Influence of a plasma on gravitational lensing by compact objects

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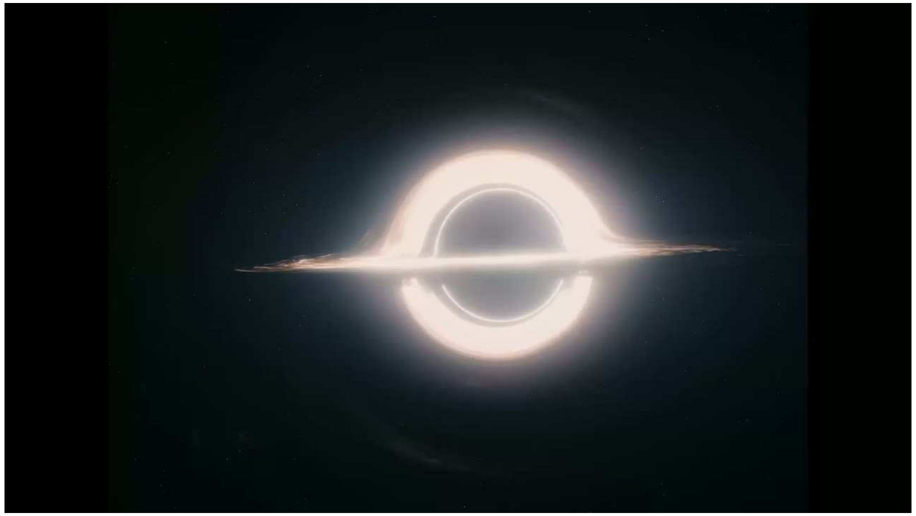


from the movie "Interstellar"

DPG Conference, Hamburg, 3 March 2016

Goal: Analytic treatment of light propagation in a plasma on general-relativistic spacetimes

1. Light propagation in a non-magnetised, pressure-free plasma [1]
2. Light deflection in a plasma on a spherically symmetric and static spacetime [2], in particular on Schwarzschild spacetime [1]
3. Influence of a plasma on the shadow of spherically symmetric compact objects [2]
4. Light deflection and shadow in a plasma on Kerr spacetime [3]

[1] VP: "Ray optics, Fermat's principle and applications to general relativity" Springer (2000)

[2] VP, O. Yu. Tsupko, G. S. Bisnovatyi-Kogan: "Influence of a plasma on the shadow of a spherically symmetric black hole" Phys. Rev. D 92, 104031 (2015)

[3] VP, O. Yu. Tsupko, G. S. Bisnovatyi-Kogan: in preparation

Also see talk by Karen Schulze-Koops at 3:35 today!

Light rays on a general-relativistic spacetime with metric $g_{ik}(x)$:

$$\dot{x}^i = \frac{\partial H(x,p)}{\partial p_i}\,, \qquad \dot{p}_i = -\frac{\partial H(x,p)}{\partial x^i}\,, \qquad H(x,p) = 0$$

In vacuo:

$$H(x,p) \; = \; \frac{1}{2} g^{ik}(x) p_i p_k$$

Light rays are lightlike geodesics of the spacetime metric g_{ik}

In a non-magnetised pressure-free plasma:

$$H(x,p) \; = \; \frac{1}{2}\Big(g^{ik}(x) p_i p_k + \omega_p(x)^2 \Big)\,,$$

plasma frequency: $\omega_p(x)^2 = \frac{e^2}{\varepsilon_0 m_e} N(x)$

e: charge of the electron, $\quad m_e$: mass of the electron
$N(x)$: number density of the electrons

Light rays are timelike geodesics of the conformally rescaled metric $\omega_p^{-2} g_{ik}$

Rigourous derivation from Maxwell's equation, even for magnetised pressure-free plasma:

R. Breuer, J. Ehlers: Proc. Roy. Soc. London, A 370, 389 (1980), A 374, 65 (1981)

for non-magnetised pressure-free plasma:

VP: "Ray Optics, Fermat's Principle and Applications to General Relativity" Springer (2000)

A plasma is a dispersive medium; propagation of light rays depend on the frequency $\omega = -p_i U^i$

For a cold non-magnetised plasma, only the plasma frequency matters, not the 4-velocity of the electrons

Light rays are characterised by a Lorentz invariant index of refraction

$$n(x,\omega)^2 = 1 - \frac{\omega_p(x)^2}{\omega^2}\,.$$

J. Synge: "Relativity: The General Theory", North-Holland (1960)

Spherically symmetric and static case

$$g_{ik}(x)dx^i dx^k = -A(r)dt^2 + B(r)dr^2 + D(r)\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)$$

