# Gravitational lensing by black holes

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Topics to be discussed:

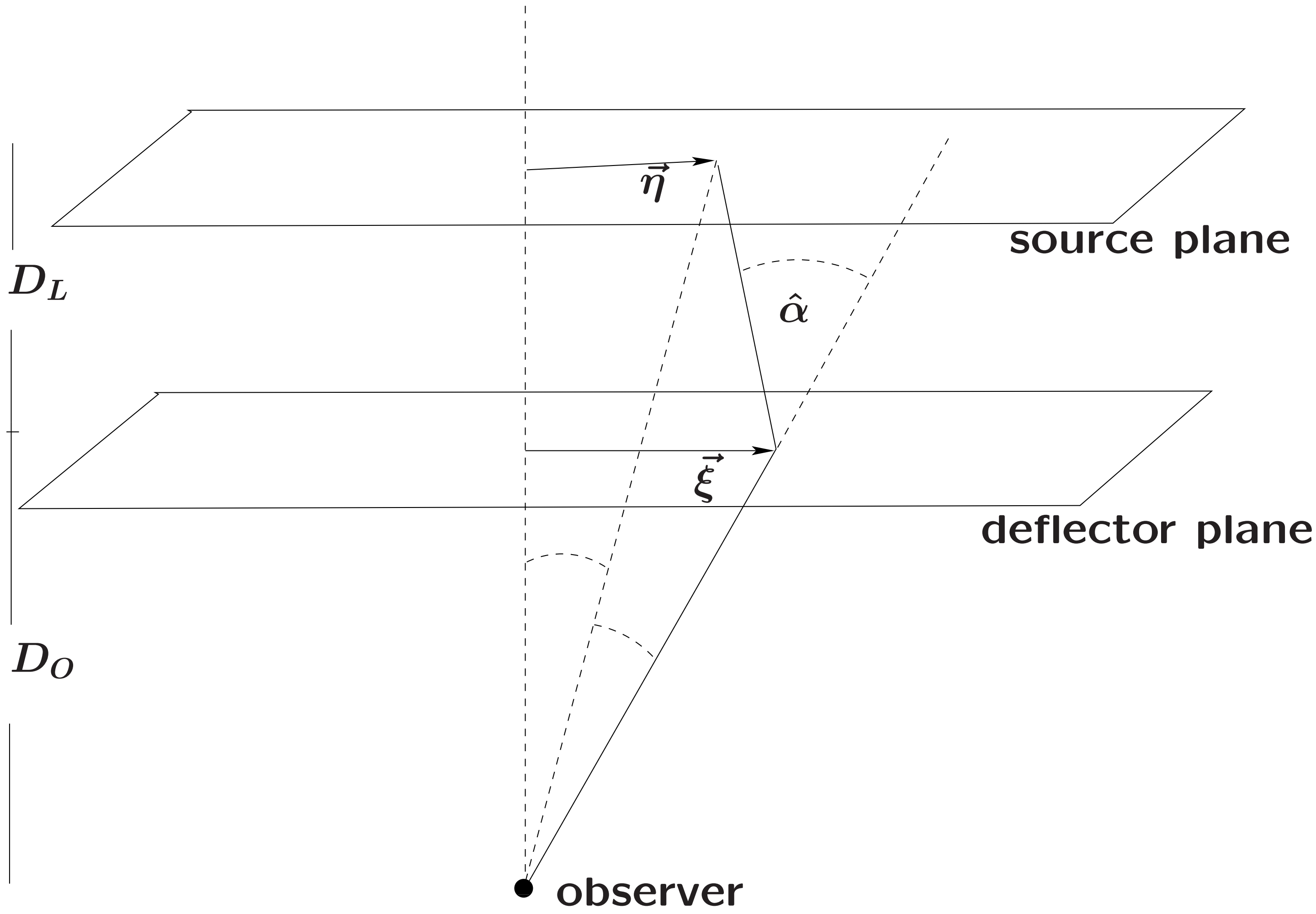
- Lensing features of black holes (multiple imaging, shadows)
- Distinguishing Schwarzschild and Kerr black holes from non-standard black holes
- Distinguishing black holes from other compact objects ("black hole imposters")

Organisation of the talk:

1. Spherically symmetric and static black holes
2. Rotating black holes

VP: "Gravitational Lensing from a Spacetime Perspective", Living Rev. Relativity 7, (2004), http://www.livingreviews.org/lrr-2004-9

