

**From fundamental problems in physics
to fundamental physics in space**

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Outline

- Space Missions which inspired the Seminar
- Big Science Questions
- Physics at its Most Fundamental Level
- Mass, Proper Time and Inertia
- Gravity and the Quantum Vacuum
- Dissipation and Quantization

Space Missions which inspired the Seminar

Gravity based Missions in Space

ACES/PHARAO (clocks in gravity, gravitational redshift)

Galileo 5 and 6 for (clocks in gravity, gravitational redshift)

GRACE Follow On (gravitational field of the Earth)

LARES (spin-orbit coupling, Lense-Thirring effect)

LISA Pathfinder (gravitational waves, black holes, big bang)

MICROSCOPE (equivalence principle)

Quantum based Missions in Space

MAIUS (Bose-Einstein condensate in space)

QUESS (entanglement between quantum in space and on ground)

Benefit of Space over Earth

- 1 - permanent microgravity
- 2 - minor environmental noise and friction
- 3 - long distances and large potential differences
- 4 - large time intervals
- 5 - high velocities
- 6 - handling of large masses

Big Science Questions

What is the structure of gravity and how does gravity relate to the non-gravitational content of the world?

Cosmological Constant

Dark Energy

Dark Matter

Particle-Mass Hierarchy

Matter-Antimatter Asymmetry

Quantum Gravity and Unification Of All Forces

loop-quantum gravity, M theory (supergravity, superstring)

Are there limitations of the quantum description of the world?

Possible outlook to the deepest but scientifically never answerable question: *Why is there something and not rather nothing?*

Measurement

Arrow of Time

Friction

Complexity

Quantum Multiverse (wave function of the whole universe?)

Why one time and three space dimensions?

Why are the fundamental physical constants as they are?

Is the Anthropic Principle indispensable?

Physics at its Most Fundamental Level

Effective Field Theory: most powerful description of Nature (also efficient for classical gravity, e.g. Goldberger and Rothstein 2006)

excellent approach: Schwinger's Source Theory (source S , action W)

$$\langle 0_+ | 0_- \rangle^S = \exp\left(\frac{i}{\hbar}W[S]\right)$$

$$|\langle 0_+ | 0_- \rangle^S|^2 = \exp\left(-\frac{2}{\hbar}\text{Im}W[S]\right) \leq 1$$

Julian Schwinger (1918 - 1994)

Gravitons and Photons: The Methodological Unification of Source Theory

Gen. Rel. Grav. 7, 251 (1976) [[Essay](#), Gravity Research Foundation 1975]

From action all physics results:

$$dW = -H(p_i, x^i, t)dt + p_i dx^i, \quad dW = -mc^2 d\tau;$$

$$dW/\hbar = -\omega(k_i, x^i, t)dt + k_i dx^i, \quad dW/\hbar = -\omega_0 d\tau, \quad \omega_0 = mc^2/\hbar$$

$$\frac{\partial H}{\partial p_i} = v^i, \quad \frac{\partial H}{\partial x^i} = -\frac{dp_i}{dt}, \quad \frac{\partial H}{\partial t} = \frac{dH}{dt} \quad (\text{process level})$$

e.g. $H(p_i, x^i, t) = \sqrt{m^2 c^4 + p_i p^i c^2}$ (EOS)

or , $\omega(k_i, x^i, t) = \sqrt{\omega_0^2 + k_i k^i c^2}$ (dispersion relation)

Electromagnetism ($\alpha \equiv \frac{e^2}{\hbar c} = \frac{1}{137.036}$, $e^* = \frac{e}{\sqrt{\alpha}} = 1.875 \times 10^{-18}$ C)

$$\frac{W[J]}{\hbar} = \frac{1}{(e^*c)^2} \frac{4\pi}{2} \int (dx)(dx') J^\mu(x) \eta_{\mu\nu} G_F(x-x') J^\nu(x')$$

assumption to be made: Lorentz invariance

consistency strictly implies $\partial_\mu J^\mu = 0$ (charge conservation)

$$\delta W[J] = \int (dx) \delta J^\mu(x) A_\mu(x)$$

$$A_\mu(x) = \frac{4\pi}{c} \int (dx') G_F(x-x') J_\mu(x') + \partial_\mu \lambda(x) \quad (\text{gauge field})$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad \partial_\mu F^{\mu\nu} = \frac{4\pi}{c} J^\nu$$