$$H(x,p) = \frac{1}{2}\left(g^{ik}(x)p_i p_k + \omega_p(r)^2\right)$$

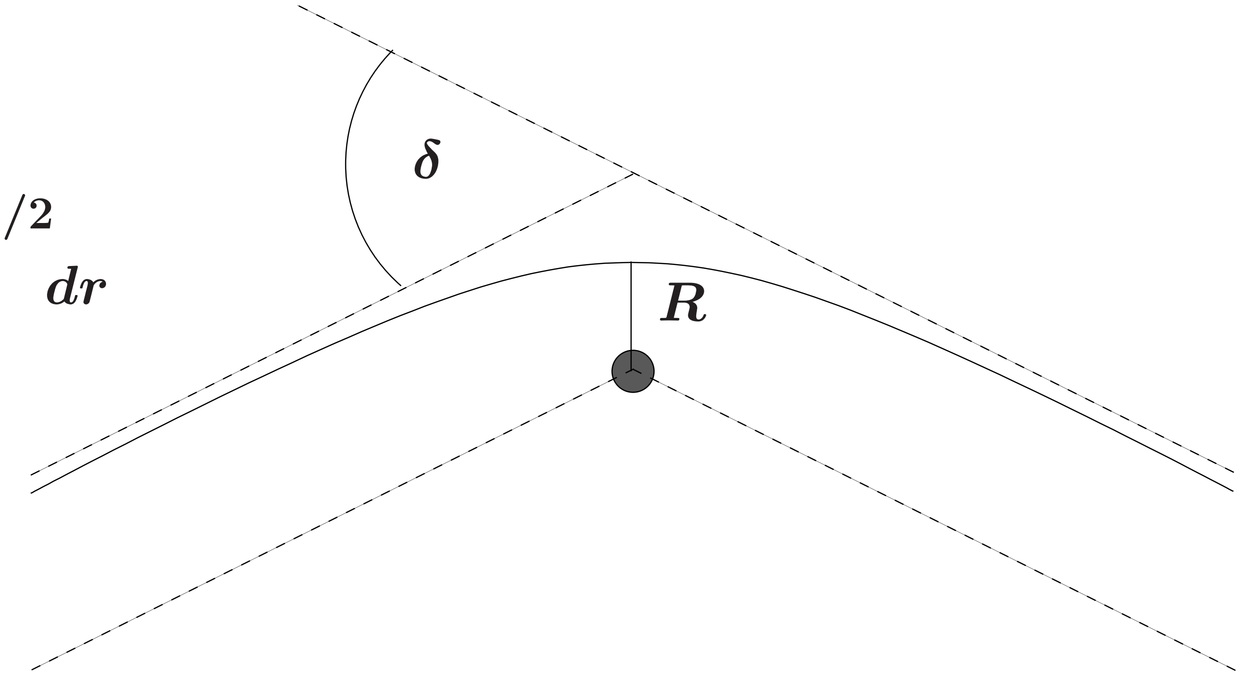
With constant of motion $\omega_0 = -p_t$, define

$$h(r)^2 = \frac{D(r)}{A(r)}\left(1 - A(r)\frac{\omega_p(r)^2}{\omega_0^2}\right)$$

Deflection angle

$$\pi + \delta =$$

$$2\int_R^\infty \frac{\sqrt{B(r)}}{\sqrt{D(r)}}\left(\frac{h(r)^2}{h(R)^2} - 1\right)^{-1/2} dr$$



VP, O. Yu. Tsupko, G. S. Bisnovatyi-Kogan: Phys. Rev. D 92, 104031 (2015)

Schwarzschild spacetime:

$$A(r) = B(r)^{-1} = 1 - \frac{2m}{r}\,, \quad D(r) = r^2\,, \quad m = \frac{GM}{c^2}$$

$$\pi + \delta = 2\int_R^\infty \left(\frac{r^2\left(\frac{r}{r-2m} - \frac{\omega_p(r)^2}{\omega_0^2}\right)}{R^2\left(\frac{R}{R-2m} - \frac{\omega_p(R)^2}{\omega_0^2}\right)} - 1 \right)^{-1/2} \frac{dr}{\sqrt{r}\sqrt{r-2m}}$$

In the weak-field approximation:

D. O. Muhleman and I. D. Johnston: Phys. Rev. Lett. 17, 455 (1966)

Exact formula:

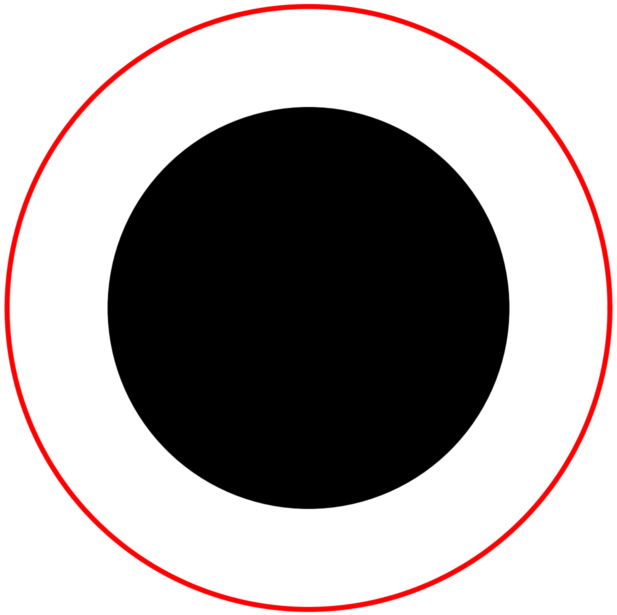
VP: “Ray optics, Fermat’s principle and applications to general relativity” Springer (2000)

Astrophysical applications:

O. Yu. Tsupko and G. S. Bisnovatyi-Kogan: Phys. Rev. D 87, 124009 (2013)
X. Er and S. Mao: Mon. Not. Roy. Astron. Soc. 437, 2180 (2013)
A. Rogers: Mon. Not. Roy. Astron. Soc. 451 4536 (2015)

