Orientational atom interferometers sensitive to gravitational waves

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We present an atom interferometer that differs from common atom interferometers as it is not based on the spatial splitting of electronic wave functions, but on orienting atoms in space. As an example we present how an orientational atom interferometer based on highly charged hydrogen–like atoms is affected by gravitational waves. We show that a monochromatic gravitational wave will cause a frequency shift that scales with the binding energy of the system rather than with its physical dimension. For a gravitational wave amplitude of \( h = 10^{-23} \) the frequency shift is of the order of 110 \( \mu \)Hz for an atom interferometer based on a 91-fold charged uranium ion. A frequency difference of this size can be resolved by current atom interferometers in 1 s.

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I. INTRODUCTION

Atom interferometry [1] has become one of the most accurate and sensitive laboratory measurement tools in science. Applications include, for example, the determination of the fine-structure constant [2], of the gravitational acceleration [3], and of gravity gradients [4]. There also exist several proposals [5] to use atom interferometry for the detection of gravitational waves. These show a couple of features that classical detectors, such as laser interferometers [6] or bar-type detectors [7], do not. The basic idea behind these proposals is to spatially split the atomic wave function. A gravitational wave will then affect the different parts of the split wave packet differently so that a differential phase shift will accumulate between these parts that can be read out by quantum interferometry techniques.

In a previous paper [8] we presented a quantum interferometer that differs significantly from current atom interferometers as it is not based on spatial splitting of wave packets but on orienting molecules in space. We showed that the application of atom interferometry techniques to the internal (rotational-vibrational) states of molecules provides a powerful tool for ultrahigh-precision spectroscopy, specifically for the investigation of anisotropic effects on molecular spectra. As an example we explained how a molecular rotational-vibrational quantum interferometer based on the hydrogen deutereide molecular ion may be used to detect gravitational waves. The concept was based on preparing a diatomic molecule in a coherent superposition of two quantum states that correspond to an orientation of the internuclear axes along two orthogonal directions in space which are perpendicular to the propagation direction of the gravitational wave. The interaction between the molecule and the gravitational wave then induces a differential energy shift between these two states that can be detected by quantum interferometric techniques. It was argued that a monochromatic gravitational wave of amplitude \( h = 10^{-19} \)

will cause a frequency shift of the order of 30 \( \mu \)Hz between appropriately prepared quantum states [8], a frequency difference likely to be detectable with the next-generation atom interferometers in 1 s [9].

These studies were motivated by the simple conclusion that anisotropic objects can per se distinguish between different directions relative to their own orientation in space. Hence molecules appeared to be optimal probes as, from a classical point of view, molecules—in contrast to atoms—are not spherically symmetric. Of course, in the quantum world molecules have to be oriented in space (i.e., have to be prepared in a superposition of specific rotational eigenstates) in order for their nonsphericity to become effective.

However, the method we proposed to orient molecules, that is, by light pulses, can also be applied to induce nonsphericity in the electronic wave function of atoms, so that atoms can also be oriented in space. The preparation, the manipulation, and the coherent control of atoms, however, is by far further developed than the corresponding control of molecules [10]. Therefore, from an experimentalist’s point of view, quantum interferometers based on the orientation of atoms deserve at least the same attention as their molecular counterparts.

It is argued quite often that atom interferometers cannot provide a reasonable sensitivity to gravitational waves [11] because their dimension is some 15 orders of magnitude smaller than the wavelength of the gravitational wave. However, the type of atom interferometer discussed here is not based on measuring the light distance between two points moving along geodesics (as it is in the case of laser interferometric detectors); rather, it is based on measuring the modification of its internal, orientational structure in response to the interaction with the gravitational wave. We therefore expect an effect that scales with the system’s binding energy. Typically, the smaller the dimension of a quantum object the larger its binding energy. Hence, the largest modification of the eigenenergy (e.g., binding energy) is expected to occur with the smallest objects. Because atom interferometers can be designed to measure energy differences rather than absolute or relative energies, they are a perfect tool for determining even the smallest modification of atomic

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or molecular spectra. Atoms or even atomic nuclei (nuclear quantum interferometers) should therefore be best suited to realize quantum-interferometer-based gravitational wave detectors.

