Shadows of black holes

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Bremen Drop Tower

Height of tower: 146 m
Free fall distance: 110 m
Free fall time: 4.7 s
With catapult: 9.3 s
Outline of talk

- Shadows of Schwarzschild black holes
- Shadows of other spherically symmetric objects
- Shadows of rotating black holes
- Perspective of observations

General reference for background material:

**Schwarzschild black hole**

\[
g_{\mu\nu} dx^{\mu} dx^{\nu} = - \left(1 - \frac{2m}{r}\right) c^2 dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right)
\]

\[
m = \frac{GM}{c^2}
\]

**Horizon:**
\[
r = 2m
\]

**Light sphere (photon sphere):**
\[
r = 3m
\]
Schwarzschild black hole

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\]

\[
m = \frac{GM}{c^2}
\]

Horizon:
\[
r = 2m
\]

Light sphere (photon sphere)
\[
r = 3m
\]
Angular radius $\alpha$ of the “shadow” of a Schwarzschild black hole:

$$\sin^2 \alpha = \frac{27 m^2}{r_O^2} \left(1 - \frac{2m}{r_O}\right)$$


$r_O = 1.05 r_S$  $r_O = 1.3 r_S$  $r_O = 3 r_S/2$  $r_O = 2.5 r_S$  $r_O = 6 r_S$
Schwarzschild black hole produces infinitely many images:
Visual appearance of a Schwarzschild black hole
Other spherically symmetric and static black holes:

- Reissner-Nordström
- Kottler (Schwarzschild-(anti)deSitter)
- Janis-Newman-Winicour
- Newman-Unti-Tamburino (NUT)
- Black holes from nonlinear electrodynamics
- Black holes from higher dimensions, braneworld scenarios, …

All of them have an unstable photon sphere \( \Rightarrow \) Qualitative lensing features are similar to Schwarzschild

Quantitative features (ratio of angular separations of images, ratio of fluxes of images) are different, see V. Bozza: Phys. Rev. D 66, 103001 (2002)

The shadow is always circular. Its angular radius depends on \( r_o \) and the parameters of the black hole.
Black hole impostor: Ellis wormhole


\[ g = -c^2 dt^2 + dr^2 + (r^2 + a^2) \left( d\theta^2 + \sin^2 \theta \, d\varphi^2 \right) \]

Angular radius \( \alpha \) of shadow: \( \sin^2 \alpha = \frac{a^2}{r_O^2 + a^2} \)
Black hole impostor: Ultracompact star

Uncharged dark star with radius between $2m$ and $3m$

Lensing features indistinguishable from Schwarzschild black hole

It seems to be that ultracompact objects are necessarily unstable, see V. Cardoso, L. Crispino, C. Macedo, H. Okawa, P. Pani: Phys. Rev. D 90, 044069 (2014)
Influence of a plasma on the shadow:


Schwarzschild spacetime, plasma frequency \( \omega_p(r)^2 = \beta_0 \omega_0^2 \left( \frac{GM}{c^2 r} \right)^{3/2} \)

\[ a : r_O = 3.3 \frac{GM}{c^2} \]
\[ b : r_O = 3.8 \frac{GM}{c^2} \]
\[ c : r_O = 5 \frac{GM}{c^2} \]
\[ d : r_O = 10 \frac{GM}{c^2} \]
\[ e : r_O = 50 \frac{GM}{c^2} \]
Rotating black holes:

Shadow no longer circular

Shape of shadow can be used for discriminating between different black holes

Shape of the shadow of a Kerr black hole for observer at infinity:


Shape and size of the shadow for black holes of the Plebański-Demiański class for observer at coordinates \((r_O, \vartheta_O)\) (analytical formulas):

Kerr metric in Boyer–Lindquist coordinates \((r, \vartheta, \varphi, t)\):

\[
g_{\mu\nu}dx^\mu dx^\nu = g(r, \vartheta)^2 \left( \frac{dr^2}{\Delta(r)} + d\vartheta^2 \right) + \frac{\sin^2 \vartheta}{g(r, \vartheta)^2} \left( a\,dt - (r^2 + a^2)d\varphi \right)^2
\]

\[
- \frac{\Delta(r)}{g(r, \vartheta)^2} \left( dt - a \sin^2 \vartheta \, d\varphi \right)^2
\]

\[
g(r, \vartheta)^2 = r^2 + a^2 \cos^2 \vartheta, \quad \Delta(r) = r^2 - 2mr + a^2.
\]

\[
m = \frac{GM}{c^2} \text{ where } M = \text{mass}, \quad a = \frac{J}{Mc} \text{ where } J = \text{spin}
\]

Plebański-Demiański black holes: Additional parameters

\[
q_e = \text{el. charge}, \quad q_m = \text{magn. charge}, \quad \ell = \text{NUT parameter},
\]

\[
\Lambda = \text{cosmol. constant}, \quad \alpha = \text{acceleration}
\]