$A_\mu(x)$ describes massless spin-1 field (photon)

$$G_F(x - x') = \frac{1}{4\pi} \delta((x - x')^2) + \frac{i}{4\pi^2} \frac{1}{(x - x')^2} = \frac{i}{4\pi^2} \frac{1}{(x - x')^2 + i0}$$

$$G_F = \bar{G} + \frac{i}{2} G^{(1)}, \quad \bar{G} = \frac{1}{2} (G_{\text{ret}} + G_{\text{adv}}) \quad G_F - G^{(-)} = G_{\text{ret}}$$

$$G_{\text{ret}} = \bar{G} - \frac{1}{2} \tilde{G}, \quad \tilde{G} = G_{\text{adv}} - G_{\text{ret}}, \quad G^{(\pm)} = \frac{1}{2} (\tilde{G} \mp iG^{(1)})$$

classical mechanics: boundary of quantum mechanics

$$(1/i\hbar)[\Phi(x), \Phi(x')] = \tilde{G}(x, x') \quad \text{classical causality for observables}$$

$$(1/i\hbar) \langle 0 | [\Phi^{(+)}(x), \Phi(x')] | 0 \rangle = G^{(+)}(x - x')$$

$$\Phi(x) = \Phi^{(+)}(x) + \Phi^{(-)}(x), \quad \Phi^{(+)}(x) | 0 \rangle = 0$$

Gravitation ($\frac{1}{M_{\text{P}}^2} = \frac{G}{\hbar c}$, $M_{\text{P}} = 2.176 \times 10^{-8}$ kg, Planck mass)

$$\frac{W[T]}{\hbar} = \frac{1}{(M_{\text{P}}c^2)^2} \frac{8\pi}{2} \int (dx)(dx') \left(T_{\mu}^{\nu}(x) G_F(x-x') T_{\nu}^{\mu}(x') - \frac{1}{2} T(x) G_F(x-x') T(x') \right)$$

$$T^{\mu\nu} = T^{\nu\mu}, \quad T = T_{\mu}^{\mu}, \quad \partial_{\mu} T_{\nu}^{\mu} = 0 \quad (\text{energy-momentum conservation})$$

$$\delta W[T] = \frac{1}{2} \int (dx) \delta T^{\mu\nu}(x) h_{\mu\nu}(x)$$

$$h_{\mu\nu}(x) = \frac{16\pi G}{c^4} \int (dx') G_F(x-x') \left(T_{\mu\nu}(x') - \frac{1}{2} \eta_{\mu\nu} T(x') \right) + \partial_{\mu} \xi_{\nu}(x) + \partial_{\nu} \xi_{\mu}(x) \quad (\text{gauge field})$$

$h_{\mu\nu}(x)$ describes massless spin-2 field (graviton)

infinitesimal coordinate transformation $\bar{x}^\mu = x^\mu + \delta x^\mu(x)$

induces $\delta h_{\mu\nu} = -(\partial_\mu \delta x_\nu + \partial_\nu \delta x_\mu)$ with gauge vector $\xi^\mu = -\delta x^\mu$

but with $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$ and $|h_{\mu\nu}(x)| \ll 1$,

it also holds: $\delta h_{\mu\nu} = -(\partial_\mu \delta x_\nu + \partial_\nu \delta x_\mu)$

Einstein Field Equations

$$G^{\mu\nu}(g, \partial g, \partial^2 g) + \Lambda g^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}(\eta \rightarrow g) \quad \underline{\text{minimal coupling}}$$

(fulfilment of Einsteinian Equivalence Principle)

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu}$$

$$\left(D_\mu G^{\mu\nu} \equiv 0, \quad D_\mu (\Lambda g^{\mu\nu}) \equiv 0 \right) \Rightarrow D_\mu T^{\mu\nu} = 0 \quad (\text{power and force})$$

With $T^{\mu\nu} = T_{\text{DM}}^{\mu\nu} + T_{\text{M}}^{\mu\nu} + T_{\text{em}}^{\mu\nu}$, it follows

$$D_{\mu}T_{\text{DM}}^{\mu\nu} = 0 \quad \text{coupling to gravity only}$$

$$D_{\mu}(T_{\text{M}}^{\mu\nu} + T_{\text{em}}^{\mu\nu}) = 0 \quad \text{or,} \quad D_{\mu}T_{\text{M}}^{\mu\nu} = -D_{\mu}T_{\text{em}}^{\mu\nu}$$

The fundamental objects in physics are current densities J^{μ} , T_{ν}^{μ} ;

also J_A^{μ} ,

$A = 1, \dots, 8$ for strong interaction, $A = 1, 2, 3$ for weak interaction

Our access to the world is through J^{μ} and T_{ν}^{μ} only.