In this paper we address this conjecture and show that it is indeed valid. To this end we transfer the concept of molecular rotational-vibrational quantum interferometry to atoms and analyze how hydrogen atoms or highly charged single-electron atomic ions may be oriented in space and in principle be used to detect gravitational waves. If size does matter, then oriented large-scale Rydberg atoms should in principle be best suited for the detection of gravitational waves. However, our discussion clearly shows that (i) highly charged hydrogen-like atoms with their single electron occupying the $2p$ states provide the highest sensitivity and (ii) the sensitivity scales like $t$ times the binding energy of the system. We find that the gravitational-wave-induced frequency shift for an appropriately prepared hydrogen atom is $130 \, \mu\text{Hz}$ at a gravitational wave amplitude of $h = 10^{-19}$ and is thus comparable to the corresponding frequency shift experienced by the hydrogen deuteride molecular ion [8]. We further show that the frequency shift scales with the square of the nuclear charge so that with a 91-fold ionized uranium atomic ion a $10^4$-fold larger shift is to be expected. As next-generation atom interferometers are likely to detect frequency shifts of $20 \, \mu\text{Hz}$ between appropriately prepared quantum states in 1 s we infer a potential sensitivity of $h_{\text{min}} = 10^{-23}$ in 1 s for an atom interferometer based on the 91-fold charged uranium ion. It should be emphasized that this paper only points out the principle lines along which a quantum-scale atom interferometer concept described here relies on orienting the atom in space and does not require any spatial splitting of the atomic wave function. This is an important advantage over common atom interferometers, as pointed out in detail in [8].

The basic idea of this quantum interferometer is motivated by the simple conclusion that nonspherical objects can per se distinguish between different directions relative to their own “internal coordinate system”. For molecules, atoms, and possibly in the future also for atomic nuclei nonsphericity as well as orientation in space can be induced by light pulses. Placed in an anisotropic environment nonspherical objects may then possess spectra that depend on their orientation with respect to the anisotropic environment.

The atom interferometric measurement scenario consists of only three steps: the preparation step, the actual measurement step, and the read-out step. Let us consider a gravitational wave propagating in the $z$ direction. In the first step the atoms will be prepared in the initial interferometer state, called the $|+\rangle$ state, which is the symmetric coherent superposition of the two states $|\psi_x\rangle$ and $|\psi_y\rangle$ that correspond to the two states of the interferometer as well as to the two orientations of the atom along the $x$ and $y$ axes, respectively. The preparation step is followed by the measurement step, in which the two quantum states $|\psi_x\rangle$ and $|\psi_y\rangle$ evolve freely in time, however, for no longer than half of a gravitational wave period. The gravitational wave propagating in the $z$ direction then modifies the spectra of the two states $|\psi_x\rangle$ and $|\psi_y\rangle$ in a different way; for example, the eigenvalue of $|\psi_x\rangle$ may be increased while the eigenvalue of $|\psi_y\rangle$ may be decreased. Hence, due to the interaction with the gravitational wave a quantum phase shift will accumulate between the two states $|\psi_x\rangle$ and $|\psi_y\rangle$. In the final step, through the final laser pulse, these states interfere and are projected onto the detection state, say the $|-\rangle$ state that is orthogonal to the initial $|+\rangle$ state and denotes the antisymmetric superposition of the interferometer states $|\psi_x\rangle$ and $|\psi_y\rangle$.

The number of atoms projected onto the detection state then carries the information about the differential quantum phase shift between the two interferometer paths. If no phase shift has accumulated, then perfect destructive interference between the $|\psi_-\rangle$ components of the two interferometer states $|\psi_x\rangle$ and $|\psi_y\rangle$ implies that no atoms are detected. In contrast, a nonvanishing phase shift partially prevents the destructive interference of the $|\psi_-\rangle$ components, so that some atoms are detected.

### III. GRAVITATIONAL WAVES AND HYDROGEN-LIKE ATOMS

In this section the perturbation of the atomic Hamiltonian due to a gravitational wave is derived. For the sake of simplicity we consider hydrogen-like atoms, that is, $(Z - 1)$-fold ionized atomic ions, where $Z$ is the atomic number. We first introduce center-of-mass coordinates, that is, the electron-nucleus distance $r$ and the reduced mass of the system $m_r = m_e m_N / (m_e + m_N)$, where $m_e$ is the electron mass and $m_N$ is the nuclear mass.