Effect of a plasma on the shadow

Recall: Shadow in vacuum of a Schwarzschild black hole



Horizon:

$$r_S = \frac{2GM}{c^2} = 2m$$

Light sphere (photon sphere)

$$\frac{3}{2} r_S = \frac{3GM}{c^2} = 3m$$

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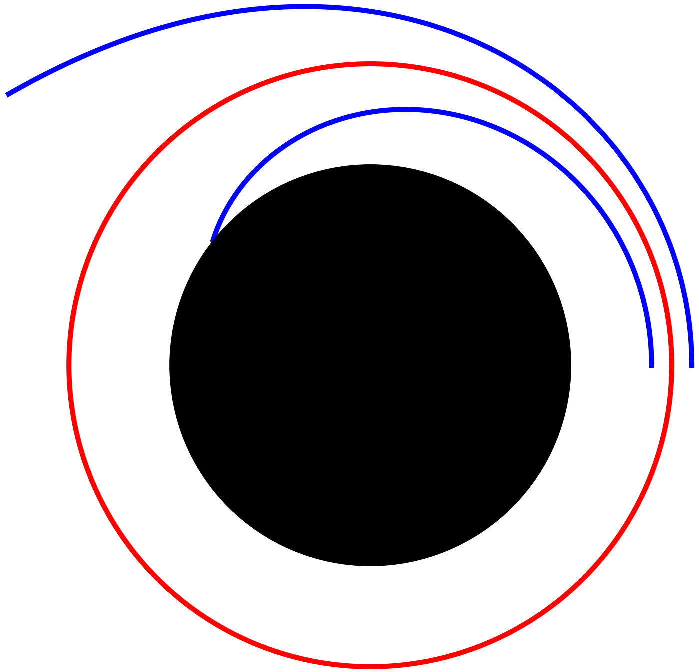

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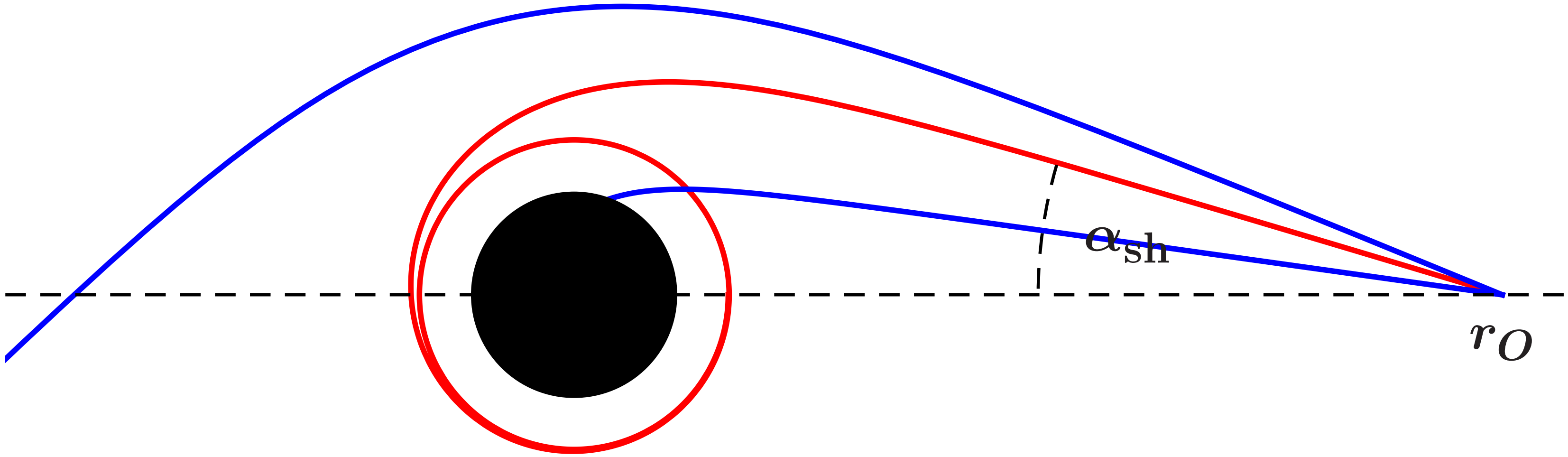
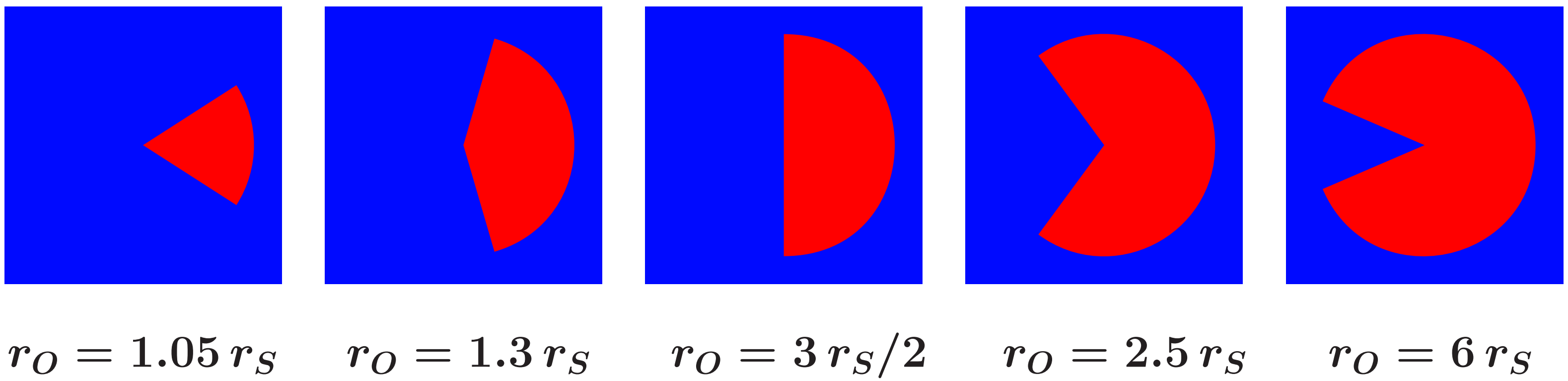