## Lens map of the weak-field formalism (S. Refsdal, 1963)



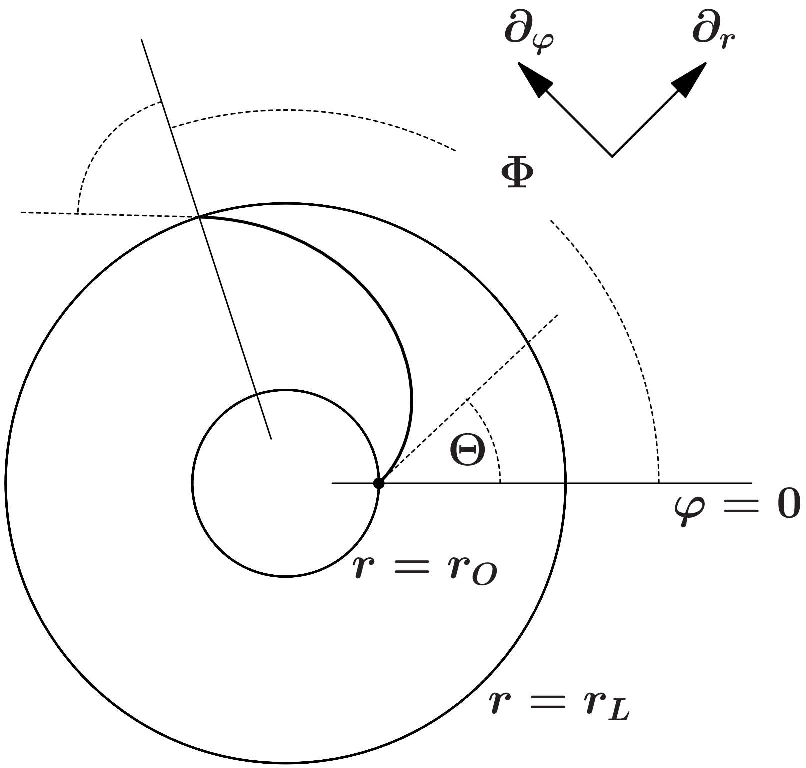

$$\vec{\eta} = \frac{D_L + D_O}{D_O}\,\vec{\xi} - D_L\,\vec{\hat{\alpha}}\,, \qquad \vec{\hat{\alpha}} = \frac{4G}{c^2}\int_{\mathbb{R}^2}\frac{(\vec{\xi}-\vec{\xi'})}{\left|\vec{\xi}-\vec{\xi'}\right|^2}\Sigma(\vec{\xi'})\,d^2\vec{\xi'}\,.$$

## 1. Spherically symmetric and static black holes

Exact lens map for spherically symmetric and static spacetimes

VP: Phys. Rev. D 69, 064917 (2004)

$$g = e^{2f(r)} \Big( - c^2 dt^2 + S(r)^2 dr^2 + R(r)^2 \big( d\vartheta^2 + \sin^2\vartheta \, d\varphi^2 \big) \Big)$$