In the unperturbed case the atomic Hamiltonian $\hat{H}_0$ is given by $\hat{H}_0 = \hat{V}_N + \hat{T}$, where $\hat{V}_N$ describes the electron-nucleus Coulomb interaction and $\hat{T}$ the kinetic energy of the reduced mass. The corresponding unperturbed wave functions $\Psi_{\text{din}}(r, \vartheta, \varphi)$, where $n, l$, and $m$ are the principal, azimuthal,
and magnetic quantum numbers, can then be separated into a radial part $R_{nl}(r)$ and an angular part $Y_{lm}(\theta, \phi)$

$$\Psi_{nlm}(r, \theta, \phi) = R_{nl}(r) Y_{lm}(\theta, \phi),$$  \hspace{1cm} (1)

where $\theta$ and $\phi$ are the azimuthal and polar angles, respectively, and $r = |\vec{r}|$ is the distance between the electron and the nucleus. According to [13] the radial wave function $R_{nl}(r)$ can be written as $R_{nl}(r) = u_{nl}(r)/r$. Here the function $u_{nl}(r)$ can be expressed with the help of the dimensionsless distance $\epsilon = \frac{Z}{a_0}$ and the function $u_{nl}(\alpha) = a^{l+1} e^{-\alpha L} L_{n+l+1}(2\alpha)$ as

$$u_{nl}(r) = A_{Zn} B_{nl} u_{nl}(\epsilon/n),$$  \hspace{1cm} (2)

where the constants $A_{Zn}$ and $B_{nl}$ are defined by

$$A_{Zn} = \sqrt{\frac{Z}{n a_0}}$$ and $B_{nl} = \sqrt{(n-l-1)! (n+l)!} \frac{2^{l+1}}{2l+1}.$  \hspace{1cm} (3)

$a_0 = \frac{4 \pi e^2}{m_e}$ is the Bohr radius, and $L_\ell$ denotes the Laguerre polynomials. Further, the spherical harmonics $Y_{lm}(\theta, \phi)$ can be expressed by the associated Legendre functions $P$ as

$$Y_{lm}(\theta, \phi) = N_{lm} P_{lm}(\cos \theta) e^{im\phi},$$  \hspace{1cm} (4)

where $N_{lm} = (-1)^{l+1/2} \frac{1}{(2l+1)^{1/2}} \sqrt{\frac{(2l)!}{4\pi} \frac{1}{n!}}.$

We now include the influence of a gravitational wave. Gravitational waves can be considered as small perturbations $\delta h_{\mu\nu}$ in a background space-time metric [14] and, thus, are described within linearized gravity. With a flat background metric $\eta_{\mu\nu}$, the metric $g_{\mu\nu}$ is then written as

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta h_{\mu\nu},$$  \hspace{1cm} (5)

where $|\delta h_{\mu\nu}| \ll 1$.

In [8] it is analyzed how gravitational waves modify the Hamiltonian of a charged point mass $m_p$ in the field of another point charge. Starting with the Klein-Gordon equation minimally coupled to gravity and to the Maxwell field we derive the interaction Hamiltonian for the nonrelativistic limit [15] in the TT (transverse and traceless) gauge [16]:

$$\hat{H}_I = \frac{\hbar^2}{2m_p} h^{ij} \partial_i \phi_j,$$  \hspace{1cm} (6)

where we consider terms linear in $h$ only [17].

We further analyze how the electric potential $A_0$ of a point charge $q$ is modified by the presence of a periodic gravitational wave $h_{\mu\nu} = h_{\mu\nu}^0 \exp(i \vec{k} \cdot \vec{x} - \omega t)$, where $h_{\mu\nu}^0$ denotes the amplitudes, $\vec{k}$ the wavevector, and $\omega$ the frequency of the gravitational wave.