Consider in the following only the Kerr metric
Lightlike geodesics:

\[ g(r, \vartheta)^2 \dot{t} = a \left( L - Ea \sin^2 \vartheta \right) + \frac{\left( r^2 + a^2 \right) \left( (r^2 + a^2)E - aL \right)}{\Delta(r)}, \]

\[ g(r, \vartheta)^2 \dot{\varphi} = \frac{L - Ea \sin^2 \vartheta}{\sin^2 \vartheta} + \frac{(r^2 + a^2)aE - a^2L}{\Delta(r)}, \]

\[ g(r, \vartheta)^4 \dot{\vartheta}^2 = K - \frac{(L - Ea \sin^2 \vartheta)^2}{\sin^2 \vartheta} =: \Theta(\vartheta), \]

\[ g(r, \vartheta)^4 r^2 = -K\Delta(r) + \left( (r^2 + a^2)E - aL \right)^2 =: R(r). \]

Spherical lightlike geodesics exist in the region where

\[ R(r) = 0, \quad R'(r) = 0, \quad \Theta(\vartheta) \geq 0. \]

\[ (2r\Delta(r) - (r - m) g(r, \vartheta)^2)^2 \leq 4a^2r^2\Delta(r) \sin^2 \vartheta \]

(unstable if \( R''(r) \geq 0 \))
Photon region for Kerr black hole with $a = 0.75 \, m$
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The shadow is determined by light rays that approach an unstable spherical lightlike geodesic.

Choose observer at \( r_0 \) and \( \vartheta_0 \)

Choose tetrad

\[
e_0 = \frac{(r^2 + a^2) \partial_t + a \partial_\varphi}{\varrho(r, \vartheta) \sqrt{\Delta(r)}} \bigg|_{(r_0, \vartheta_0)}
\]

\[
e_1 = \frac{1}{\varrho(r, \vartheta)} \partial_\vartheta \bigg|_{(r_0, \vartheta_0)}
\]

\[
e_2 = -\frac{(\partial_\varphi + a \sin^2 \vartheta \partial_t)}{\sqrt{\varrho(r, \vartheta)^2 \sin \vartheta}} \bigg|_{(r_0, \vartheta_0)}
\]

\[
e_3 = -\frac{\sqrt{\Delta(r)}}{\varrho(r, \vartheta)} \partial_r \bigg|_{(r_0, \vartheta_0)}
\]

Observer with other 4-velocity: Aberration

celestial coordinates at observer \((\theta, \psi)\)

direction towards black hole

observer at \((r_O, \vartheta_O)\)

light ray tangent
constants of motion \( (K_E = \frac{K}{E^2}, L_E = \frac{L}{E} - a) \)

\[
\sin \theta = \frac{\sqrt{\Delta(r)} \ K_E}{r^2 - aL_E} \bigg|_{r=r_O} \quad \sin \psi = \frac{L_E + a \cos^2 \vartheta + 2 \ell \cos \vartheta}{\sqrt{K_E} \sin \vartheta} \bigg|_{\vartheta=\vartheta_O}
\]

\[
K_E = \frac{16r^2\Delta(r)}{(\Delta'(r))^2} \bigg|_{r=r_p} \quad aL_E = \left( r^2 - \frac{4r\Delta(r)}{\Delta'(r)} \right) \bigg|_{r=r_p}
\]

Analytic formula for shadow allows to extract parameters of the spacetime from the shape of the shadow.

Vertical angular radius \( \alpha_v \) of the shadow:

\[
\sin^2 \alpha_v = \frac{27m^2r_O^2(a^2 + r_O(r_O - 2m))}{r_O^6 + 6a^2r_O^4 + 3a^2(4a^2 - 9m^2)r_O^2 + 8a^6} = \frac{27m^2}{r_O^2} \left( 1 + O(m/r_O) \right)
\]

Up to terms of order \( O(m/r_O) \), Synge’s formula is still correct for the vertical diameter of the shadow.
Shadow of black hole with $a = m$ for observer at $r_O = 5m$
Perspectives of observations

Object at the centre of our galaxy:
Mass = $4 \times 10^6 \, M_\odot$
Distance = 8 kpc
Synge’s formula gives for the diameter of the shadow $\approx 54 \mu\text{as}$
(corresponds to a grapefruit on the moon)

Object at the centre of M87:
Mass = $3 \times 10^9 \, M_\odot$
Distance = 16 Mpc
Synge’s formula gives for the diameter of the shadow $\approx 9 \mu\text{as}$
Kerr shadow with emission region and scattering taken into account:

no scattering \[ \lambda = 0.6 \text{ mm} \]
\[ \lambda = 1.3 \text{ mm} \]


Observations should be done at sub-millimeter wavelengths
T. Müller (2012)
from the Movie “Interstellar” (2014)
Projects to view the shadow with sub-millimeter VLBI:

**Event Horizon Telescope (EHT),**

Using ALMA, NOEMA, LMT, CARMA, South Pole Telescope …
BlackHoleCam

H. Falcke, L. Rezzolla, M. Kramer
Millimetron ($\approx 2025$)

Good chance to see the shadow of the centre of our galaxy within a few years