$$T_{\text{M}}^{\mu\nu} = m_a u_a^{\mu} u_a^{\nu} \frac{d\tau_a}{dt} \delta(\mathbf{r} - \mathbf{r}_a), \quad \text{Mass drops out from } D_{\mu}T^{\mu\nu} = 0$$

fulfilment of Equivalence Principle (weak, Einsteinian, strong)

fundamental appearance of Mass : $T_{M\mu}^{\mu} = m_a c^2 \frac{d\tau_a}{dt} \delta(\mathbf{r} - \mathbf{r}_a)$

$$T_{(\text{Dirac})\mu}^{\mu} = \frac{1}{2} m_a c^2 \bar{\Psi}_a \Psi_a$$

$$T_{(\text{Dirac})}^{\mu\nu} = \frac{1}{4} i\hbar c \bar{\Psi}_a (\gamma_a^{\mu} D^{\nu} + \gamma_a^{\nu} D^{\mu}) \Psi_a, \quad (i\hbar \gamma_a^{\mu} D_{\mu} - m_a c) \Psi_a = 0$$

Mass, Proper Time and Inertia

$$W_{Mm} = - c^2 \int_1^2 m_a d\tau_a$$

no Mass \rightarrow no Scale \rightarrow no Proper Time

m_a : rest mass, τ_a : proper time, e_a : electric charge of particle a

$$W_{Me} = -\frac{1}{c} \int_1^2 A_\mu J_a^\mu d^3x dt = -\frac{1}{c} \int_1^2 A_\mu e_a dx_a^\mu$$

Electric Charge \rightarrow matter-field interaction

dynamics via total matter action: $W_M = W_{Mm} + W_{Me}$

Inertia

$$W_M = \int_1^2 L_M dt \rightarrow \text{EOM: } \frac{d}{dt} \frac{\partial L_M}{\partial \mathbf{v}_a} = \frac{\partial L_M}{\partial \mathbf{r}_a} \rightarrow$$

$$m_a \frac{d}{dt} \frac{\mathbf{v}_a}{\sqrt{1 - \frac{v_a^2}{c^2}}} = e_a \left(\mathbf{E} + \frac{\mathbf{v}_a}{c} \times \mathbf{B} \right), \quad \text{equiv } \partial_\mu T_M^{\mu i} = - \partial_\mu T_{\text{em}}^{\mu i}$$

$$T_{\text{em}}^{\mu\nu} = \frac{1}{4\pi} (-F^{\mu\beta} F^\nu{}_\beta + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}), \quad T_{\text{em}\mu}^\mu = 0$$

$$W_{\text{em}} = -\frac{1}{16\pi c} \int (dx) F^{\alpha\beta} F_{\alpha\beta}$$

Inertia does not need other bodies or gravity to exist,
fully present in Minkowski space already

Gravity and the Quantum Vacuum

$$W_g = -\frac{c^3}{16\pi G} \int (dx)(-g)^{1/2} \left(2\Lambda + R + c_1 R^2 + c_2 R^{(\mu\nu)} R_{(\mu\nu)} + c_3 R^{[\alpha\beta][\gamma\delta]} R_{[\alpha\beta][\gamma\delta]} \right)$$

$$W_{\text{em}} = -\frac{1}{16\pi c} \int (dx)(-g)^{1/2} F^{[\alpha\beta]} F_{[\alpha\beta]}$$

Gauß-Bonnet theorem, valid in 4 spacetime dimensions

QFT in BH spacetimes: Hawking radiation, because of horizons

Where is Mass coming from?

$$W_{(\text{Dirac})} = \int (dx)(-g)^{1/2} \left(i\hbar \bar{\Psi} \gamma^\mu (D_\mu + (e/c)A_\mu) \Psi - m(\bar{\Psi}\Psi) \right)$$

no Mass m but coupling to a Higgs scalar field Φ :

$$W_{(\text{Dirac})} = \int (dx) (-g)^{1/2} \left(i\hbar \bar{\Psi} \gamma^\mu (D_\mu + (e/c) A_\mu) \Psi - \Phi (\bar{\Psi} \Psi) \right)$$

vacuum expectation value: $\langle 0 | \Phi | 0 \rangle = m$ (sp. sym. breaking)

The vacuum state is globally defined: A kind of Mach's Principle

Similar procedure with Cosmological Constant:

$$W = \dots - m^2 \Phi^2$$

$\langle 0 | \Phi^2 | 0 \rangle = C$, thus $\Lambda \sim m^2 C$ vacuum energy density

Conformal Factor

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$m^2 c^2 = g^{\mu\nu} \left(\partial_\mu W + \frac{e}{c} A_\mu \right) \left(\partial_\nu W + \frac{e}{c} A_\nu \right)$$

for light propagation: $0 = g_{\mu\nu} dx^\mu dx^\nu$ and $g^{\mu\nu} (\partial_\mu W)(\partial_\nu W) = 0$

no fixation of Ψ in $\Psi g_{\mu\nu}$ or $(1/\Psi)g^{\mu\nu}$

Mass breaks conformal invariance!

