Light effects near black holes

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1. Spherically symmetric black holes
   – Schwarzschild black holes
   – Comparison with other spherically symmetric black holes and black hole impostors
2. Kerr black holes (Plebański-Demiański black holes)
3. Influence of a plasma

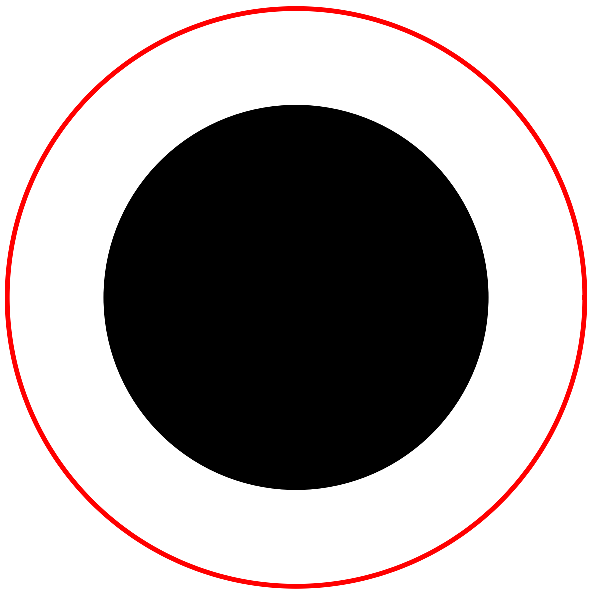
VP: "Gravitational Lensing from a Spacetime Perspective",
http://www.livingreviews.org/lrr-2004-9

341st WE-Heraeus seminar, Bad Honnef, 28 April 2017

Schwarzschild black hole

$$g_{\mu\nu}dx^\mu dx^\nu = -\left(1 - \frac{2m}{r}\right) c^2 dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} + r^2 \left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)$$

$$m = \frac{GM}{c^2}$$



Horizon:

$$r = r_S = 2m$$

Light sphere (photon sphere)

$$r = r_p = 3m$$

Schwarzschild black hole

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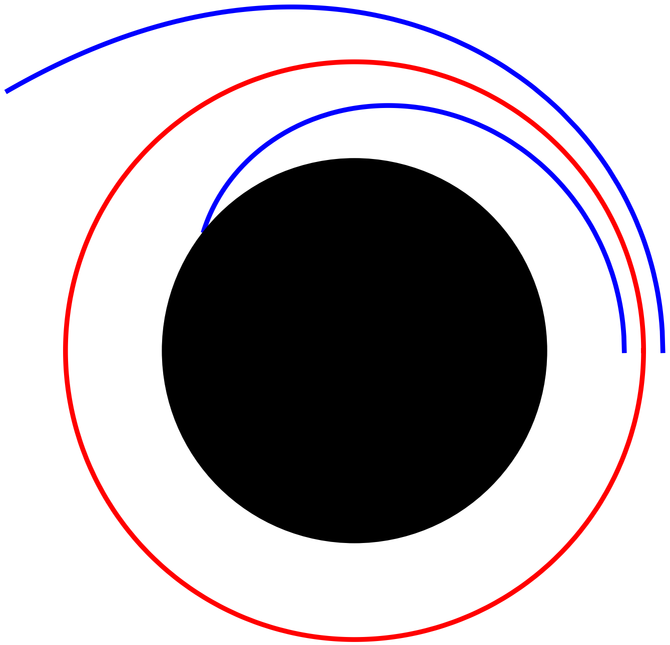

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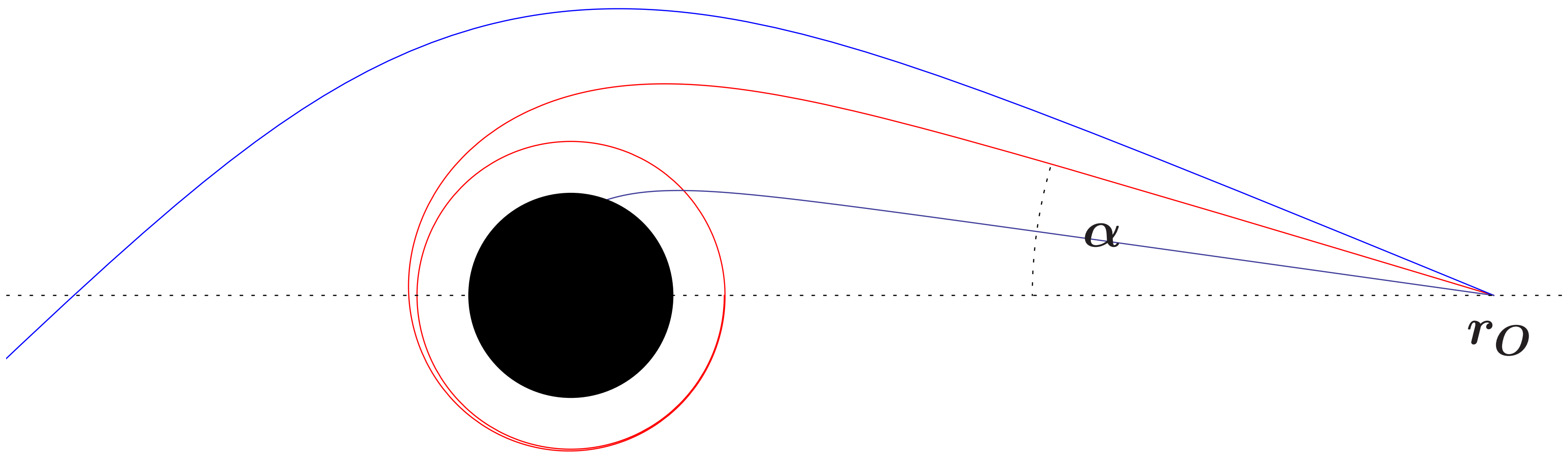
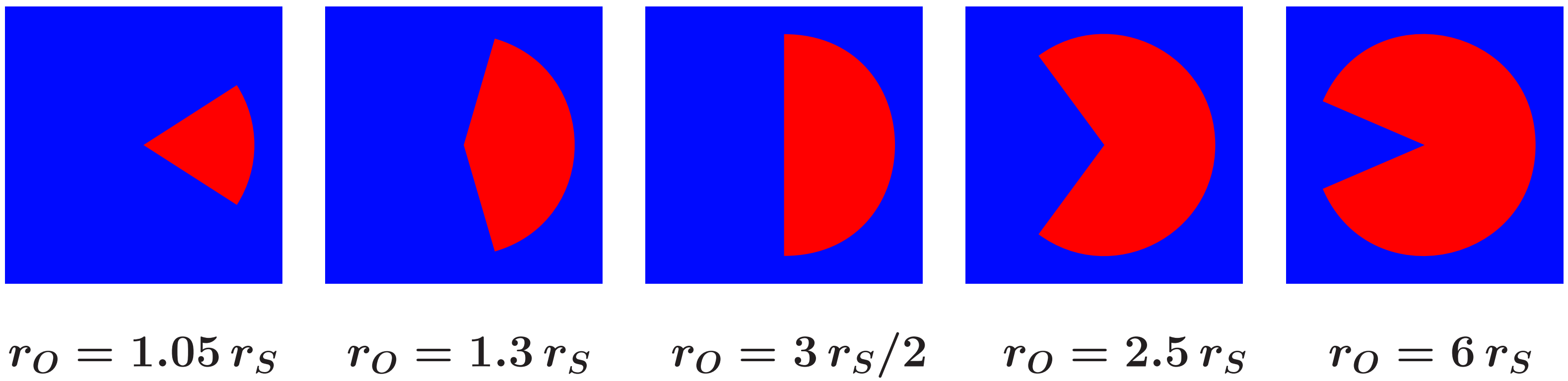


Angular radius α of the "shadow" of a Schwarzschild black hole:

$$\sin^2\alpha \;=\; \frac{27\,m^2}{r_O^2}\Big(1-\frac{2m}{r_O}\Big)$$

J. L. Synge, Mon. Not. R. Astr. Soc. 131, 463 (1966)



Perspectives of observations

Object at the centre of our galaxy:

Mass $= 4.3 \times 10^6\, M_\odot$

Distance $= 8.3\,\mathrm{kpc}$

Synge's formula gives for the diameter of the shadow $\approx 54\mu$as

(corresponds to a grapefruit on the moon)