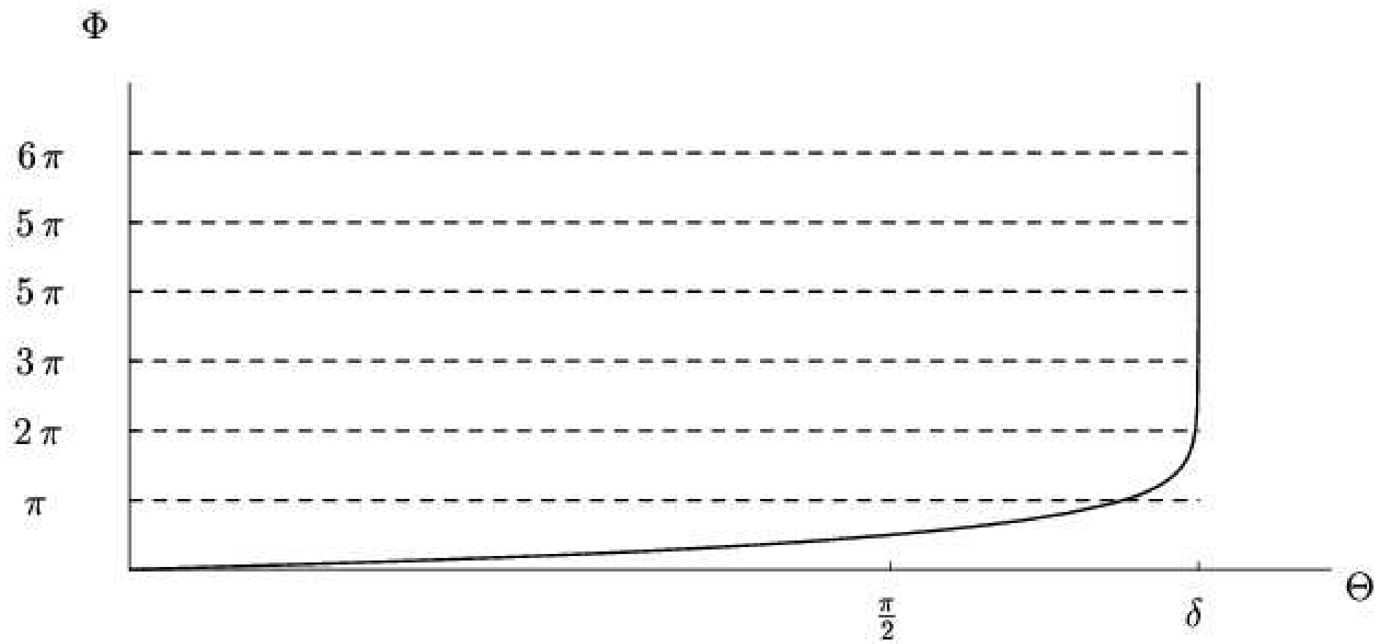



$$\Phi = R(r_O) \sin\Theta \int_{r_O}^{r_L} \frac{S(r)\, dr}{R(r) \sqrt{R(r)^2 - R(r_O)^2 \sin^2\Theta}}$$

## Schwarzschild spacetime

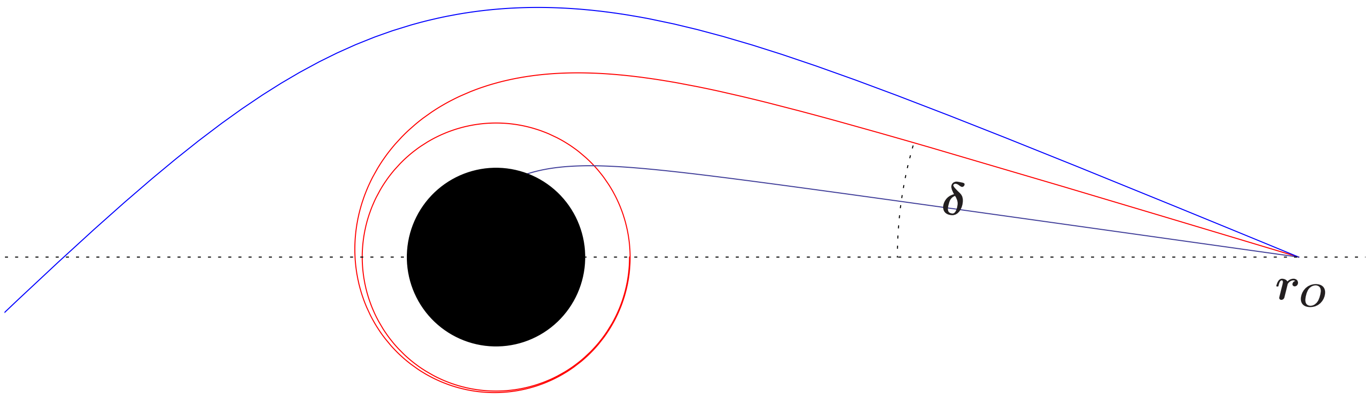
$$g = -\left(1 - \frac{r_S}{r}\right) c^2 dt^2 + \left(1 - \frac{r_S}{r}\right)^{-1} dr^2 + r^2 \left(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2\right) \; , \qquad r_S = \frac{2GM}{c^2}$$

$$S(r)^{-1} = 1 - \frac{r_S}{r}, \qquad R(r) = \frac{r}{\sqrt{1 - \frac{r_S}{r}}}$$