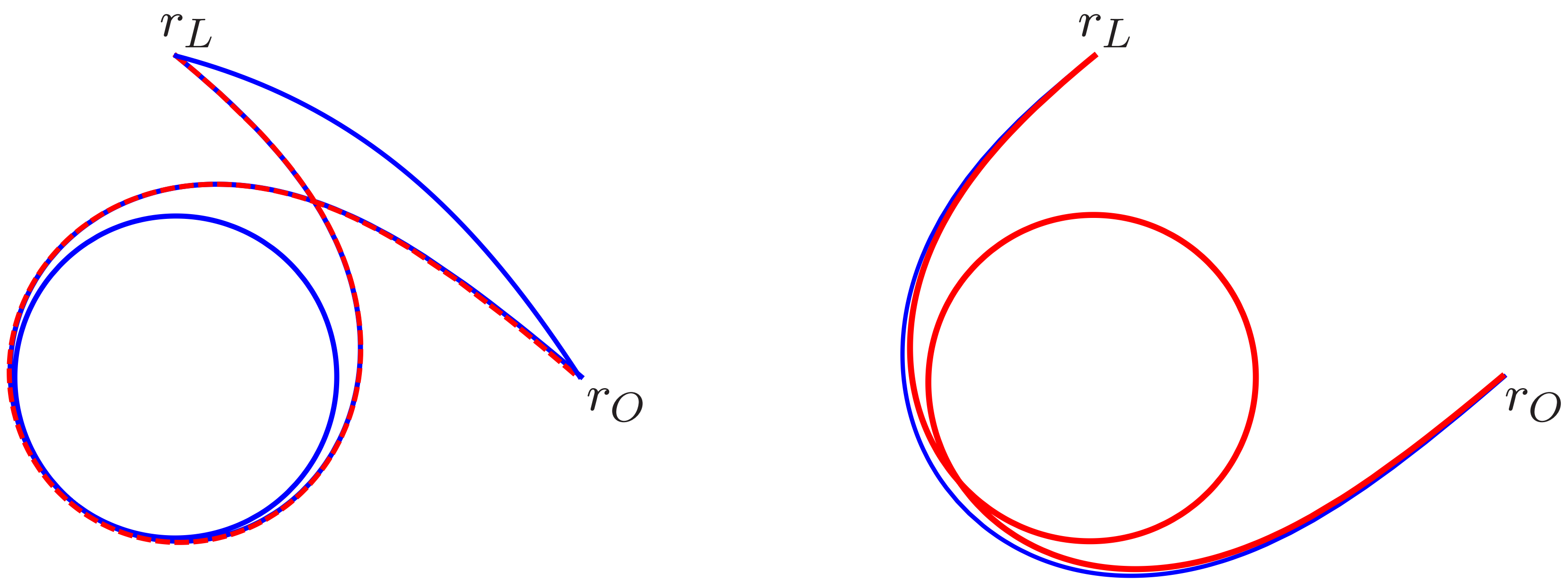
Angular radius $\alpha_{\rm sh}$ of the "shadow" of a Schwarzschild black hole:

$$\sin^2\alpha_{\rm sh} = \frac{27\, r_S^2\, (r_O - r_S)}{4\, r_O^3} = \frac{27\, m^2}{r_O^2}\Big(1 - \frac{2m}{r_O}\Big)$$

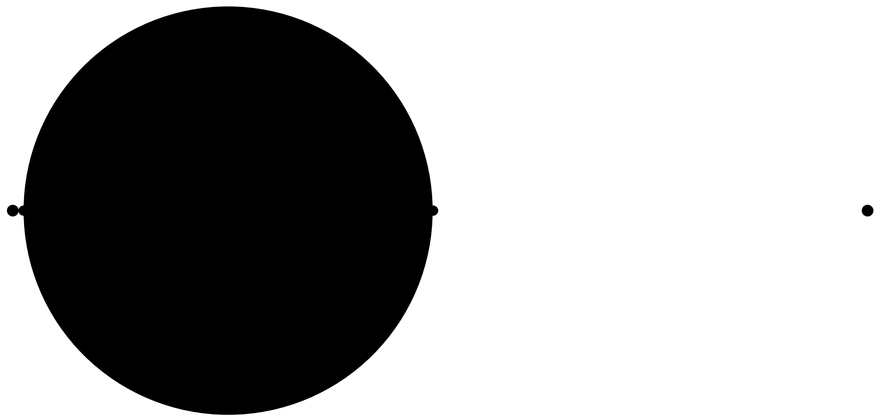
J. L. Synge, Mon. Not. R. Astr. Soc. 131, 463 (1966)



Schwarzschild black hole produces infinitely many images:



Imaging of a point source by a Schwarzschild black hole



Perspectives of observations

Object at the centre of our galaxy:

Mass $= 4 \times 10^6 \, M_\odot$

Distance $= 8 \, \mathrm{kpc}$

Angular diameter of the shadow by Synge's formula $\approx 54\mu$as

(corresponds to a grapefruit on the moon)

