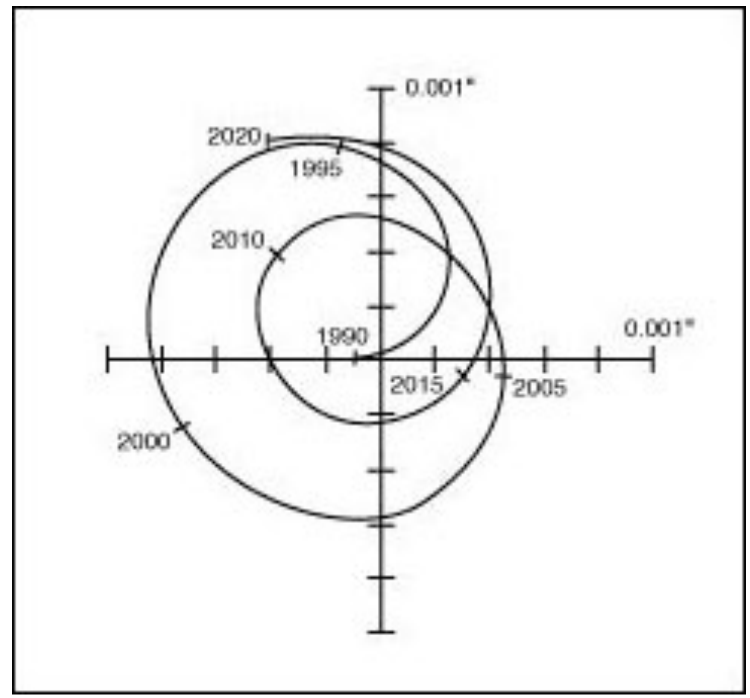
The *a priori* properties of the target

- We want to image Earth 2.0, around a G star, which is not transiting:
  - Once habitability is confirmed ("big TPF" for spectra), the next step is to image it.
- We will rely on astrometry, RV, spectroscopy, and direct imaging to obtain:
  - orbital ephemeris: to ~mas accuracy and precision;
  - rotation: from temporal monitoring of the spectroscopy;
  - atmosphere: temperature, structure, chemical composition, and albedo, from non-spatially-resolved spectroscopy;
  - understanding of cloud & surface properties from Doppler imaging.
- This information will help us to point the s/c:
  - Time to reach 550 AU ~10 years, enough to observe the parent star's location ~100 times with 1 μas precision, so that its position would be known to 0.1 μas;
  - The parent star's position would be known to ~45 km at a distance of 30 pc;
  - Orbital period to <1% $\Rightarrow$ the semi-major axis is known to ~0.7% (~1 million km);
  - If face-on, the radial distance to ~1 million km, with tangential error ~6 larger;
  - Earth's diameter is 13,000 km, so we will search the (80 × 500) grid on the sky;
  - Once SGLFM detects the planet $\Rightarrow$ scan a smaller area to define the "edges".

The solar wobble



Center of the Sun shown as dots monthly from 1944 to 2020 with actual size of the sun shown at its average position, during this time period



Astrometric displacement of the Sun due to Jupiter as at it would be observed from 10 parsecs, or about 33 light-years.

Direct Multipixel Imaging and Spectroscopy of an Exoplanet with a Solar Gravity Lens Focus (SGLF) Mission

An imaging mission to SGLF appears to be feasible, but needs further study

Concept

- SGLF provides a major gain (~10^{11} at 1um), resolution of 10^{-9} arcsec in a narrow FOV;
- A 1-m telescope at ~750AU has a collecting area equivalent ~80 km aperture in space;
- A mission to the SGLF could image Earth 2.0 up to 30pc away with resolution to ~10km to see surface features;
- A small s/c with electric propulsion (or solar sails) can reach the SGLF in <35-40 yrs.

Proposed Study and Approach

- Define baseline design, sub-syst components;
- Define mission science goals & requirements;
- Develop system and subsystem requirements;
- Study mission architecture and con-ops;
- Assessment of feasibility (cluster) small-sats;
- Identify technology development needs;
- Study instruments & systems: power, comm, pointing, s/c, autonomy, coronagraph, nav, propulsion, raster scan in the image plane, etc.

Benefits

- A breakthrough mission concept to resolve a habitable exoplanet at modest cost/time;
- Could find seasonal changes, oceans, continents, life signatures on an exo-Earth;
- Small-sat & fast exit from the solar system;
- Electric propulsion for raster-scanning the image using tethered s/c (or cluster);
- SLGF is valuable for other astrophysics and cosmology targets.

Earth with resolution of (1000 × 1000) pixels.