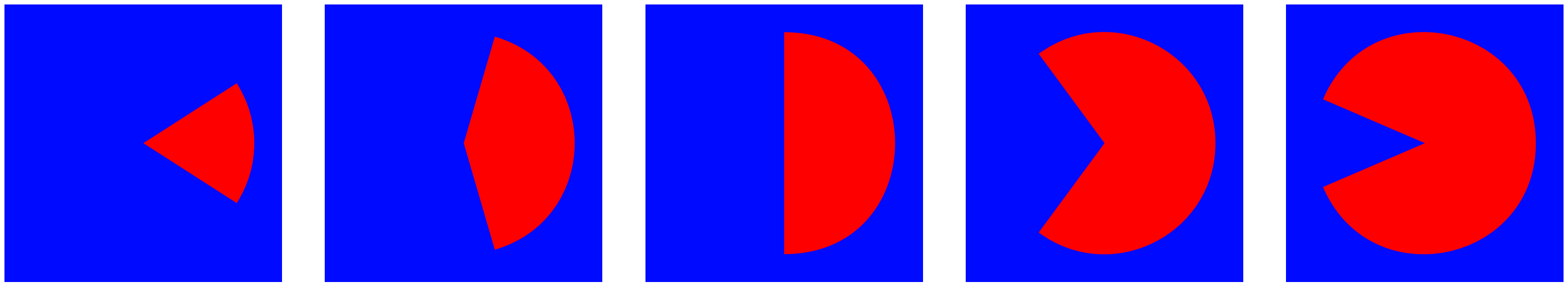

**Lens map $\Theta \mapsto \Phi$ for $r_O = 2.5\, r_S$ and $r_L = 5\, r_S$**

**Infinite sequence of images converges towards $\delta$**




**Angular radius $\delta$ of the "shadow" of a Schwarzschild black hole:**

$$\sin^2\delta = \frac{27\, r_S^2 (r_O - r_S)}{4\, r_O^3}\,, \qquad r_S = 2\,m = \frac{2GM}{c^2}$$



$r_O = 1.05\, r_S$ $\quad$ $r_O = 1.3\, r_S$ $\quad$ $r_O = 3\, r_S/2$ $\quad$ $r_O = 2.5\, r_S$ $\quad$ $r_O = 6\, r_S$

Other black holes:

- Reissner-Nordström
- Janis-Newman-Winicour
- Newman-Unti-Tamburino (NUT)
- Black holes from nonlinear electrodynamics
- Black holes from higher dimensions, braneworld scenarios, ...

All of them have an unstable photon sphere $\Longrightarrow$ Qualitative lensing features are similar to Schwarzschild

Quantitative features (ratio of angular separations of images, ratio of fluxes of images) are different

V.Bozza: Phys. Rev. D 66, 103001 (2002)

If higher-order images are seen, we can distinguish a Schwarzschild black hole from other black holes

## Black hole imposter: Ellis wormhole

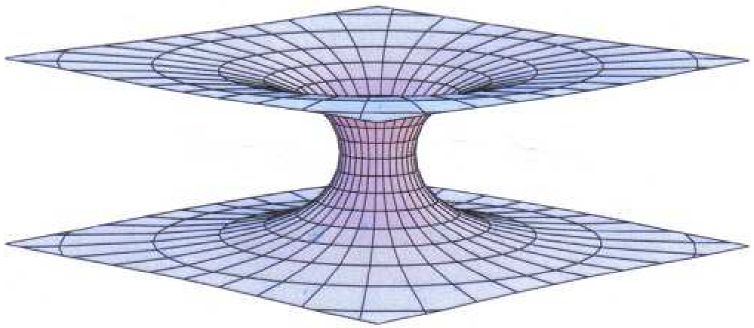
H. Ellis: J. Math. Phys. 14, 104 (1973)

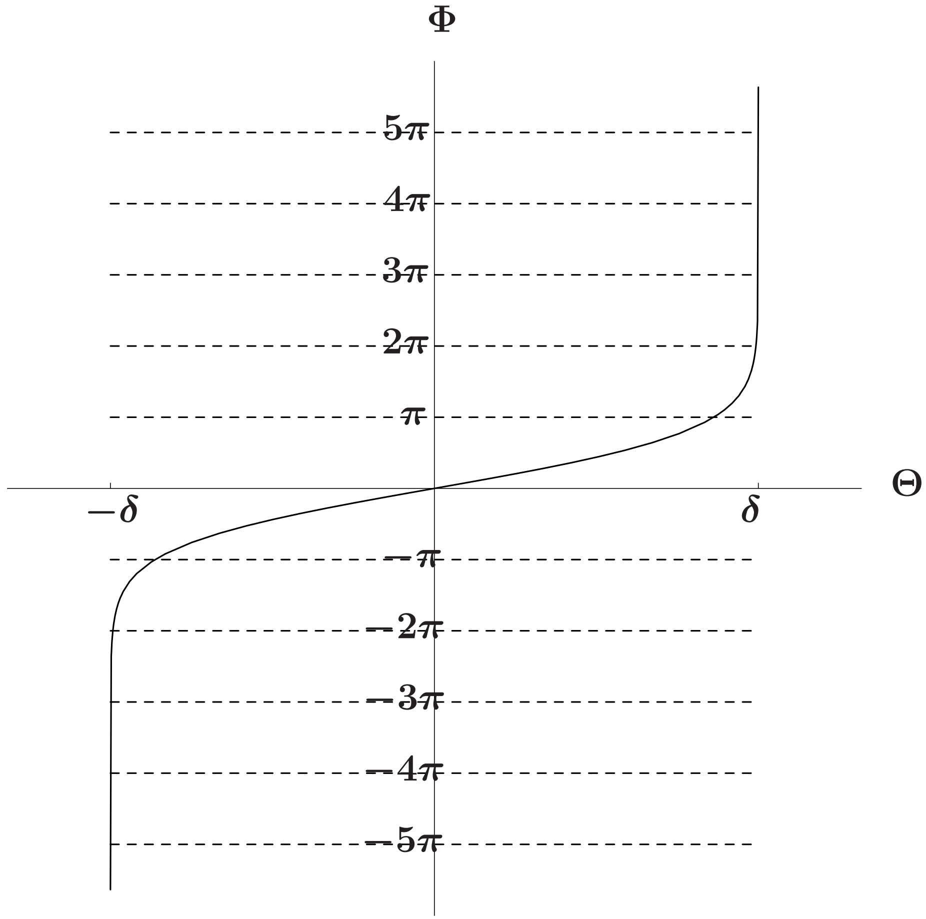
$$g = -c^2 dt^2 + dr^2 + \left(r^2 + a^2\right)\left(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2\right)$$