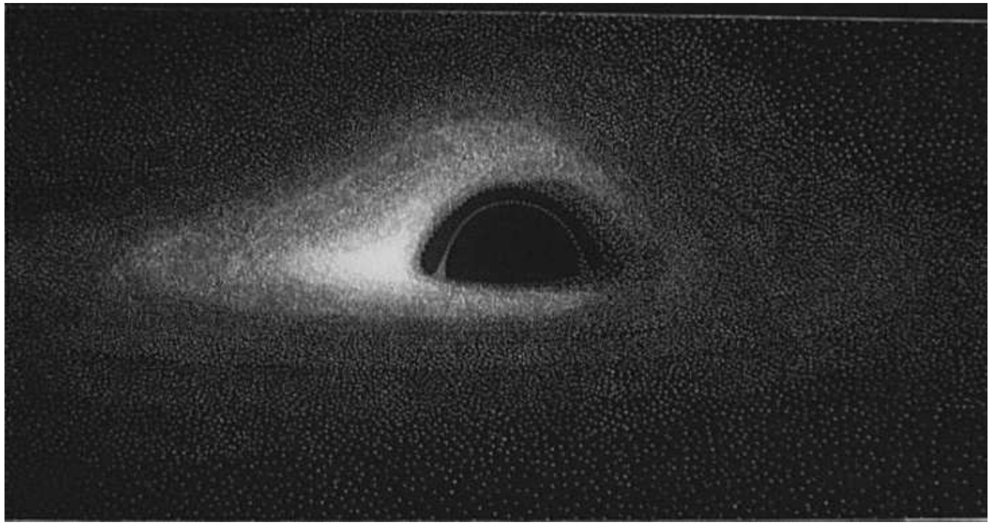
Object at the centre of M87:

Mass $= 3 \times 10^9 \, M_\odot$

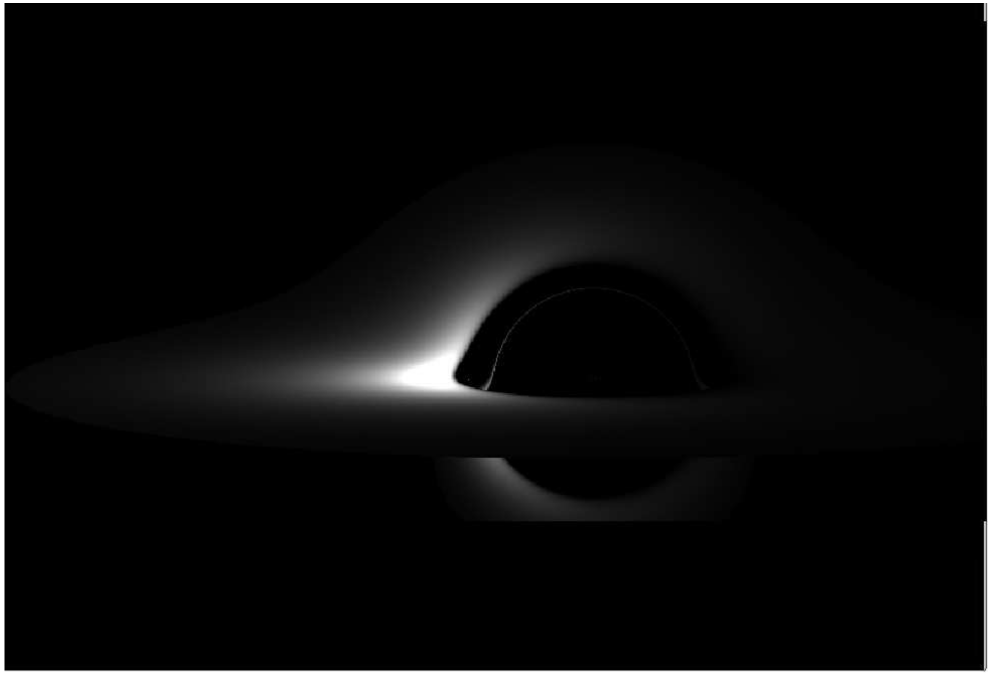
Distance $= 16 \, \mathrm{Mpc}$

Angular diameter of the shadow by Synge's formula $\approx 20\mu$as

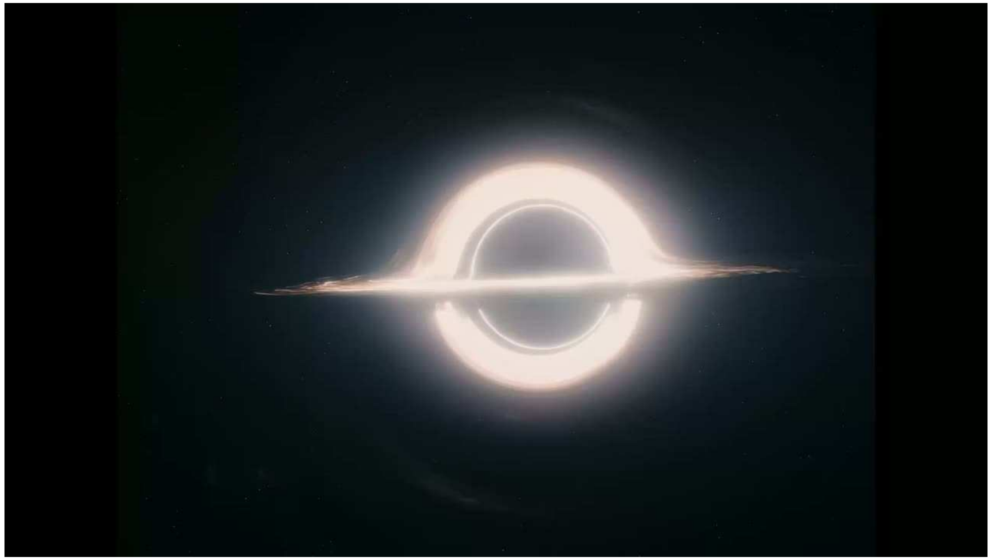
Perhaps observable soon with VLBI (Event Horizon Telescope, BlackHoleCam)



J.-P. Luminet (1979)