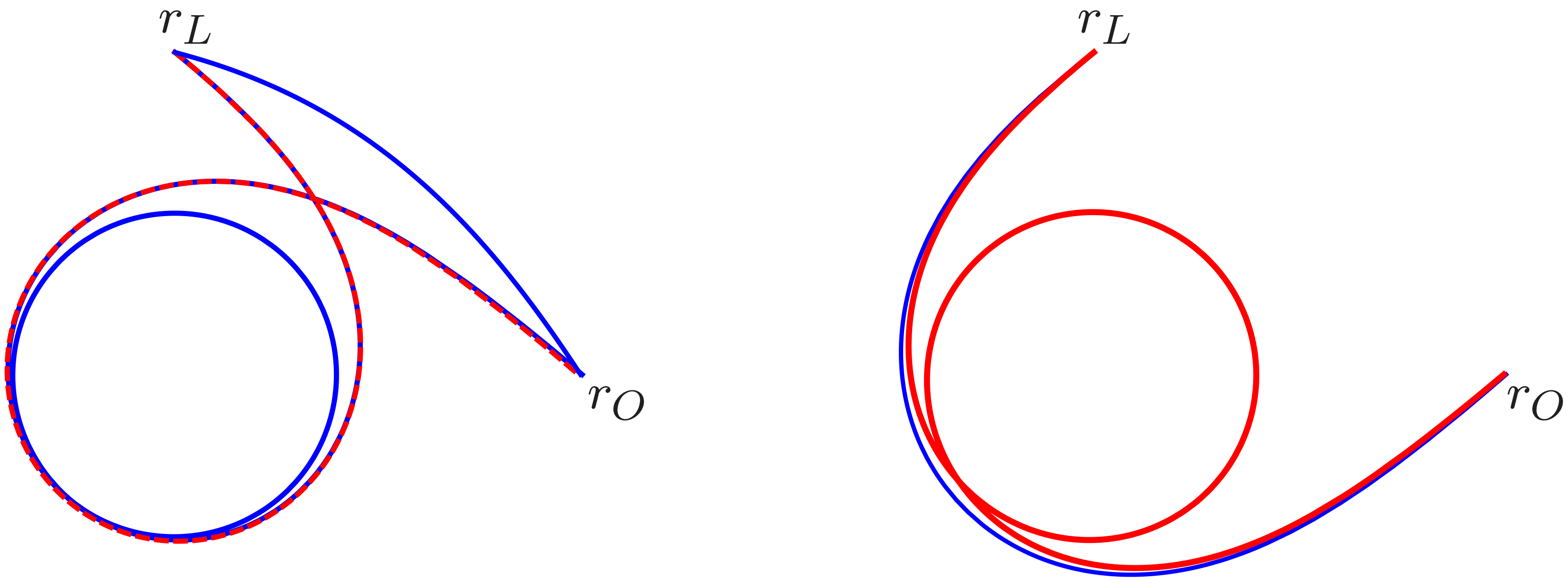
Object at the centre of M87:

Mass $= 3 \times 10^9\, M_\odot$

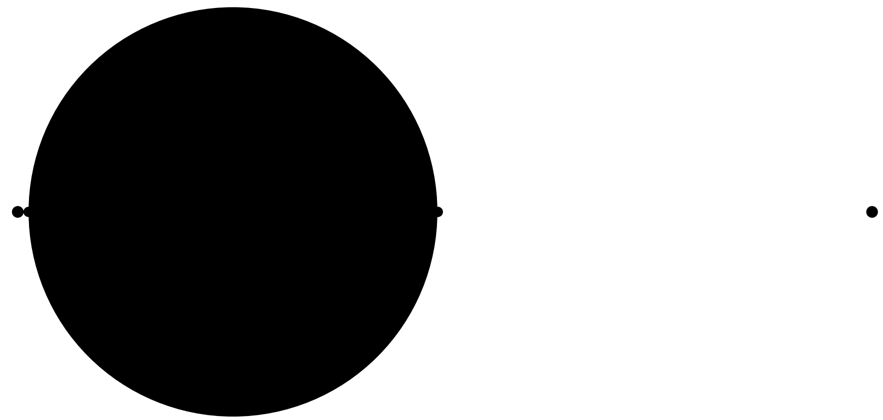
Distance $= 16\,\mathrm{Mpc}$

Synge's formula gives for the diameter of the shadow $\approx 20\mu$as

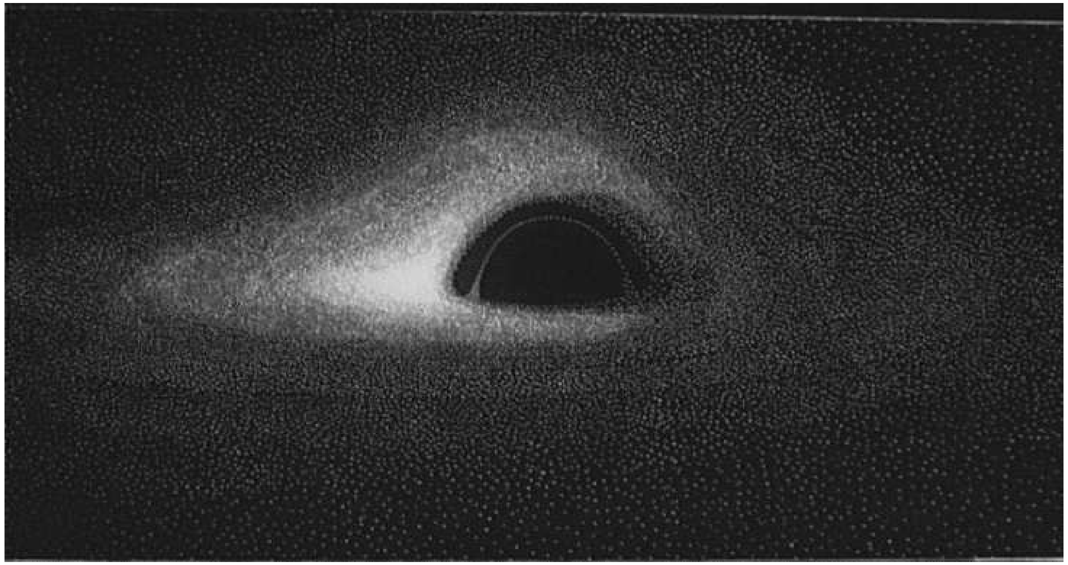
Schwarzschild black hole produces infinitely many images:



Two infinite sequences of images for every light source:

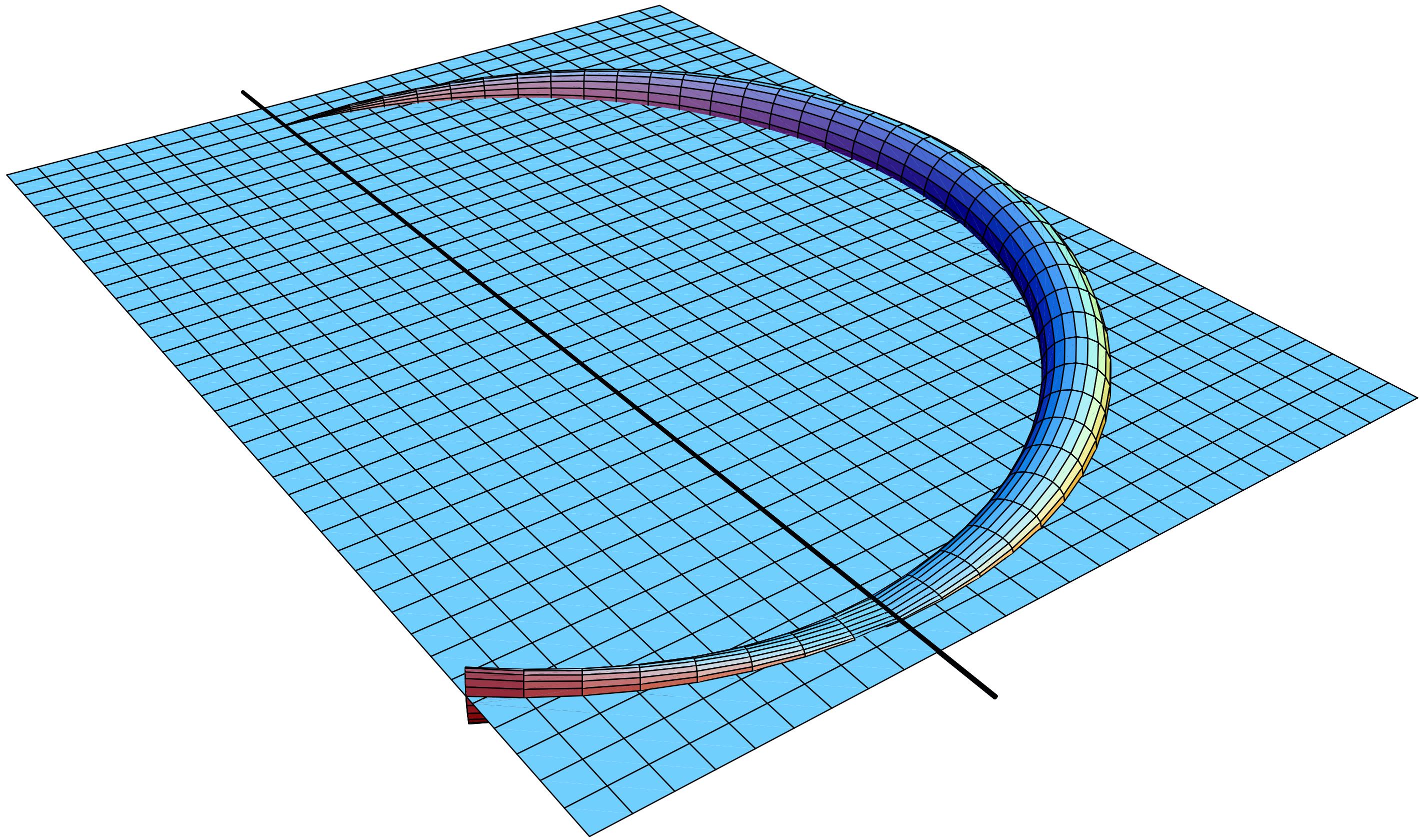


Visual appearance of a Schwarzschild black hole with a thin accretion disc

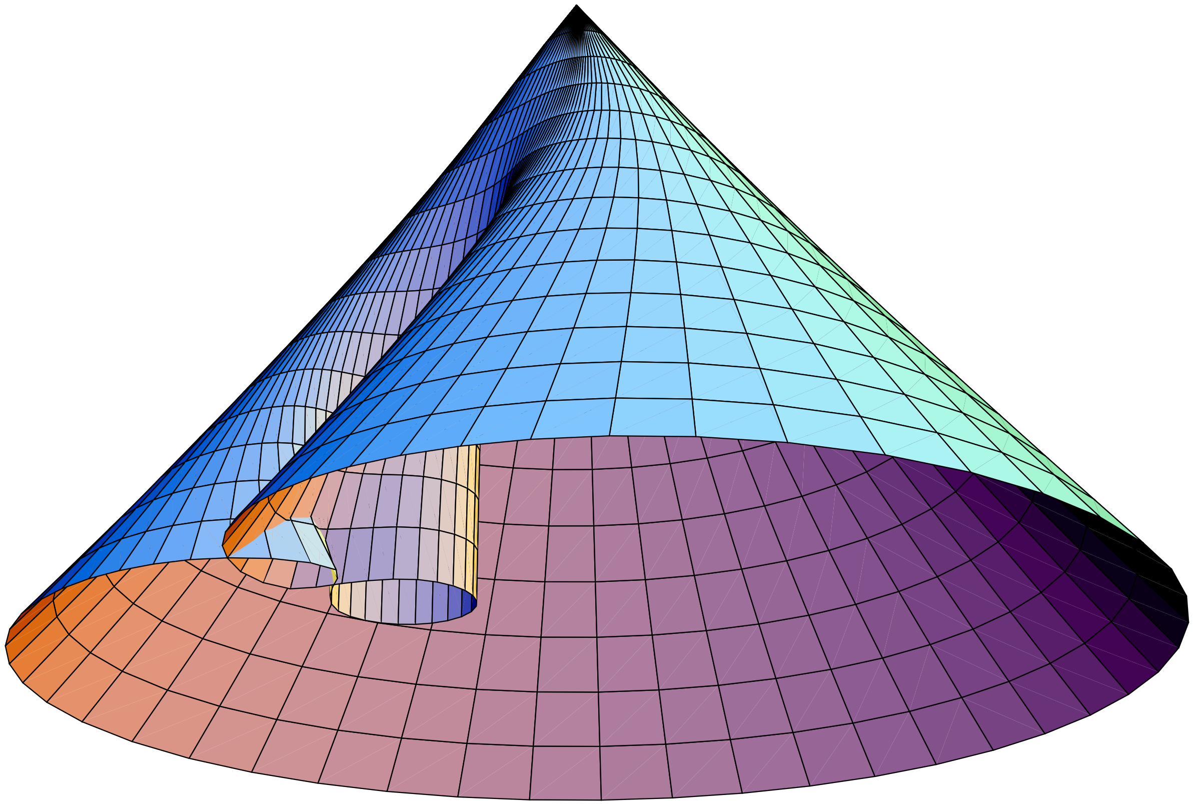


(picture from J.-P. Luminet, Astron. Astrophys. 75, 228 (1979))

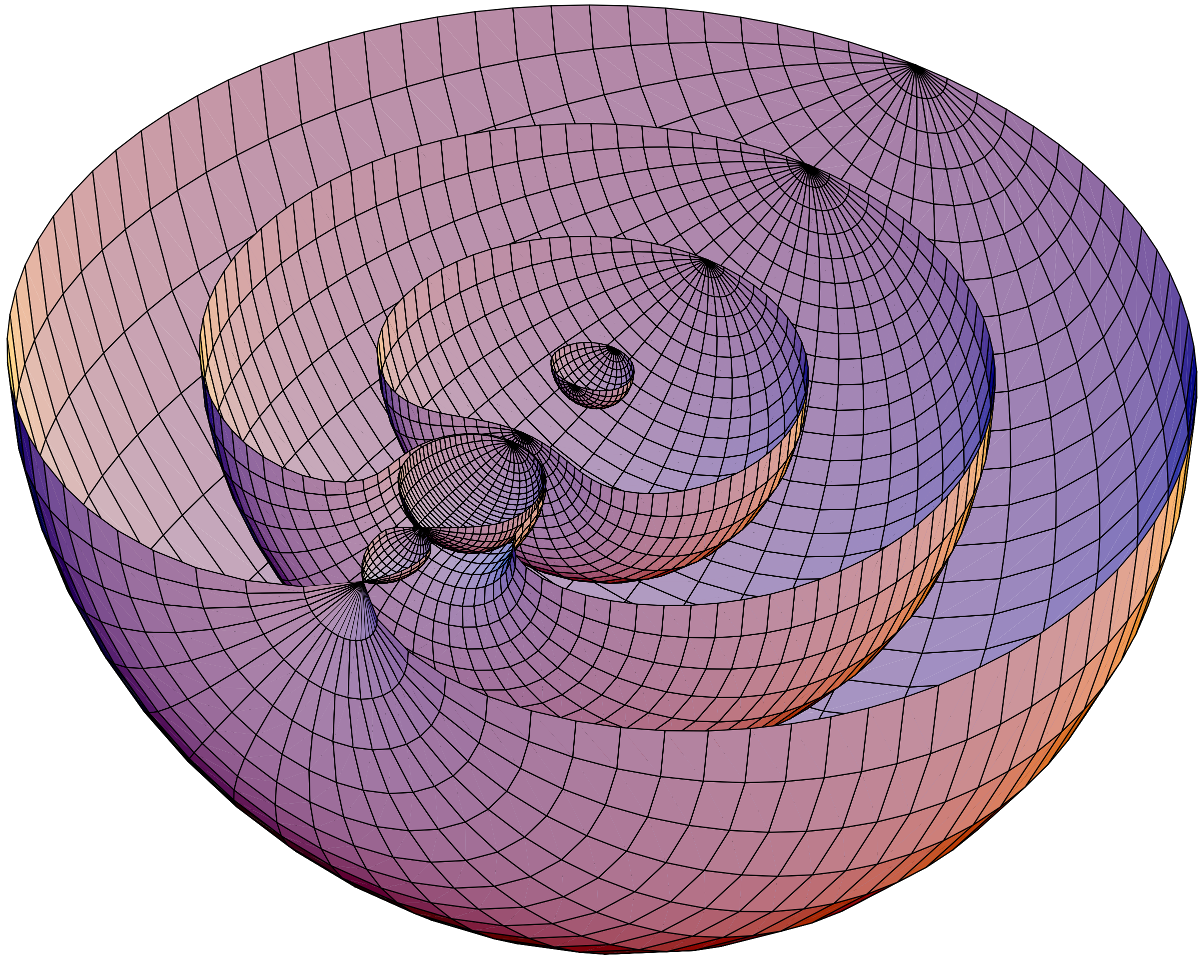
Radial and tangential caustic



Light-cone in Schwarzschild spacetime



Time slices (“wave fronts’) of light-cone:



Other spherically symmetric and static black holes:

- Reissner-Nordström
- Kottler (Schwarzschild-(anti)deSitter)
- Janis-Newman-Winicour
- Newman-Unti-Tamburino (NUT)
- Black holes from nonlinear electrodynamics
- Black holes from higher dimensions, braneworld scenarios, ...

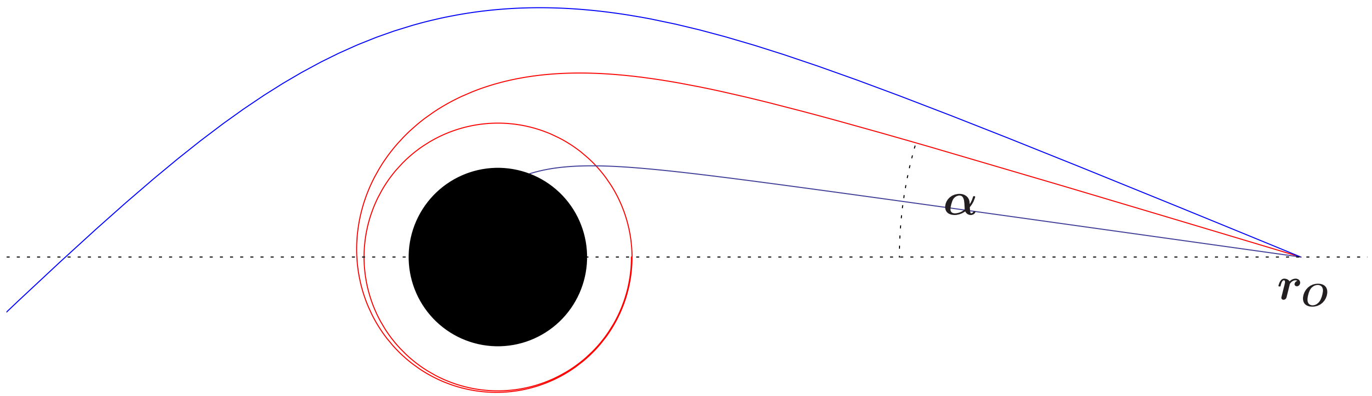
All of them have an unstable photon sphere $\Longrightarrow$ Qualitative lensing features are similar to Schwarzschild

Quantitative features (ratio of angular separations of images, ratio of fluxes of images) are different, see V. Bozza: Phys. Rev. D 66, 103001 (2002)

The shadow is always circular. Its angular radius depends on r_O and the parameters of the black hole.