Under the assumption that the influence of a gravitational wave is adiabatic (small frequency) and hence quasiconstant (long wavelength), the electric potential becomes quasistatic. To first order we find the electric potential $A_0$ to be

$$A_0 = \frac{q}{4\pi \epsilon_0 r} \left(1 - \frac{x^i h_{ij}^0 x^j}{2r^2} e^{i \vec{k} \cdot \vec{x} - \omega t}\right).$$  \hspace{1cm} (7)

From these results it can be concluded that both terms in the Hamiltonian $\hat{H}_0$, the electron-nucleus Coulomb interaction $\hat{V}_{E,N}$ as well as the kinetic energy of the reduced mass $\hat{T}$, are modified by the presence of a gravitational wave. For the sake of simplicity we consider a polarized gravitational wave propagating in the $z$ direction so that $h_{xx} = -h_{yy} = h$ are the only nonvanishing components. With the help of Eq. (7) the modification of the electron-nucleus Coulomb interaction in the center-of-mass system is then given by

$$\delta \hat{V}_{E,N} = h \frac{e^2 Z}{4\pi \epsilon_0} \frac{x^2 - y^2}{2r^3}.$$  \hspace{1cm} (8)

In spherical coordinates this expression transforms to

$$\delta \hat{V}_{E,N} = h E_R Z^2 \frac{1}{2\epsilon} \cos(2\varphi)(1 - \cos(2\theta)), $$  \hspace{1cm} (9)

where $E_R = (m_e e^4)/[(4\pi \epsilon_0)^2 2\hbar^2]$ is the Rydberg energy.

The modification of the kinetic energy of the reduced mass $m_r$ can be derived with the help of Eq. (6) as

$$\delta \hat{T} = h \frac{\hbar^2}{2m_r} \left(\delta \hat{r}^2 - \delta \hat{\theta}^2\right),$$  \hspace{1cm} (10)

or, in spherical coordinates,

$$\delta \hat{T} = h \frac{m_e}{m_r} E_R Z^2 \delta \hat{T}'',$$  \hspace{1cm} (11)

where $\delta \hat{T}'$ is defined as

$$\delta \hat{T}' = \left(\sin \theta \sin \varphi \partial_\theta + \frac{\cos \theta \cos \varphi}{\epsilon} \partial_\theta - \frac{\sin \varphi}{\epsilon \sin \theta} \partial_\varphi\right)^2 \left(\sin \varphi \partial_\varphi + \frac{\cos \varphi}{\epsilon \sin \theta} \partial_\varphi\right)^2.$$

IV. CONSTRUCTION OF THE ATOM INTERFEROMETER

We can now evaluate the matrix elements of the two perturbation operators $\delta \hat{V}_{E,N}$ and $\delta \hat{T}$ in the basis $|nlm\rangle$ of the unperturbed system. Let $\delta A$ denote one of the perturbation operators. The matrix elements can then be calculated according to

$$\langle nlm | \delta A | n'l'm' \rangle = \int_0^\infty dr d\delta d\varphi r^2 \sin \theta \frac{\bar{Y}_{lm}(\theta, \phi)}{R_{nl}(r)}$$

$$\times R_{nl}(r) \delta A \bar{Y}_{lm'}(\theta, \phi) R_{nl'}(r)$$(13)

and the perturbation operator can be diagonalized to optimize the atom interferometer for maximum sensitivity to gravitational waves. The matrix elements $\langle nlm | \delta V_{E,N} | n'l'm' \rangle$ corresponding to the perturbation operator for the modified Coulomb interaction $\delta \hat{V}_{E,N}$ [see Eq. (9)] are given by

$$\langle nlm | \delta \hat{V}_{E,N} | n'l'm' \rangle = h E_R Z^2 \frac{1}{\sqrt{nm}} N_{lm} N_{lm'} B_{nl} B_{nl'}$$

$$\times \int_0^\infty d\epsilon \tilde{u}_{nl}(\epsilon/n) \tilde{u}_{nl'}(\epsilon/n') \frac{1}{2\epsilon}$$

$$\times \int_0^\infty d\varphi \bar{\mathcal{P}}_{|lm|}(\cos \theta) \mathcal{P}_{|lm'|}(\cos \theta) \sin \theta [1 - \cos(2\vartheta)]$$