T. Müller (2012)

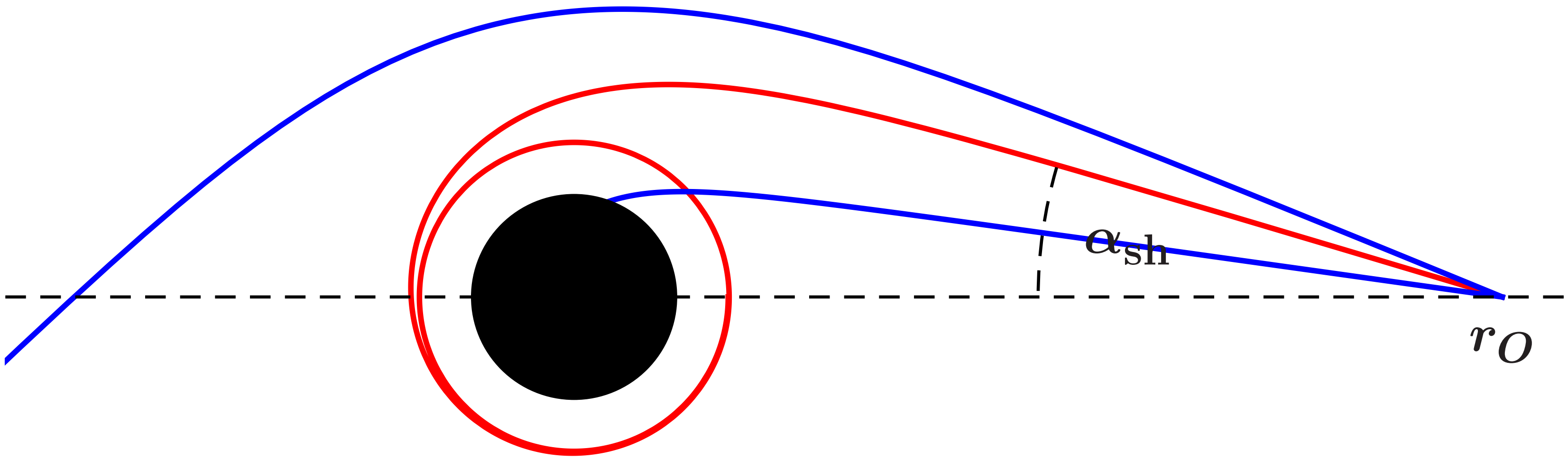


Interstellar (2014)

Plasma on spherical symmetric and static spacetime

Condition for photon sphere:

$$\frac{d}{dr} h(r)^2 \Big|_{r=r_{\rm ph}} = 0\,, \quad h(r)^2 = \frac{D(r)}{A(r)} \left(1 - A(r) \frac{\omega_p(r)^2}{\omega_0^2} \right)$$



Angular radius $\alpha_{\rm sh}$ of shadow:

$$\sin^2 \alpha_{\rm sh} = \frac{h(r_{\rm ph})^2}{h(r_{\rm O})^2}$$

VP, O. Yu. Tsupko, G. S. Bisnovatyi-Kogan: Phys. Rev. D 92, 104031 (2015)

Example 1: Accretion of dust onto Schwarzschild

$$g = -\Big(1 - \frac{2m}{r}\Big) dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} + r^2 \Big(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2 \Big) , \quad \frac{\omega_p(r)^2}{\omega_0^2} = \beta_0 \left(\frac{m}{r} \right)^{3/2}$$