Dissipation and Quantization

W and L shall be hermitean

Time Inversion: no unitary (canonical) symmetry transformation

$$W = \int_{-a}^{+a} dt L(t) \rightarrow \int_{+a}^{-a} dt L(t) = - \int_{-a}^{+a} dt L(t) = -W$$

Motion Reversion: unitary (canonical) symmetry transformation

$$W = \int_{-a}^{+a} dt L(t) \rightarrow \int_{-a}^{+a} dt L(-t) = \int_{-a}^{+a} dt L(t) = W$$

Time Inversion: anti-unitary symmetry transformation

$$iW = i \int_{-a}^{+a} dt L(t) \rightarrow \left(i \int_{+a}^{-a} dt L(t) \right)^\dagger = i \int_{-a}^{+a} dt L(t) = iW$$

Dissipation and Quantization:

The arrow of time and squeezed coherent states

Damped harmonic oscillator (A):

$$M\ddot{x} + R\dot{x} + \kappa x = 0$$

Bateman dual system (B):

$$M\ddot{y} - R\dot{y} + \kappa y = 0, \quad y = 0 \text{ (classically)}, \quad y \neq 0 \text{ (quantum noise)}$$

Hamiltonian:

$$H = H_0 + H_I$$

$$H_0 = \hbar\Omega(A^\dagger A - B^\dagger B), \quad H_I = i\hbar\Gamma(A^\dagger B^\dagger - AB), \quad [H_0, H_I] = 0$$

$$\Omega = \left(\frac{\kappa}{M} - \Gamma^2 \right)^{1/2}, \quad \Gamma = \frac{R}{2M}$$

$$|0\rangle = |n_A = 0, n_B = 0\rangle = |0\rangle \otimes |0\rangle$$

$$|0(t)\rangle = e^{-it\frac{H}{\hbar}}|0\rangle = e^{-it\frac{H_I}{\hbar}}|0\rangle = \frac{1}{\cosh(\Gamma t)} e^{\tanh(\Gamma t)A^\dagger B^\dagger}|0\rangle$$

$$\langle 0(t)|0(t)\rangle = 1$$

$$\langle 0(t)|0\rangle = \frac{1}{\cosh(\Gamma t)} \rightarrow 0 \quad \text{for } t \rightarrow \infty$$

QFT: inequivalent representations of CCR

$$|0(t)\rangle = \exp\left(-\frac{V}{(2\pi)^3} \int d^3k \ln \cosh(\Gamma_k t)\right) \exp\left(\int d^3k \tanh(\Gamma_k t) A_k^\dagger B_k^\dagger\right) |0\rangle$$

$$\langle 0(t)|0\rangle = \exp\left(-\frac{V}{(2\pi)^3} \int d^3k \ln \cosh(\Gamma_k t)\right) \rightarrow 0 \quad \text{for } V \rightarrow \infty \quad \text{for all(!) } t$$

$|0(t)\rangle$ is a two-mode time-dependent generalized coherent state, where the modes A and B are entangled.

squeezed state modes:

$$\alpha_k = (A_k + B_k)/\sqrt{2}, \quad \beta_k = (A_k - B_k)/\sqrt{2}$$

squeezing parameter: $\theta_k = \Gamma_k t$

$$\alpha_k = X_k + iY_k, \quad [X_k, Y_k] = i/2$$

$$\Delta X_k^2(\theta_k) = \Delta X_k^2 \exp(2\theta_k), \quad \Delta Y_k^2(\theta_k) = \Delta Y_k^2 \exp(-2\theta_k)$$

Conclusions

General Relativity is the fundamental theory of gravity

Research in Space complements research on Earth substantially

Quantum Field Theory is crucial for comprehending Nature

Testing and applying QFT in Space goes to its extremes:

quantum processes under weakest noise conditions

long-range space-time quantum correlations of large objects

quantum correlations in curved spacetime

Literature:

M. Blasone, P. Jizba and G. Vitiello, *Quantum Field Theory and its Macroscopic Manifestations*, Imperial College Press, London 2011

References added:

J. Schwinger, *Sources and Gravitons*, Phys. Rev. 173, 1264 (1968)

B.S. DeWitt, *Dynamical Theory of Groups and Fields*, as book by Gordon and Breach, New York 1965, or as quite a long article in “Relativity, Groups and Topology”, C. DeWitt and B. DeWitt, Gordon and Breach, New York 1964 ([one of the most beneficial publications in theoretical fundamental physics](#))

W.D. Goldberger and I.Z. Rothstein, *An Effective Field Theory for Gravity for Extended Objects*, PRD 73, 104029 (2006) (also see arXiv)