$$\times \int_0^{2\pi} d\varphi e^{-i(m-m')\varphi} \cos(2\varphi).$$

Similarly, the matrix elements $\langle nlm | \delta \hat{T} | n'l'm' \rangle$ corresponding to the perturbation operator for the kinetic energy of the
reduced mass, $\delta \hat{T}$ [see Eq. (11)], can be calculated in spherical coordinates:

$$
\langle nlm | \delta \hat{T} | n'l'm' \rangle = \frac{\hbar Z}{m_r} \left[ \frac{1}{\sqrt{nn'}} N_m N_{n' m'} B_{nl} B_{n'l'} \right. \\
\times \int_0^\infty \int_0^\pi \int_0^{2\pi} d\epsilon \, d\theta \, d\varphi \left( \sin \theta \, \hat{u}_{nl}(\epsilon/n) \mathcal{P}_{nlm}^{\prime}(\cos \theta) \right. \\
\times e^{-im'\varphi} \left. \delta \hat{T} \epsilon \hat{u}_{n'l'}(\epsilon/n') \mathcal{P}_{n'l'm'}^{\prime}(\cos \theta) e^{im'\varphi} \right], \tag{15}
$$

which can be evaluated numerically.

Due to the relatively small frequencies of gravitational waves (kiloherz range), gravitational waves cannot resonantly couple nondegenerate atomic states (i.e., states with different principal quantum number $n$ or angular momentum $l$). The rotating wave approximation therefore simplifies the perturbation operators to $\delta \hat{A} = \delta a_{n,m} \cdot \delta l_{l',m'}$.

Figure 1 shows the matrix elements of the total perturbation operator $\delta \hat{H} = \delta V_{zn} + \delta \hat{T}$ in units of $\hbar E_k Z^2$. Note that all matrix elements have been multiplied by $(-1)$ to ease the graphical viewing. The gravitational wave effectively only couples states $|nlm \rangle$ and $|nl(m \pm 2) \rangle$. The selection rule $|\Delta m| = 2$ is expected because of the quadrupole nature of gravitational waves.

The corresponding spectrum of eigenvalues is shown in Fig. 2 for principal quantum numbers $n = 2, \ldots, 6$ in units of $\hbar E_k Z^2$. The different energy corrections given for a specific principal quantum number $n$ correspond to the related azimuthal quantum numbers $l = 0, \ldots, (n-1)$; For every $l$, $2l + 1$ eigenstates and corresponding energy corrections are evaluated. The dashed line indicates the well-known $n^{-2}$ scaling of the atomic hydrogen binding energy. Figure 2 clearly shows that the calculated energy corrections scale with this $n^{-2}$ behavior. We therefore conclude that the energy corrections scale with the binding energy rather than with the physical size of the quantum object. One may therefore speculate that nuclear quantum interferometers, which would be based on a coherent superposition of nuclear quantum states, should—in principle—provide even higher sensitivity owing to their even smaller physical dimension and even larger binding energy.

This important result may be surprising for those who have large laser interferometers in mind with dimensions ranging between 1 and $10^6$ km (e.g., GEO, LIGO, or LISA [18]). However, the atom interferometric gravitational wave detector presented here is quite different from these laser interferometers: It does not measure the light distance between two points moving along geodesics; rather, it determines the modification of the internal structure of a quantum object due to the interaction with a gravitational wave. It hence resembles neither laser interferometers nor bar detectors but rather it defines a new type of gravitational wave detector.

Further, the analysis points out that the size of the differential energy shift between the states $|1_+ \rangle$ and $|1_- \rangle$ is of the order of $0.4 \hbar E_k Z^2$. If we consider 91-fold ionized atomic ions (i.e., $Z = 92$) a gravitational wave with amplitude $h = 10^{-3}$ will cause a frequency shift of $\approx 110 \mu Hz$. Whereas the current atom interferometers can detect shifts of the order of $\approx 100 \mu Hz$ given sufficiently long integration times, the next-generation atom interferometers will be able to detect a shift of $20 \mu Hz$ in 1s [9].