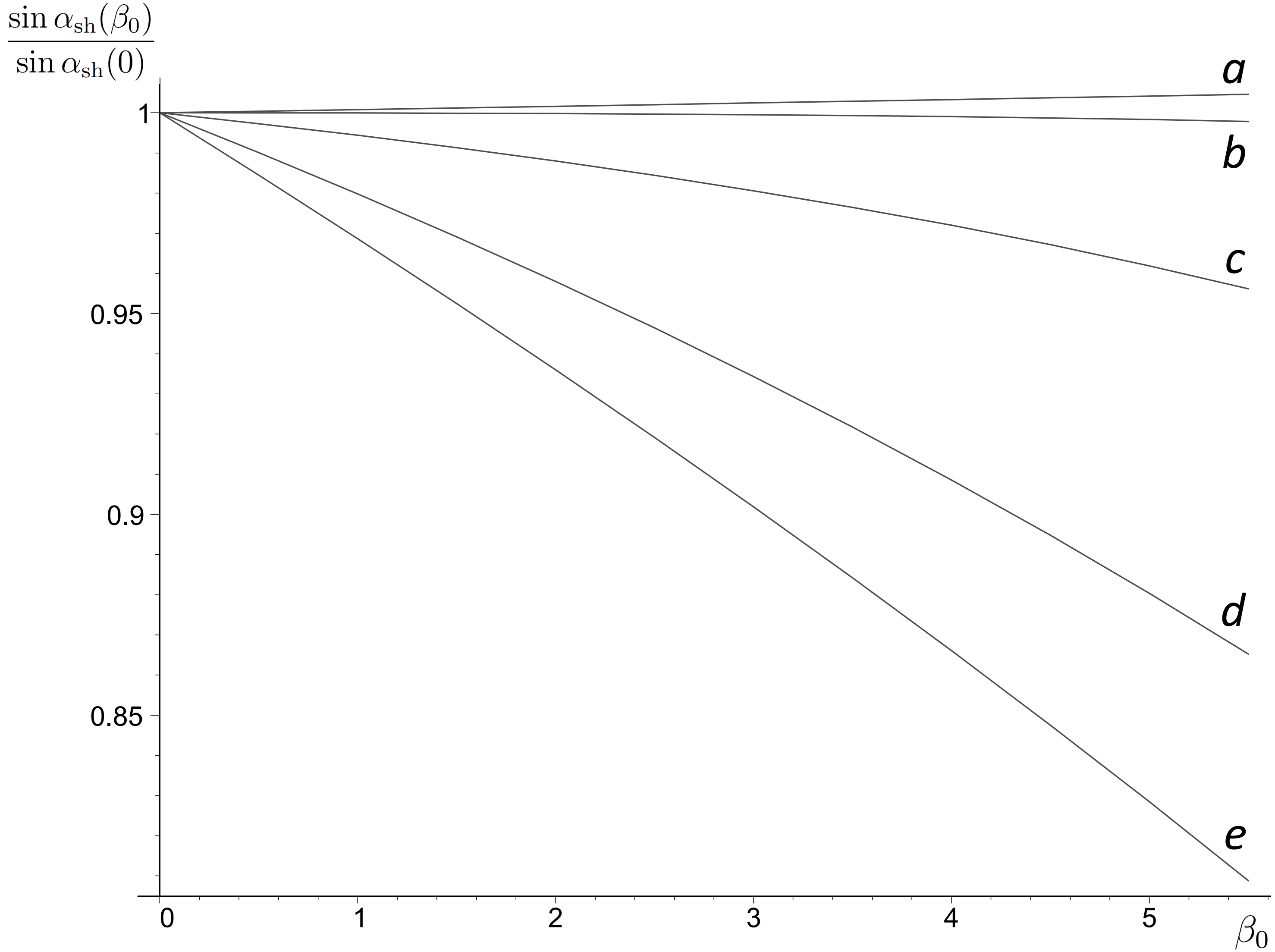
$a : r_O = 3.3\, m$

$b : r_O = 3.8\, m$

$c : r_O = 5\, m$

$d : r_O = 10\, m$

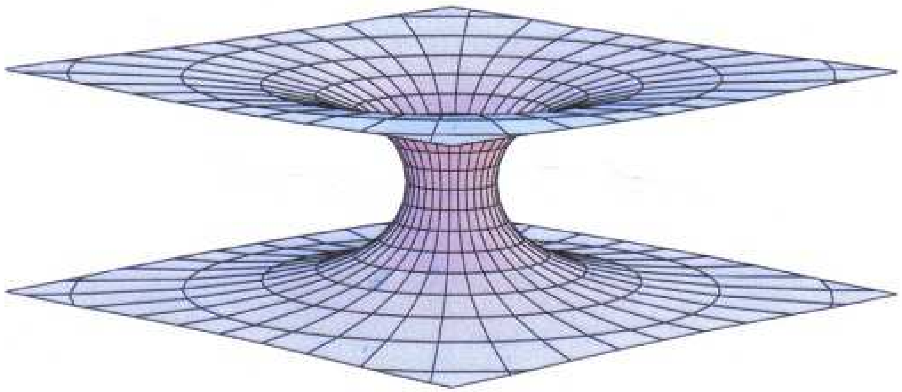
$e : r_O = 50\, m$



Example 2: Ellis wormhole

$$g = -dt^2 + dr^2 + \left(r^2 + a^2\right)\left(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2\right)$$

H. G. Ellis, J. Math. Phys. 14, 104 (1973)



Condition for photon sphere:

$$r\left(1 - \frac{\omega_p(r)^2}{\omega_0^2}\right) = \left(r^2 + a^2\right)\frac{\omega_p(r)\omega_p'(r)}{\omega_0^2}$$

Angular radius of shadow:

$$\sin^2\alpha_{\mathrm{sh}} = \frac{\left(r_{\mathrm{ph}}^2 + a^2\right)}{\left(r_{\mathrm{O}}^2 + a^2\right)}\frac{\left(\omega_0^2 - \omega_p(r_{\mathrm{ph}})^2\right)}{\left(\omega_0^2 - \omega_p(r_{\mathrm{O}})^2\right)}$$

Homogeneous plasma $(\omega_p(r) = \mathrm{const.})$:

$$r_{\mathrm{ph}} = 0 \, , \quad \sin^2\alpha_{\mathrm{sh}} = \frac{a^2}{r_{\mathrm{O}}^2 + a^2}$$

Light propagation in a plasma on Kerr spacetime

$$g_{ik}dx^i dx^k = \varrho(r,\vartheta)^2 \left(\frac{dr^2}{\Delta(r)} + d\vartheta^2 \right) + \frac{\sin^2\vartheta}{\varrho(r,\vartheta)^2} \Big(a\, dt - (r^2 + a^2) d\varphi \Big)^2$$

$$- \frac{\Delta(r)}{\varrho(r,\vartheta)^2} \Big(dt - a \sin^2\vartheta\, d\varphi \Big)^2$$

$$\varrho(r,\vartheta)^2 = r^2 + a^2 \cos^2\vartheta, \qquad \Delta(r) = r^2 - 2mr + a^2 \,.$$

$m = \dfrac{GM}{c^2}$ where M =mass , $\quad a = \dfrac{J}{Mc}$ where J =spin

Bending angle for light rays in the equatorial plane for an r dependent plasma frequency:

VP: "Ray optics, Fermat's principle and applications to general relativity" Springer (2000)

Light deflection of a slowly rotating Kerr black hole with constant plasma density:

V. Morozova, B. Ahmedov, and A. Tursunov, Astrophys. Space Sci. 346, 513 (2013)

Still to be determined:

Bending of light in the general case

The shadow of a Kerr black hole in a plasma

Recall: Shadow of a Kerr black hole in vacuum

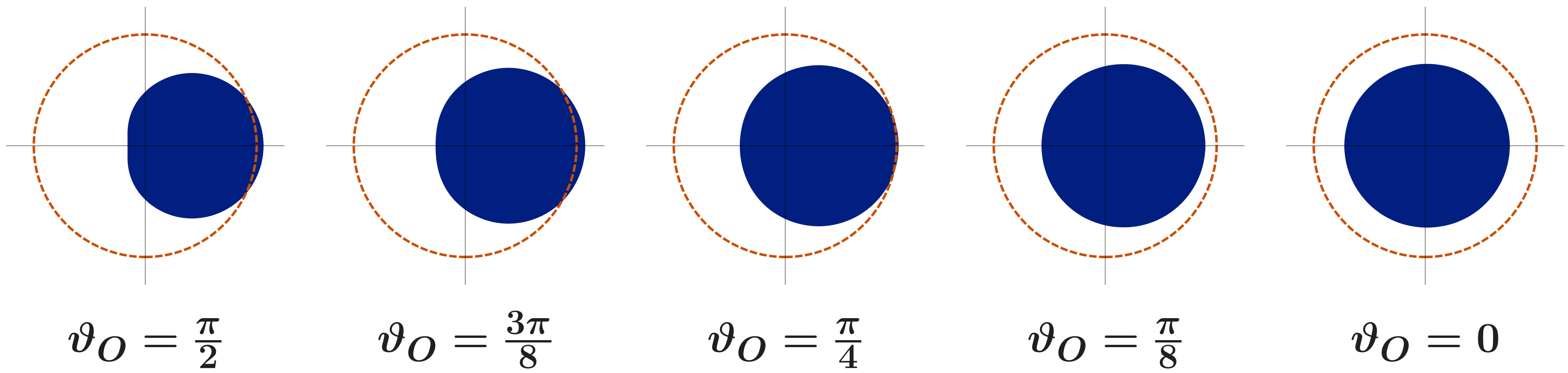
Shape of the shadow of a Kerr black hole for observer at infinity:

J. Bardeen in C. DeWitt and B. DeWitt (eds.): "Black holes" Gordon & Breach (1973)

Shape and size of the shadow of Kerr black holes (and other black holes) for observer at coordinates (r_O, ϑ_O) (analytical formulas):

A. Grenzebach, VP, C. Lämmerzahl: Phys. Rev. D 89, 124004 (2014), Int. J. Mod. Phys. D 24, 1542024 (2015)

Shadow of black hole with $a = m$ for observer at $r_O = 5m$

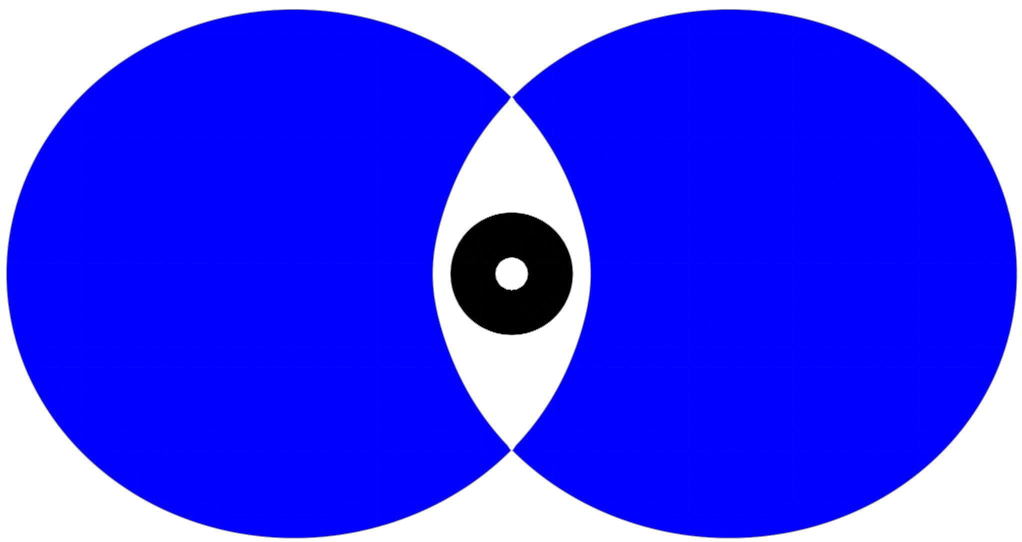


Analytical formula for the shadow is based on the following facts:

The equation for lightlike geodesics is completely integrable (because of the Carter constant)

There is a "photon region" filled with spherical lightlike geodesics (which generalises the photon sphere in Schwarzschild at $r = 3m$)

Photon region for $a = 0.75\, m$



Boundary curve of shadow corresponds to light rays that approach a spherical lightlike geodesic

Analytical formula for the shadow follows from identifying constants of motion of a light ray with the constants of motion of its limit curve

This gives, in particular, a formula for the vertical angular radius α_v of the shadow:

$$\sin^2 \alpha_v = \frac{27m^2 r_O^2 \big(a^2 + r_O(r_O - 2m)\big)}{r_O^6 + 6a^2 r_O^4 + 3a^2(4a^2 - 9m^2) r_O^2 + 8a^6}$$

$$= \frac{27m^2}{r_O^2} \Big(1 + O(m/r_O)\Big) \, , \qquad \vartheta_O = \frac{\pi}{2}$$

Up to terms of order $O(m/r_O)$, Synge's formula is still correct for the vertical diameter of the shadow

Generalisation to the plasma case requires:

Find out for which form of the plasma frequency $\omega_p(r, \vartheta)$ a generalised Carter constant exists

Determine, for these cases, the photon region

Derive, thereupon, an analytical formula for the shadow

VP, O. Yu. Tsupko, G. S. Bisnovatyi-Kogan: work in progress