Black hole impostor: Ultracompact star

Dark star with radius between $2m$ and $3m$



Lensing features indistinguishable from Schwarzschild black hole

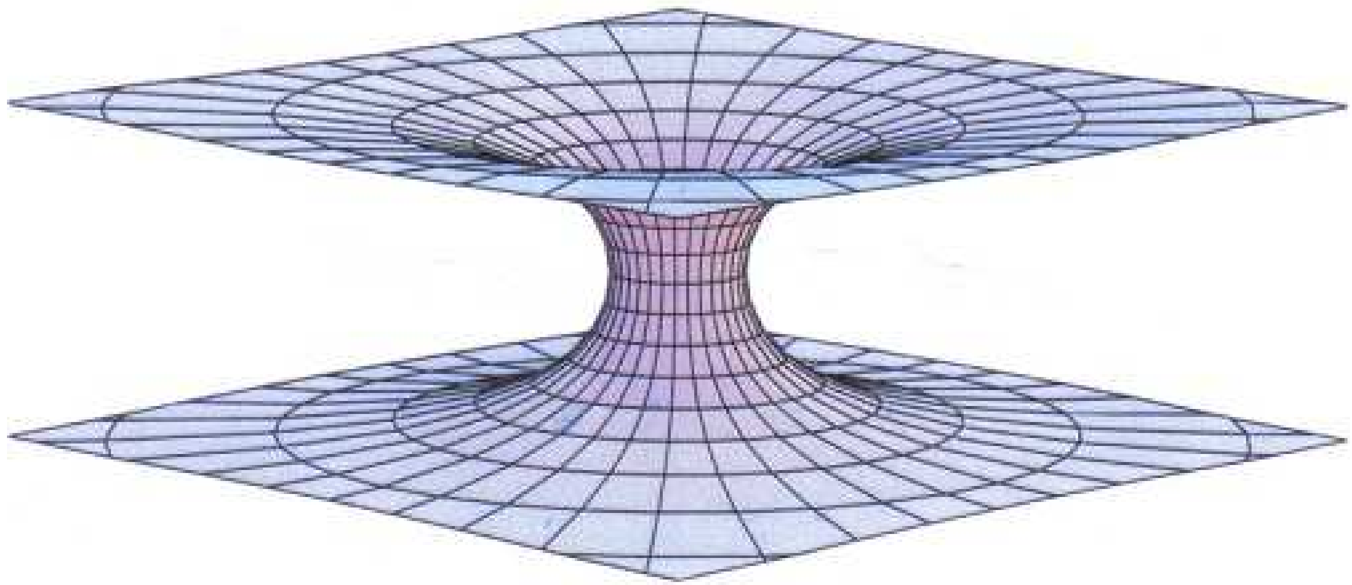
Ultracompact objects are unstable, see

V. Cardoso, L. Crispino, C. Macedo, H. Okawa, P. Pani: Phys. Rev. D 90, 044069 (2014)

Black hole impostor: Ellis wormhole

H. Ellis: J. Math. Phys. 14, 104 (1973)

$$g = -c^2 dt^2 + dr^2 + (r^2 + a^2)\left(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2\right)$$



Angular radius α of shadow: $\sin^2\alpha = \dfrac{a^2}{r_O^2 + a^2}$

Kerr black holes
(More general: Plebański-Demiański black holes)

Shadow no longer circular

Shape of shadow can be used for discriminating between Kerr and other black holes

Shape of the shadow of a Kerr black hole for observer at infinity:

J. Bardeen in C. DeWitt and B. DeWitt (eds.): "Black holes" Gordon & Breach (1973)

cf. S. Chandrasekhar: "The mathematical theory of black holes" Oxford UP (1983)

Shape and size of the shadow for black holes of the Plebański-Demiański class for observer at coordinates (r_O, ϑ_O) (analytical formulas):

A. Grenzebach, VP, C. Lämmerzahl: Phys. Rev. D 89, 124004 (2014), Int. J. Mod. Phys. D 24, 1542024 (2015)

A. Grenzebach: "The shadow of black holes. An analytic description." Springer Briefs in Physics, Springer, Heidelberg (2016)

Kerr metric in Boyer–Lindquist coordinates $(r, \vartheta, \varphi, t)$:

$$g_{\mu\nu}dx^\mu dx^\nu = \varrho(r,\vartheta)^2 \left(\frac{dr^2}{\Delta(r)} + d\vartheta^2\right) + \frac{\sin^2\vartheta}{\varrho(r,\vartheta)^2}\Big(a\,dt - (r^2 + a^2)d\varphi\Big)^2$$

$$- \frac{\Delta(r)}{\varrho(r,\vartheta)^2}\Big(dt - a\sin^2\vartheta\, d\varphi\Big)^2$$

$$\varrho(r,\vartheta)^2 = r^2 + a^2\cos^2\vartheta, \qquad \Delta(r) = r^2 - 2mr + a^2 \,.$$

$m = \dfrac{GM}{c^2}$ where M =mass , $a = \dfrac{J}{Mc}$ where J =spin

Plebański-Demiański black holes: Additional parameters

q_e =el. charge , q_m =magn. charge , ℓ =NUT parameter ,

Λ =cosmol. constant , α =acceleration

Consider in the following only the Kerr metric

Lightlike geodesics:

$$\varrho(r,\vartheta)^2\dot{t} = a\left(L - Ea\,\sin^2\vartheta\right) + \frac{(r^2+a^2)\left((r^2+a^2)E - aL\right)}{\Delta(r)},$$

$$\varrho(r,\vartheta)^2\dot{\varphi} = \frac{L - Ea\,\sin^2\vartheta}{\sin^2\vartheta} + \frac{(r^2+a^2)aE - a^2L}{\Delta(r)},$$

$$\varrho(r,\vartheta)^4\dot{\vartheta}^2 = K - \frac{(L - Ea\,\sin^2\vartheta)^2}{\sin^2\vartheta} =: \Theta(\vartheta),$$

$$\varrho(r,\vartheta)^4\dot{r}^2 = -K\Delta(r) + \left((r^2+a^2)E - aL\right)^2 =: R(r).$$

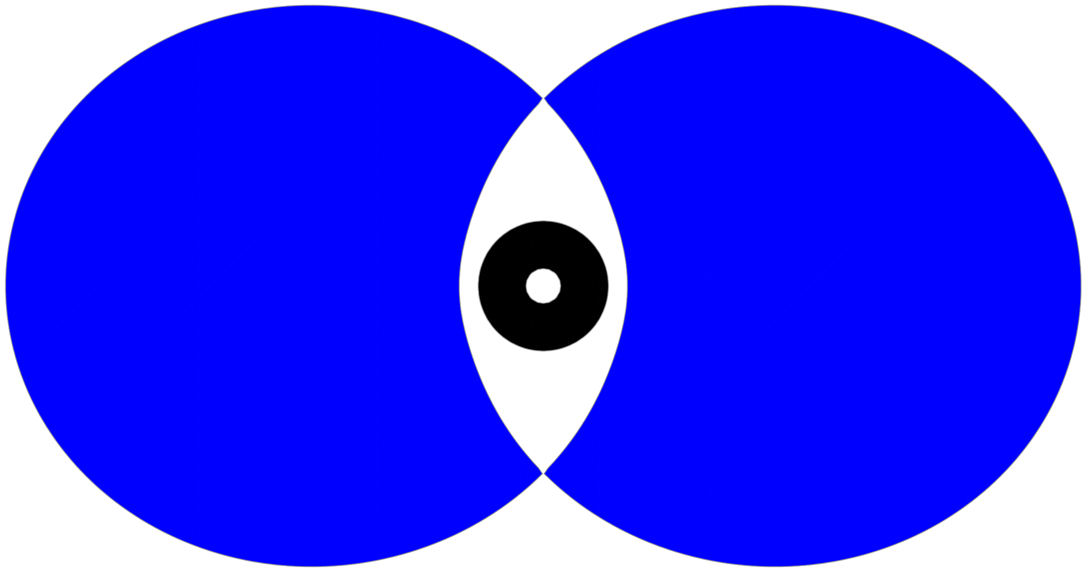
Spherical lightlike geodesics exist in the region where

$$R(r) = 0\,, \quad R'(r) = 0\,, \quad \Theta(\vartheta) \geq 0\,.$$

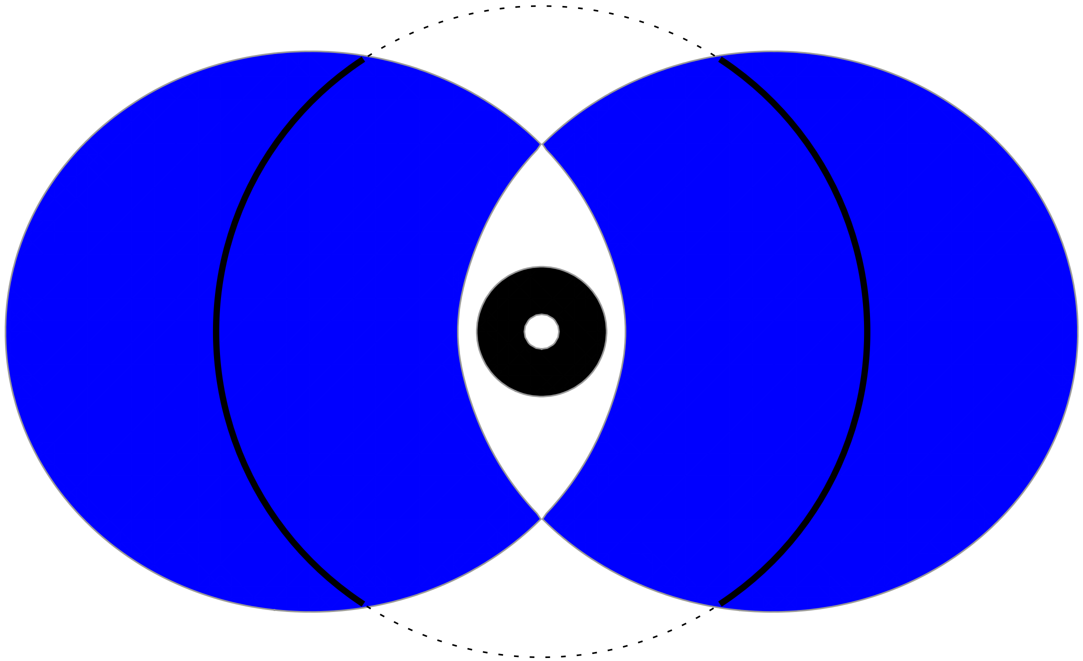
$$\left(2r\Delta(r) - (r-m)\,\varrho(r,\vartheta)^2\right)^2 \leq 4a^2r^2\Delta(r)\,\sin^2\vartheta$$