We next determine the eigenstates which are optimal for gravitational wave detection. For principal quantum numbers $n \geq 2$ a coherent superposition of states with azimuthal quantum number $l = 1$ and magnetic quantum number $m = \pm 1$ defines the states that experience the largest differential

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energy shift. For the principal quantum number \( n = 2 \) these states are labeled \( |1_+\rangle \) and \( |1_−\rangle \) in Fig. 2. The corresponding state spectrum, that is, the coefficients \( c^+_m = \langle 1m|1_+\rangle \) and \( c^-_m = \langle 1m|1_−\rangle \) defining the perturbed eigenstate in the basis of the unperturbed eigenstates, are the real numbers given in Fig. 3. The state spectra of the \( l = 1 \) subspace for larger principal quantum numbers \( n = 3, 4, \ldots \) are alike. For \( n \geq 2 \) and \( l = 1 \) the two states corresponding to the two paths of the atom interferometer are obviously \( |1_+\rangle = 1/\sqrt{2}(|1, 1\rangle - |1, −1\rangle) \) and \( |1_−\rangle = 1/\sqrt{2}(|1, −1\rangle + |1, 1\rangle) \). Only for the \( l = 1 \) subspace does the initial state of the atom interferometer evaluate to an unperturbed eigenstate (e.g., \( |+\rangle = |1_+\rangle + |1_−\rangle \)) evaluates to \( (l, m) = (1, −1) \).

As another example we consider the \( n = 6, l = 5 \) subspace. The corresponding optimal eigenstates are \( |5_+\rangle \) and \( |5_−\rangle \). The corresponding state spectrum is shown in Fig. 4. It is obvious that now the initial interferometer state \( |+\rangle = |5_+\rangle + |5_−\rangle \) no longer corresponds to an unperturbed eigenstate. We recognize further that more states have to be coupled coherently for the \( l = 5 \) than for the \( l = 1 \) interferometer, which requires larger experimental efforts. The \( l = 5 \) subspaces for larger principal quantum numbers \( n = 7, 8, \ldots \) are alike.

We finally want to check whether the intuitive idea of aligning the atomic wave functions to perform gravitational wave detection is consistent with our findings. To this end we calculate the spherical part of the probability distribution \( |\langle r|\psi\rangle|^2 \) for the eigenstates \( |5_+\rangle, |5_0\rangle, \) and \( |5_−\rangle \) and plot them in spherical coordinates in Fig. 5. This visualizes the probability distribution for the localization of the electron. The two states \( |5_+\rangle \) and \( |5_−\rangle \) correspond to an alignment of the axis in the \( x-y \) plane, which is normal to the propagation direction of the gravitational wave. As suggested earlier in the qualitative discussion, the largest energy shift occurs for atoms that are aligned along the \( x \) or the \( y \) axis, that is, along one of the two polarization directions of the gravitational wave. Consequently, the maximum differential energy shift is observed between states which are aligned along these axes. Figure 5 also shows that states such as \( |5_0\rangle \), which are not shifted in energy, do not show any alignment along the \( x \) or \( y \) direction. We conclude that our findings are in perfect agreement with our intuitive understanding.

The most simple but nevertheless most sensitive gravitational wave detector can now be implemented along the lines discussed in Sec. II. To this end atoms are prepared in the \( |+\rangle = |1, −1\rangle \) state, which is the symmetric superposition of the \( |1_+\rangle \) and \( |1_−\rangle \) states which correspond to the two paths of the interferometer. In the final step of the interferometer sequence the differential phase shift accumulated during free evolution of the interferometer states can then be read out by projecting the quantum interferometer onto the \( |1, 1\rangle \) state which is orthogonal to the initial \( |1, −1\rangle \) state. The
number of atoms projected onto this detection state carries the information about the differential quantum phase shift accumulated between the two interferometer paths.

Atom interferometers bear several advantages [5] over "classical" gravitational wave detectors (i.e., over laser interferometric [6] and bar-type detectors [7]). These advantages are based on the quantum physical nature of the microscopic probes (see [8] for a detailed discussion): First, perfectly identical copies of sensors exist. Second, the internal structure of atoms is understood very well and their interaction with the environment can be controlled accurately. Third, quantum objects are typically localized very well. These features do not apply to macroscopic sensors that are built from mirrors or beam splitters.