$$S(r) = 1$$

$$R(r) = \sqrt{r^2 + a^2}$$

$$\sin^2\delta = \frac{a^2}{r_O^2 + a^2}$$



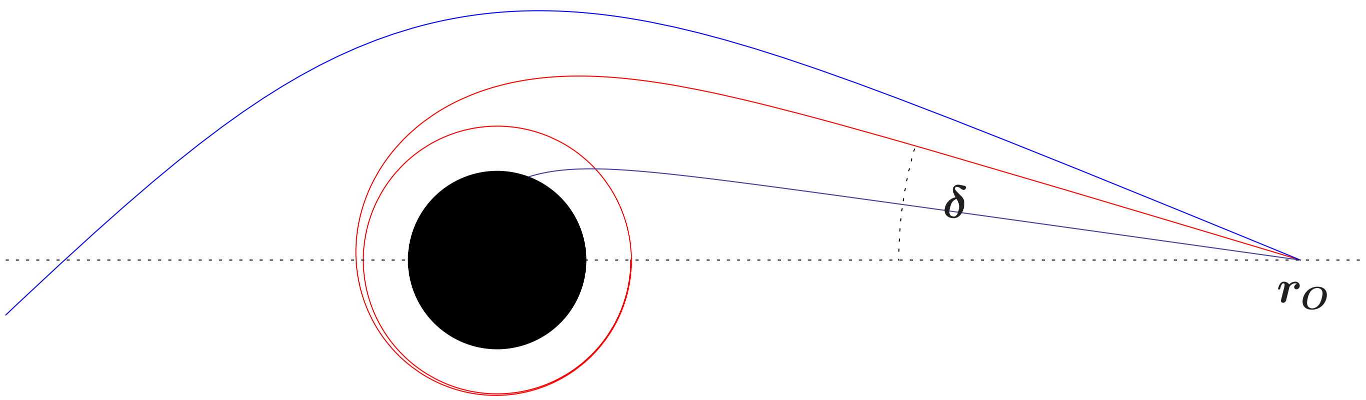



$$r_L < 0 < r_O$$

**Qualitatively the same, quantitatively different from black holes**

Black hole imposter: Ultracompact star

Uncharged dark star with radius between $2m$ and $3m$




Lensing features indistinguishable from Schwarzschild black hole

Claim: ultracompact objects cannot exist

V. Cardoso, L. Crispino, C. Macedo, H. Okawa, P. Pani: arXiv:1406.5510

## 1. Rotating black holes

Shadow no longer circular

Shape of shadow can be used for discriminating between different black holes

### Shadow of Kerr black hole:

J. Bardeen in C. DeWitt and B. DeWitt (eds.): "Black Holes" Gordon & Breach (1973)

### Shadow in Plebański spacetimes:

A. Grenzebach, VP, C. Lämmerzahl: Phys. Rev. D 89, 124004 (2014)

**Plebański metric in Boyer–Lindquist coordinates $(r, \vartheta, \varphi, t)$:**

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = \Sigma(\frac{1}{\Delta_r}dr^2 + \frac{1}{\Delta_\vartheta}d\vartheta^2) + \frac{1}{\Sigma}((\Sigma + a\chi)^2\Delta_\vartheta \sin^2\vartheta - \Delta_r\chi^2)d\varphi^2$$

$$+\frac{2}{\Sigma}(\Delta_r\chi - a(\Sigma + a\chi)\Delta_\vartheta \sin^2\vartheta)dtd\varphi - \frac{1}{\Sigma}(\Delta_r - a^2\Delta_\vartheta \sin^2\vartheta)dt^2$$