(unstable if $R''(r) \geq 0$)

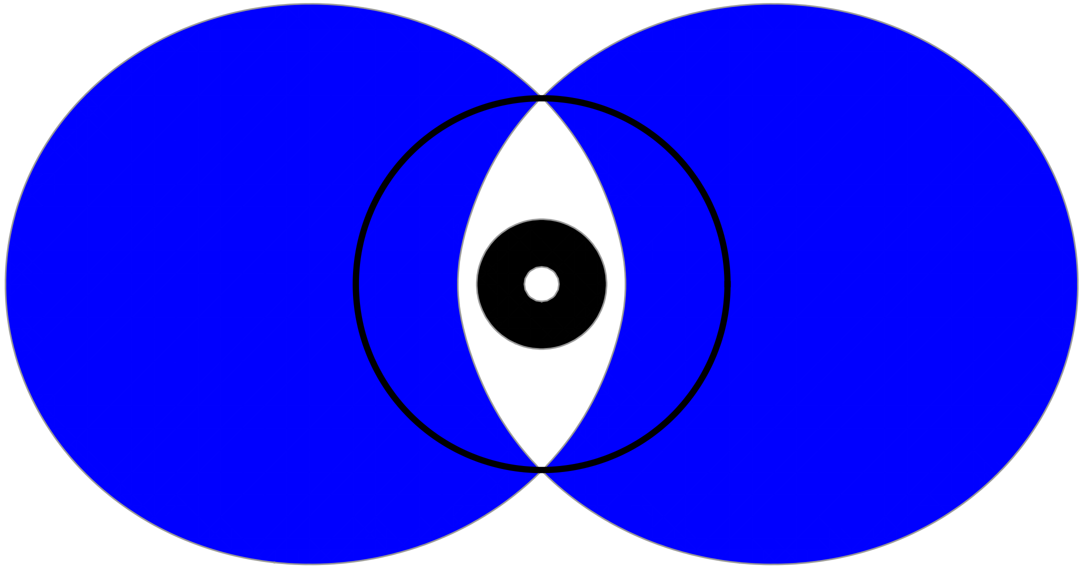
Photon region for Kerr black hole with $a = 0.75\,m$



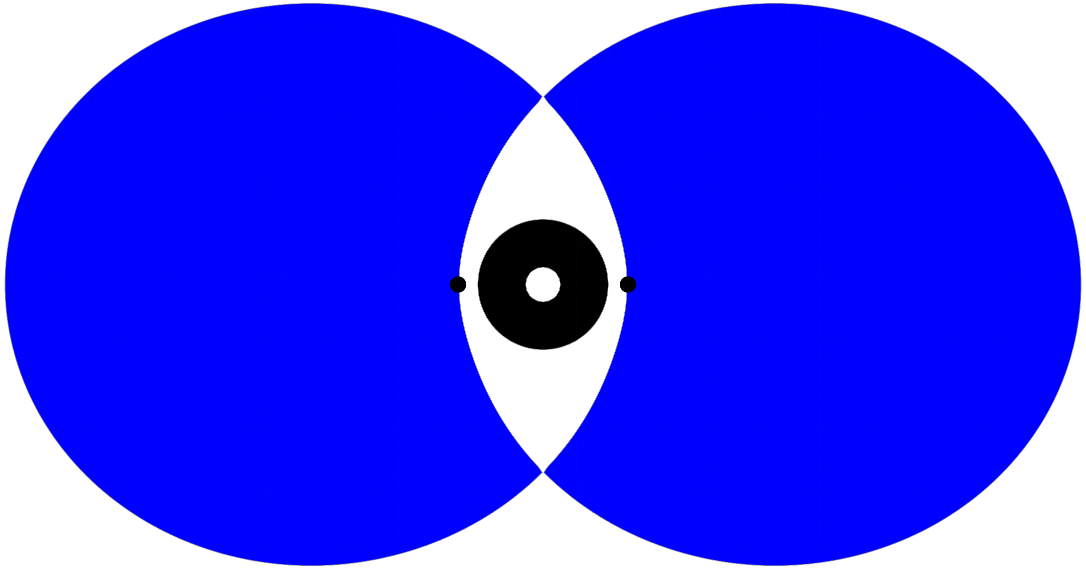
Photon region for Kerr black hole with $a = 0.75\,m$



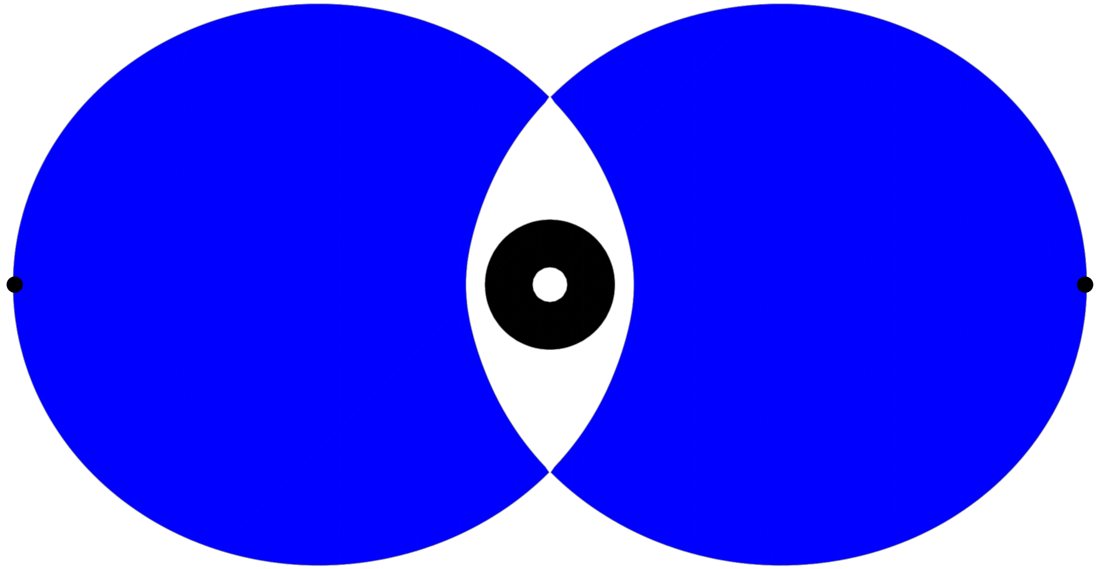
Photon region for Kerr black hole with $a = 0.75\,m$



Photon region for Kerr black hole with $a = 0.75\,m$



Photon region for Kerr black hole with $a = 0.75\, m$



The shadow is determined by light rays that approach an unstable spherical lightlike geodesic.

Choose observer at r_O and ϑ_O

Choose tetrad

$$e_0 = \frac{(r^2+a^2)\partial_t + a\partial_\varphi}{\varrho(r,\vartheta)\sqrt{\Delta(r)}}\Big|_{(r_O,\vartheta_O)}$$

$$e_1 = \frac{1}{\varrho(r,\vartheta)}\,\partial_\vartheta\Big|_{(r_O,\vartheta_O)}$$