Moreover, the high accuracy that atom interferometers provide is based on two nonclassical features: First, quantum interferometer states can be manipulated coherently. Hence, the storage time limit of gravitational wave detectors can simply be overcome by coherently swapping the two arms of the quantum interferometer. The signal-to-noise ratio can therefore increase linearly with integration time. Detection frequency and bandwidth can be chosen at will because all detector internal frequencies (hyperfine structure, fine structure, and electronic energies) are many orders of magnitude larger than typical gravitational wave frequencies (microhertz to many tens of kilohertz). In principle, the maximum detection frequency will be limited by the time that is required to implement a single interferometric measurement cycle (i.e., mostly to apply the laser pulses and to let the wave packets evolve freely). It therefore seems likely that atom interferometric gravitational wave detectors will typically be operated at low Fourier frequencies, say at 1 Hz and below. However, this is a technological rather than a principle issue. Further, in practice, technical noise entering through various ports of a given implementation will determine the optimum detection frequency and bandwidth. It should also be noted that light pulses can be used to reconfigure an interferometer. In this way, the orientation and the spectral or polarization sensitivity of the quantum gravitational wave detector can be reconfigured within milliseconds. This is possible with classical gravitational wave detectors. Second, quantum mechanics allows for the simultaneous implementation of several identical copies of sensors within one quantum object. This unique feature provides almost perfect rejection of the common-mode phase evolution as well as of many systematic effects and noise.

V. CONCLUSION

In this article we presented an atom interferometer that has the potential to provide gravitational wave detection with ultra-high sensitivity. It should however be emphasized that current quantum optics and laser technology does not yet provide a straightforward implementation of the measurement principle proposed. It differs from current atom interferometers used for precision measurements in fundamental physics as it is not based on spatially splitting wave packets but on orienting atoms in space.

The basic idea is to use coherent superpositions of electronic states in order to (i) create nonspherical electronic wave functions and (ii) align the nonspherical wave functions in space along the two polarization directions of a gravitational wave. The two quantum states which correspond to an alignment along these two axes define the two paths of the atom interferometer. The interaction with the gravitational wave then introduces a quantum phase shift between these states which can be read out by means of quantum interferometric methods.

As an example we referred to highly charged hydrogen-like atoms. We showed that the maximum differential energy shift between appropriately prepared quantum states is of the order of $0.4 \hbar E_R Z^2$. For 91-fold ionized atomic ions and a gravitational wave with amplitude $h = 10^{-23}$ this evaluates to a frequency shift of $\approx 110 \mu$Hz. Current atom interferometers resolve shifts at the level of 100 $\mu$Hz for sufficiently long integration times, and the next-generation interferometers will resolve shifts of 20 $\mu$Hz in 1 s. Spectroscopy and coherent manipulation of highly charged, hydrogen-like ions can in principle be performed with optical radiation if hyperfine transitions are used to couple the interferometer states [19].

It should be emphasized that the application to gravitational wave detection discussed here only serves as an example for the quantum interferometric measurement principle presented here. This atom interferometer may also be applied in other fundamental physics experiments (e.g., in the search for an anisotropy of the Coulomb force [20] or in experimental tests of the standard model extension [21]).

Our findings may have far-reaching consequences for quantum-interferometer-based gravitational wave detection in general. We pointed out that the potential sensitivity of quantum interferometers used for gravitational wave detection scales with the binding energy of the system rather than with its physical dimension. Therefore a $(Z - 1)$-fold charged atomic ion with a single strongly localized $2p$ electron provides a larger sensitivity than a "large" Rydberg atom. If our findings would also apply to systems that are controlled by nuclear forces rather than by Coulomb forces, orientational quantum interferometers based on coherent superposition of nuclear states may provide even larger sensitivity owing to their huge binding energy. Of course, these interferometers are out of reach for the experimentalist at present as no means exist so far to coherently control nuclear quantum states.

Although molecular interferometry provides less sensitivity than highly charged atom interferometers due to their limited binding energy they still deserve attention. Molecular spectra are by far more complex than atomic spectra, which impedes experimental manipulation of molecular states. However, the complex structure provides the basis for the construction of complex quantum interferometers that could be optimized for the rejection of systematic effects.

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