$$\Sigma = r^2 + (\ell + a\cos\vartheta)^2$$

$$\chi = a\sin^2\vartheta - 2\ell\cos\vartheta$$

$$\Delta_r = r^2 - 2mr + a^2 - \ell^2 + {q_e}^2 + {q_m}^2$$

$$-\Lambda((a^2 - \ell^2)\ell^2 + (\tfrac{1}{3}a^2 + 2\ell^2)r^2 + \tfrac{1}{3}r^4)$$

$$\Delta_\vartheta = 1 + \Lambda(\tfrac{4}{3}a\ell\cos\vartheta + \tfrac{1}{3}a^2\cos^2\vartheta)$$

**Lightlike geodesics:**

$$\Sigma^2 \dot{\vartheta}^2 = \Delta_\vartheta K - \frac{(\chi E - L)^2}{\sin^2 \vartheta} =: \Theta(\vartheta)$$

$$\Sigma^2 \dot{r}^2 = \big( (\Sigma + a\chi) E - aL \big)^2 - \Delta_r K =: R(r)$$

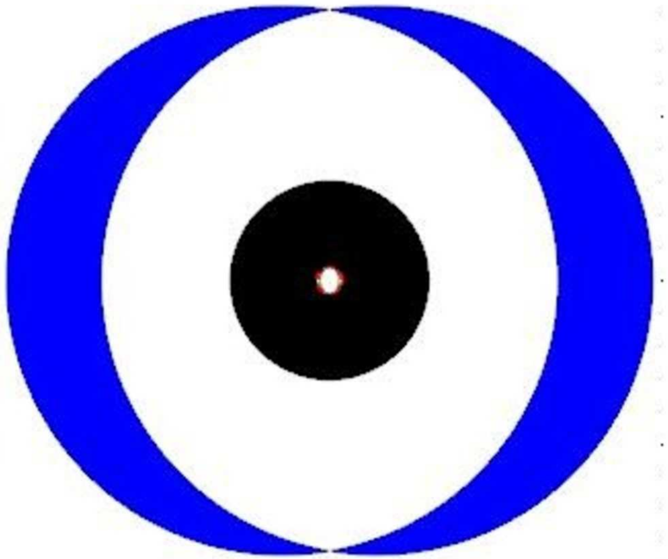
**Spherical lightlike geodesics exist in the region where**

$$R(r) = 0 \, , \quad R'(r) = 0 \, , \quad \Theta(\vartheta) \geq 0 \, .$$

$$\big( 4r\Delta_r - \Sigma \Delta_r' \big)^2 \leq 16 a^2 r^2 \Delta_r \Delta_\vartheta \sin^2 \vartheta$$ **(“photon region”)**

**(unstable if $R''(r) \geq 0$)**

## Photon region for Kerr black hole with $a = 0.15\,m$



The shadow is determined by light rays that approach an unstable spherical lightlike geodesic.