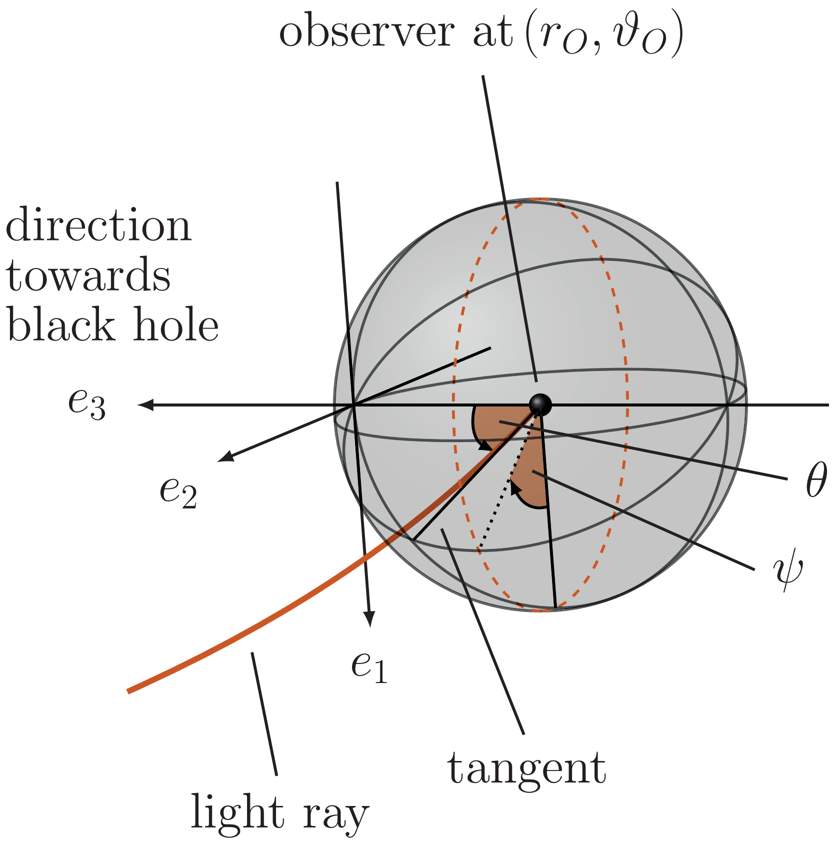
$$e_2 = -\frac{(\partial_\varphi + a\sin^2\!\vartheta\,\partial_t)}{\sqrt{\varrho(r,\vartheta)^2}\sin\vartheta}\Big|_{(r_O,\vartheta_O)}$$

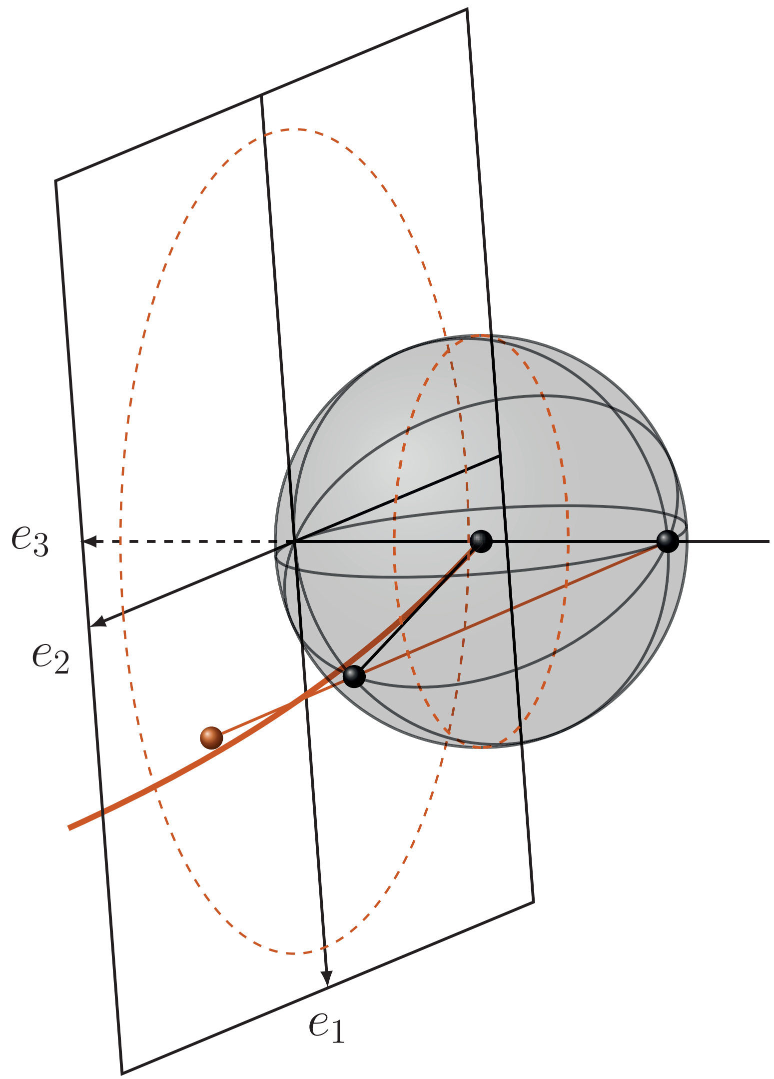
$$e_3 = -\frac{\sqrt{\Delta(r)}}{\varrho(r,\vartheta)}\,\partial_r\Big|_{(r_O,\vartheta_O)}$$

Observer with other 4-velocity: Aberration

A. Grenzebach, in D. Puetzfeld, C. Laemmerzahl, B.Schutz (eds.): “Equations of motion in relativistic gravity” Springer (2015)

celestial coordinates at observer (θ, ψ)





constants of motion $\left(K_E = \frac{K}{E^2}, L_E = \frac{L}{E} - a \right)$

$$\sin\theta = \left. \frac{\sqrt{\Delta(r)\ K_E}}{r^2 - aL_E} \right|_{r=r_O}, \qquad \sin\psi = \left. \frac{L_E + a\cos^2\vartheta + 2\ell\cos\vartheta}{\sqrt{K_E}\ \sin\vartheta} \right|_{\vartheta=\vartheta_O}$$

$$K_E = \left. \frac{16r^2\Delta(r)}{(\Delta'(r))^2} \right|_{r=r_p}, \qquad aL_E = \left. \left(r^2 - \frac{4r\Delta(r)}{\Delta'(r)} \right) \right|_{r=r_p}$$

Analytic formula for shadow allows to extract parameters of the spacetime from the shape of the shadow

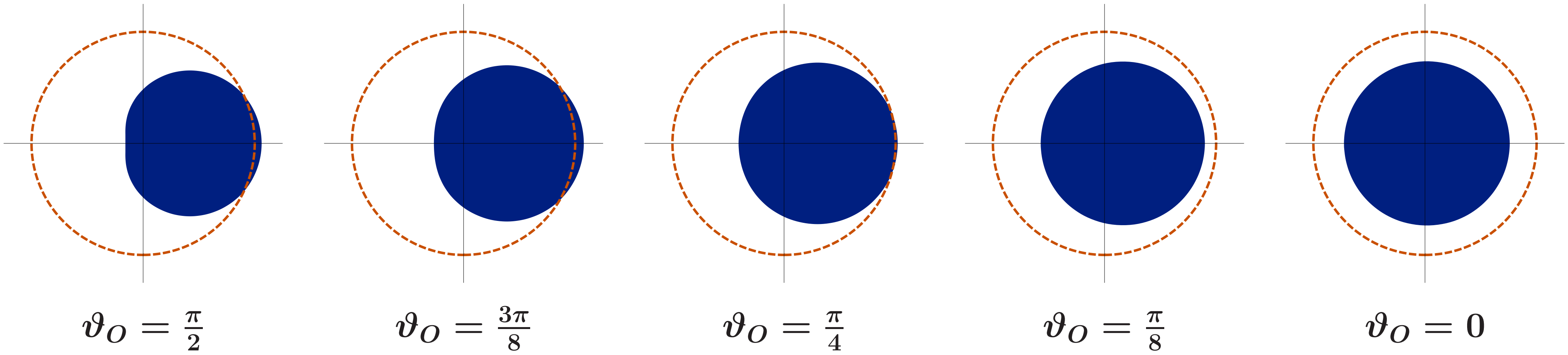
Vertical angular radius α_v of the shadow ($\vartheta = \pi/2$)

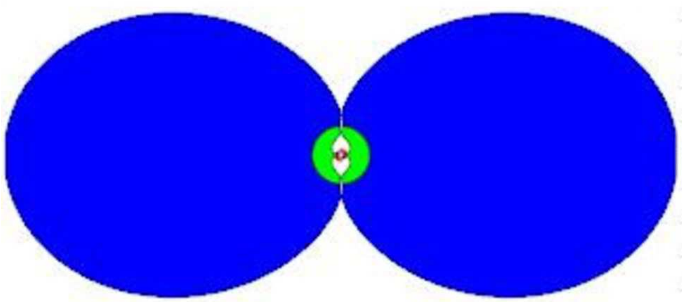
$$\sin^2\alpha_v = \frac{27m^2r_O^2\big(a^2 + r_O(r_O - 2m)\big)}{r_O^6 + 6a^2r_O^4 + 3a^2(4a^2 - 9m^2)r_O^2 + 8a^6} = \frac{27m^2}{r_O^2}\Big(1 + O(m/r_O)\Big)$$

A. Grenzebach, VP, C. Lämmerzahl: Int. J. Mod. Phys. D 24, 1542024 (2015)

Up to terms of order $O(m/r_O)$, Synge's formula is still correct for the vertical diameter of the shadow