Choose observer at $r_O$ and $\vartheta_O$

celestial coordinates at observer

$$(\theta, \psi)$$

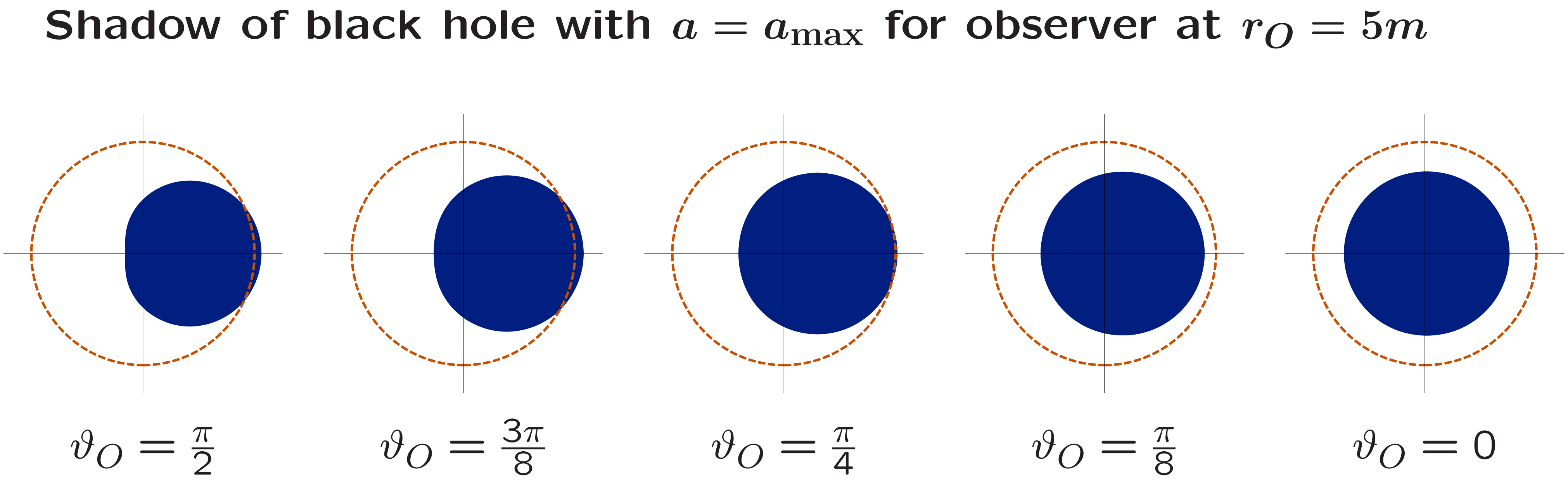
constants of motion

$$\left(K_E = \frac{K}{E^2}, \widetilde{L}_E = \frac{L}{E} - a\right)$$

$$\cos\theta = \frac{\sqrt{\Delta_r K_E}}{r^2 + \ell^2 - a\widetilde{L}_E}\bigg|_{r=r_O}, \qquad \sin\psi = \frac{\widetilde{L}_E + a\cos^2\vartheta + 2\ell\cos\vartheta}{\sqrt{\Delta_\vartheta K_E}\sin\vartheta}\bigg|_{\vartheta=\vartheta_O}$$

$$K_E = \frac{16 r^2 \Delta_r}{(\Delta_r')^2}\bigg|_{r=r_p}, \qquad a\widetilde{L}_E = \left(r^2 + \ell^2 - \frac{4r\Delta_r}{\Delta_r'}\right)\bigg|_{r=r_p}$$

Analytic formula for shadow allows to extract parameters of the spacetime from the shape of the shadow


Shadow of black hole with $a = a_{\mathrm{max}}$ for observer at $r_O = 5m$
$\vartheta_O = \frac{\pi}{2}$
$\vartheta_O = \frac{3\pi}{8}$
$\vartheta_O = \frac{\pi}{4}$
$\vartheta_O = \frac{\pi}{8}$
$\vartheta_O = 0$

## Object at the centre of our galaxy:

Mass $= 4 \times 10^6\, M_\odot$

Distance $= 8.3\,\mathrm{kpc}$

If it is a Schwarzschild black hole, the diameter of the shadow should be $\approx 56\mu$as

(corresponds to a grapefruit on the moon)

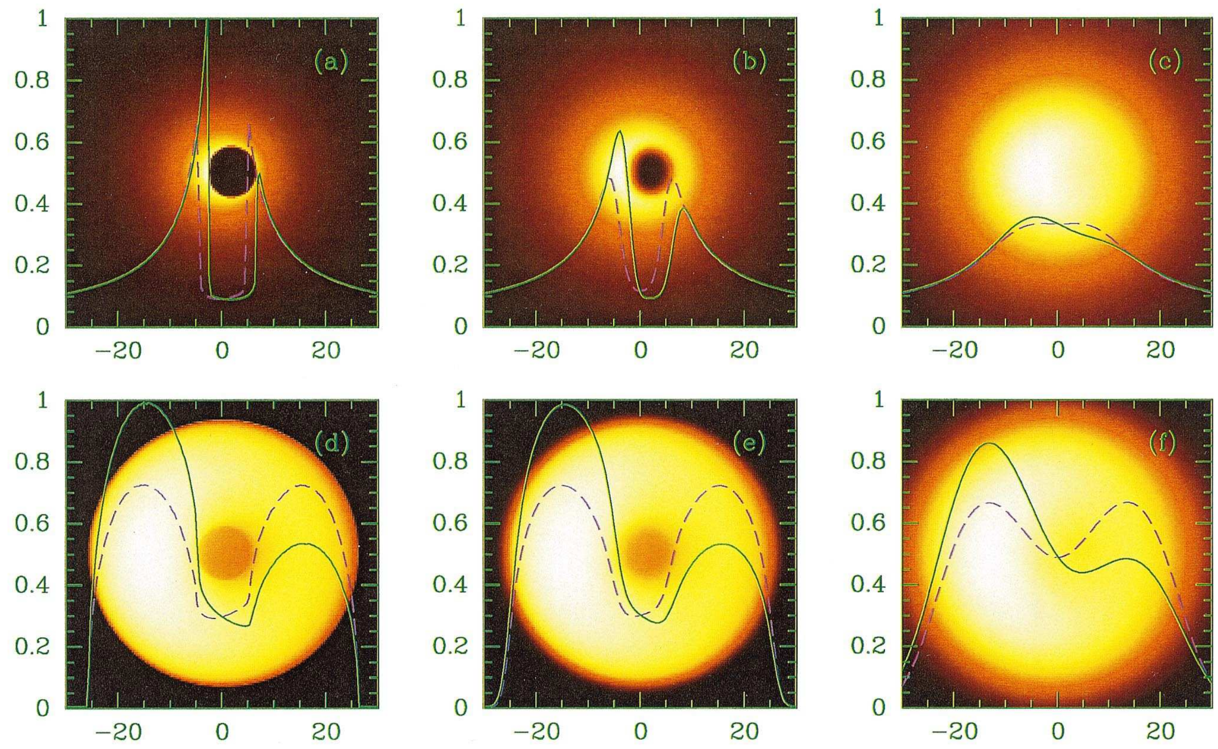
## Object at the centre of M87:

Mass $= 3 \times 10^9\, M_\odot$

Distance $= 16\,\mathrm{Mpc}$

If it is Schwarzschild black hole, the diameter of the shadow should be $\approx 9\mu$as

# Kerr shadow with emission region and scattering taken into account:



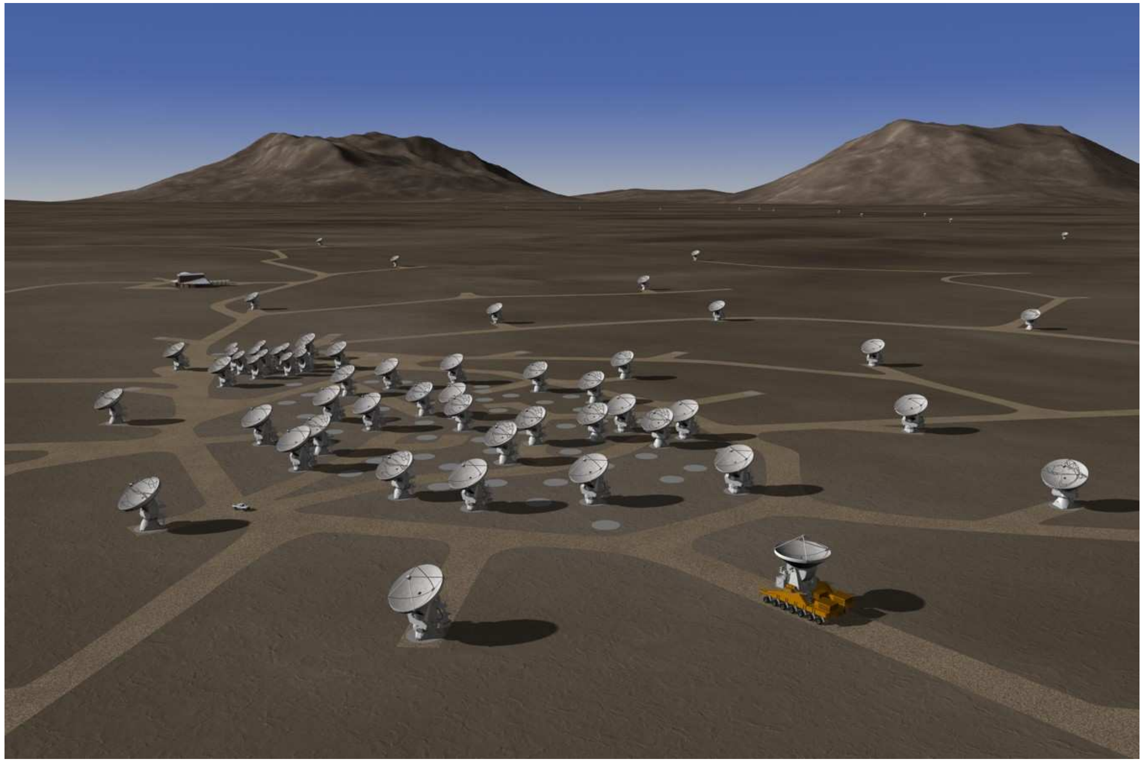

H. Falcke, F. Melia, E. Agol: Astrophys. J. 528, L13 (2000)

Observations should be done at sub-millimeter wavelength

Two projects to view the shadow with sub-millimeter VLBI:

Event Horizon Telescope (EHT), BlackHoleCam

Using ALMA, NOEMA, ...



ALMA

Good chance to see the shadow of the centre of our galaxy within a few years