Shadow of black hole with $a = m$ for observer at $r_O = 5m$

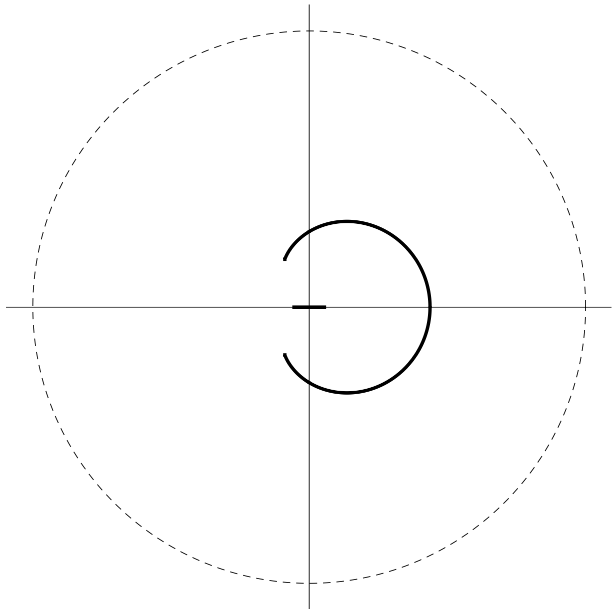




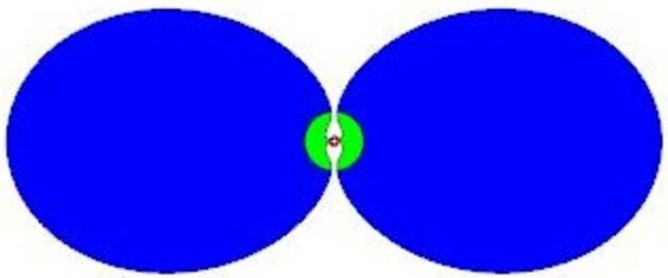
$a = 1.1\,m$

$r_O = 6\,m$

$\vartheta_O = \pi/2$



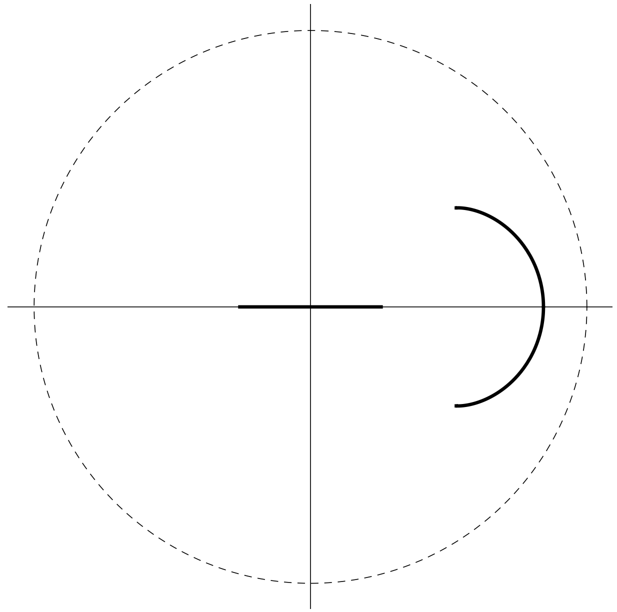
Cf. A. deVries: Class. Quantum Grav. 17, 123 (2000)



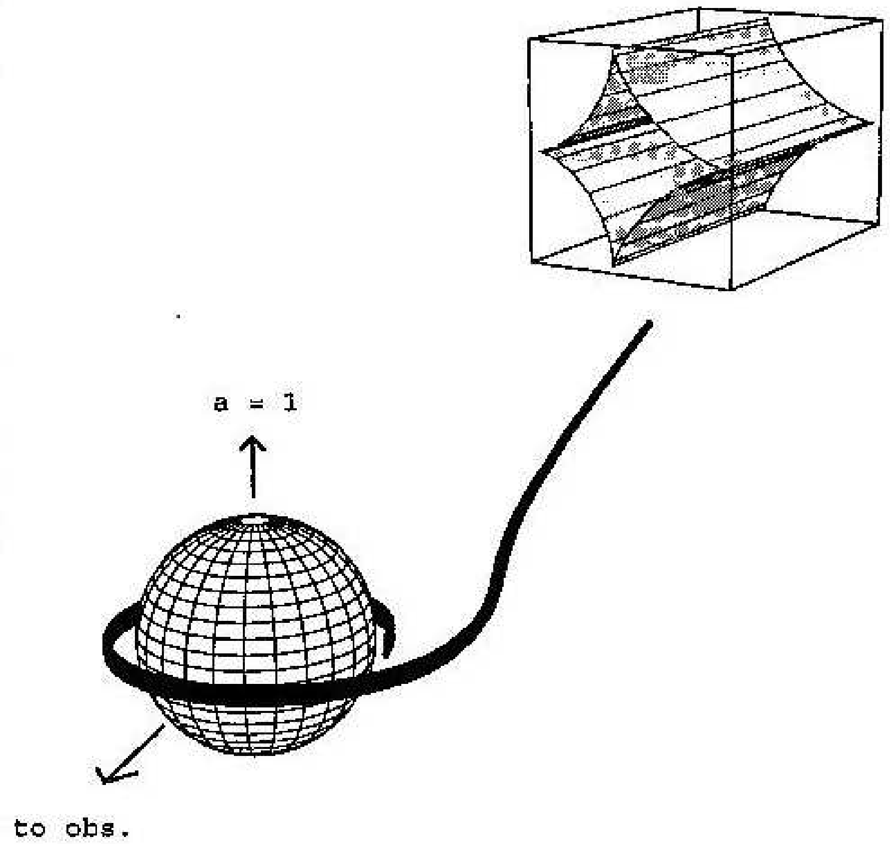
$a = 2\,m$

$r_O = 8\,m$

$\vartheta_O = \pi/2$



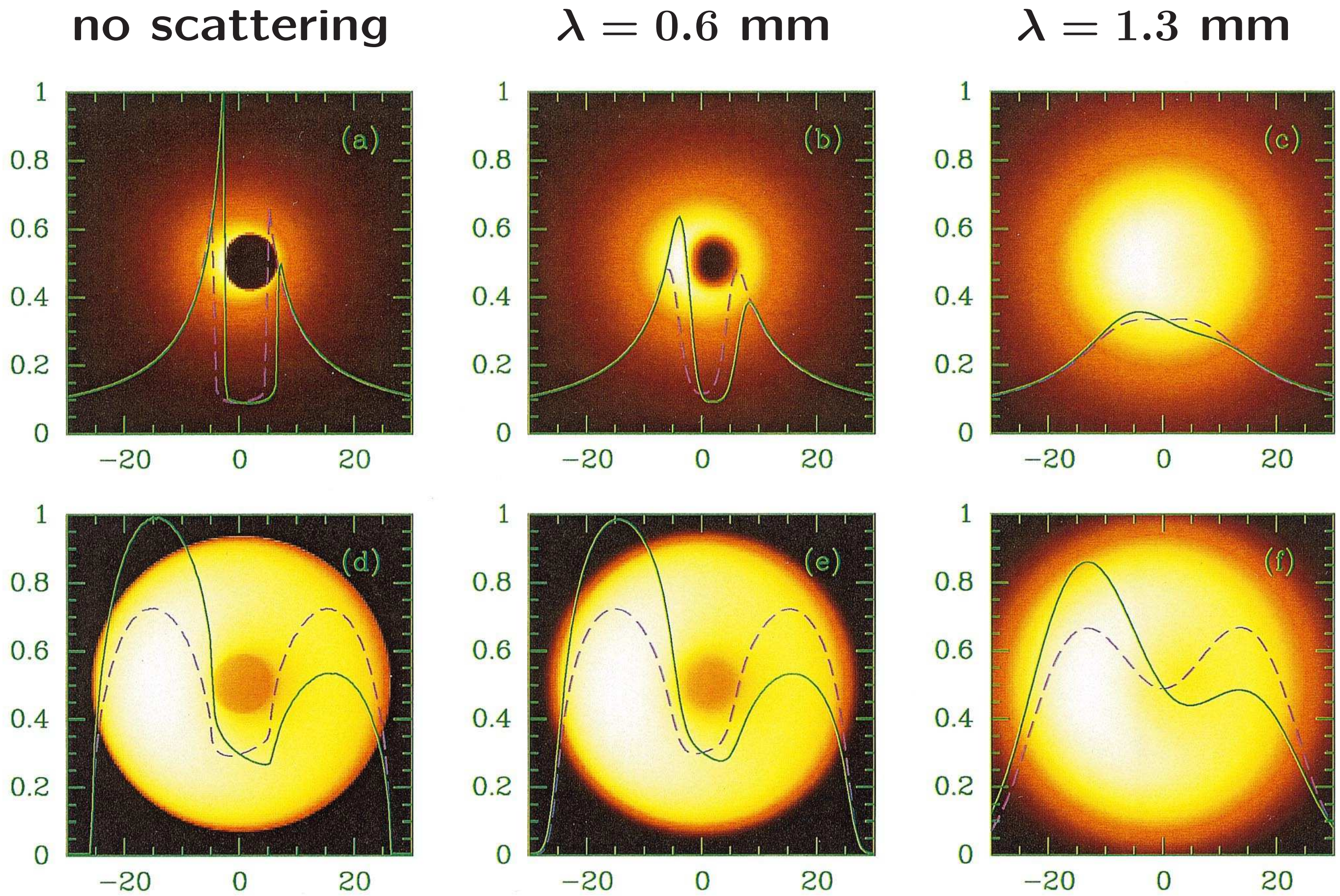
Caustics in Kerr spacetime



R.Blandford, K.Rauch: Astrophys. J. 421, 46 (1994)

V.Bozza: Phys. Rev. D 78, 063014 (2008)

Kerr shadow with emission region and scattering taken into account:



H. Falcke, F. Melia, E. Agol: Astrophys. J. 528, L13 (2000)

Observations should be done at (sub-)millimeter wavelengths

Influence of a plasma on light rays

Hamilton formalism for light rays in a pressureless non-magnetised plasma:

$$\dot{x}^\mu = \frac{\partial H(x,p)}{\partial p_\mu}\,, \qquad \dot{p}_\mu = -\frac{\partial H(x,p)}{\partial x^\mu}\,, \qquad H(x,p) = 0$$

$$H(x,p) \; = \; \frac{1}{2}\Big(g^{\mu\nu}(x) p_\mu p_\nu + \omega_p(x)^2 \Big)\,,$$

plasma frequency: $\omega_p(x)^2 = \frac{e^2}{\varepsilon_0 m_e} N(x)$

e: charge of the electron, m_e: mass of the electron

$N(x)$: number density of the electrons

Rigourous derivation from Maxwell's equation, even for magnetised pressure-free plasma:

R. Breuer, J. Ehlers: Proc. Roy. Soc. London, A 370, 389 (1980), A 374, 65 (1981)

for non-magnetised pressure-free plasma:

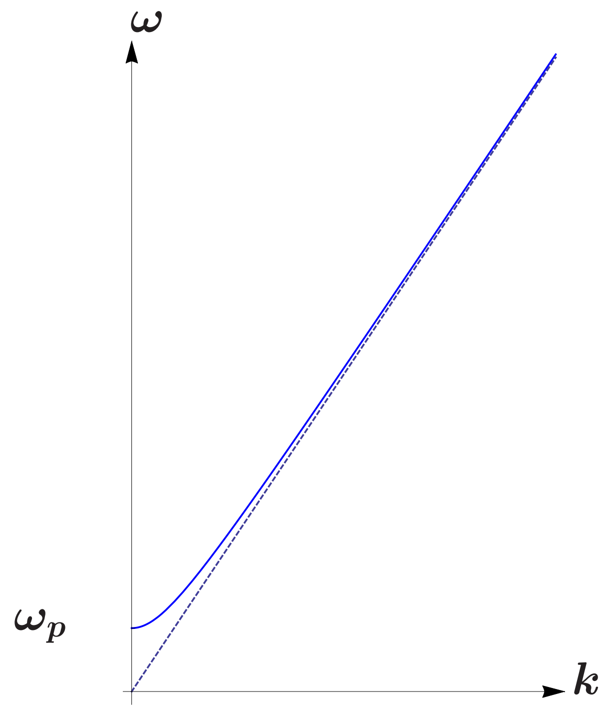
VP: "Ray Optics, Fermat's Principle and Applications to General Relativity" Springer (2000)

A plasma is a dispersive medium; propagation of light rays depend on the frequency:

$$p^{\mu} = -\frac{1}{c}\,\omega(x)\,U^{\mu}(x) + k^{\mu}(x)\,,$$

Dispersion relation ($H = 0$)

$$\omega = \sqrt{\omega_p^2 + c^2 k^2}$$



For a cold non-magnetised plasma, only the plasma frequency matters, not the 4-velocity of the electrons

Spherically symmetric and static case

- Bending angle in Schwarzschild with plasma:

  In the weak-field approximation:

  D. O. Muhleman and I. D. Johnston: Phys. Rev. Lett. 17, 455 (1966)

  Exact formula:

  VP: “Ray optics, Fermat’s principle and applications to general relativity” Springer (2000)

  G. S. Bisnovatyi-Kogan and O. Yu. Tsupko: Gravitation and Cosmology, 15, 20 (2009), Mon. Not. Roy. Astr. Soc. 404, 1790 (2010), Phys. Rev. D 87, 124009 (2013)

- Effect of a plasma on the shadow (talk by Oleg Tsupko):

  VP, O. Yu. Tsupko, G. S. Bisnovatyi-Kogan: Phys. Rev. D 92, 104031 (2015)

  Examples: Schwarzschild, Ellis wormhole

Influence of a plasma on the Kerr shadow

VP, O. Yu. Tsupko: Phys. Rev. D, to appear (2017)

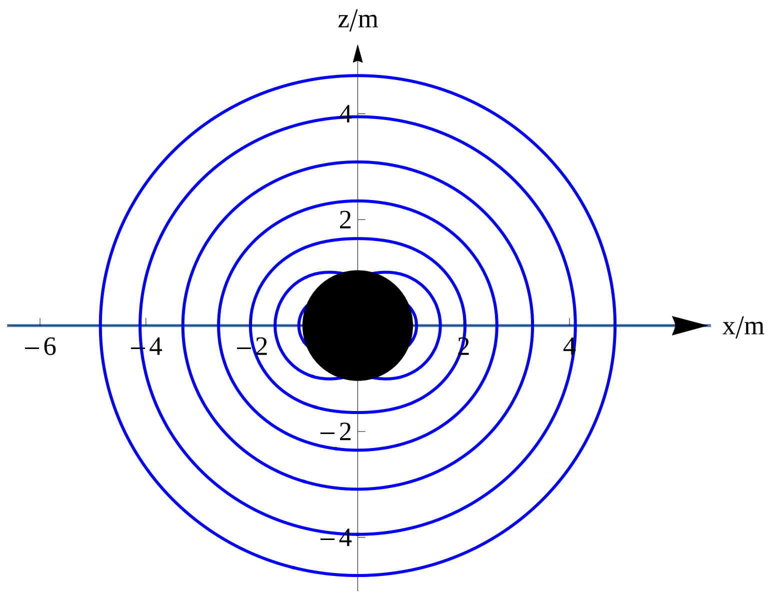
Condition for separability:

$$\omega_p(r,\vartheta)^2 = \frac{f_r(r) + f_\vartheta(\vartheta)}{r^2 + a^2\cos^2\vartheta}$$

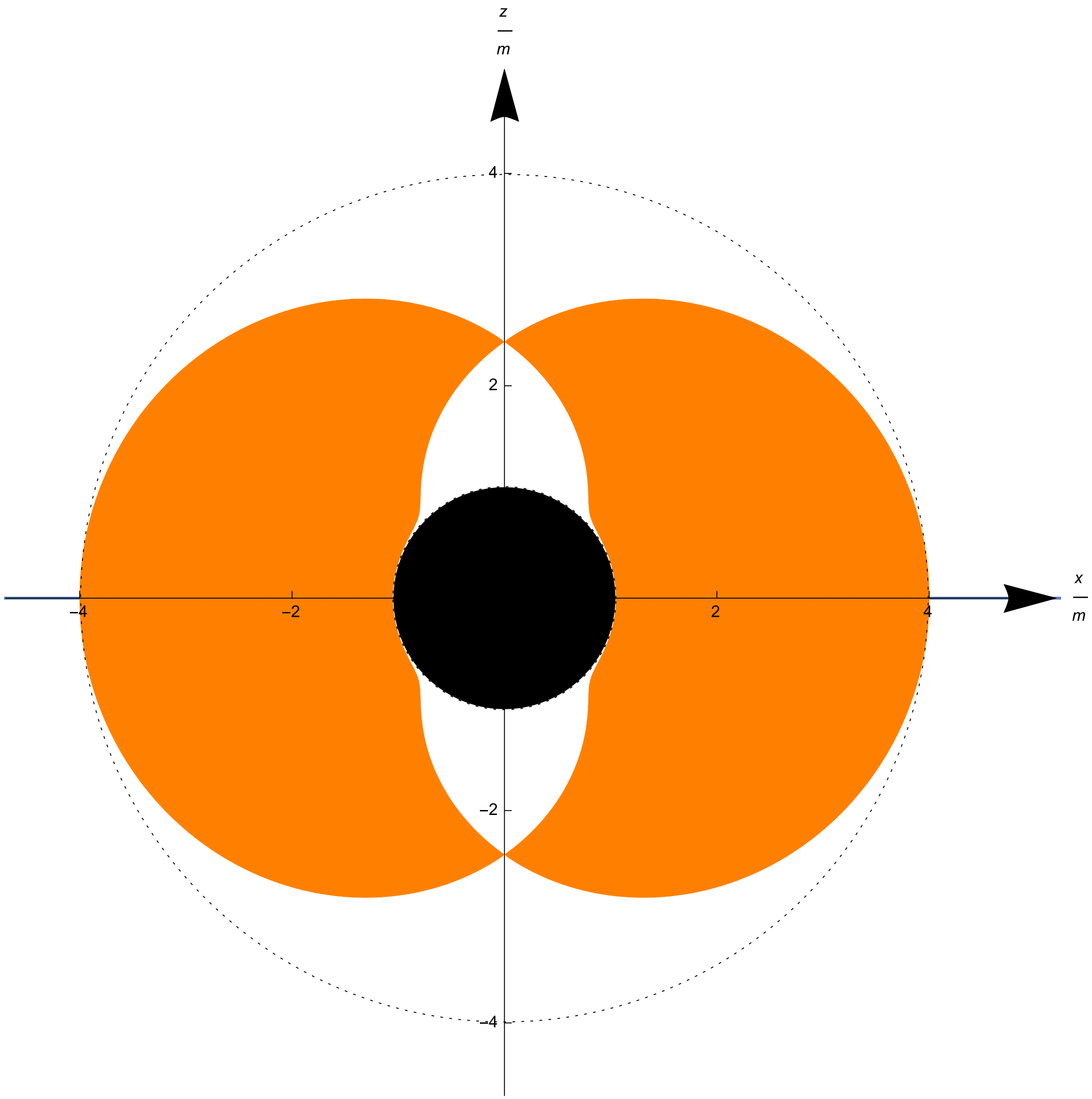
Necessary for analytical construction of the shadow

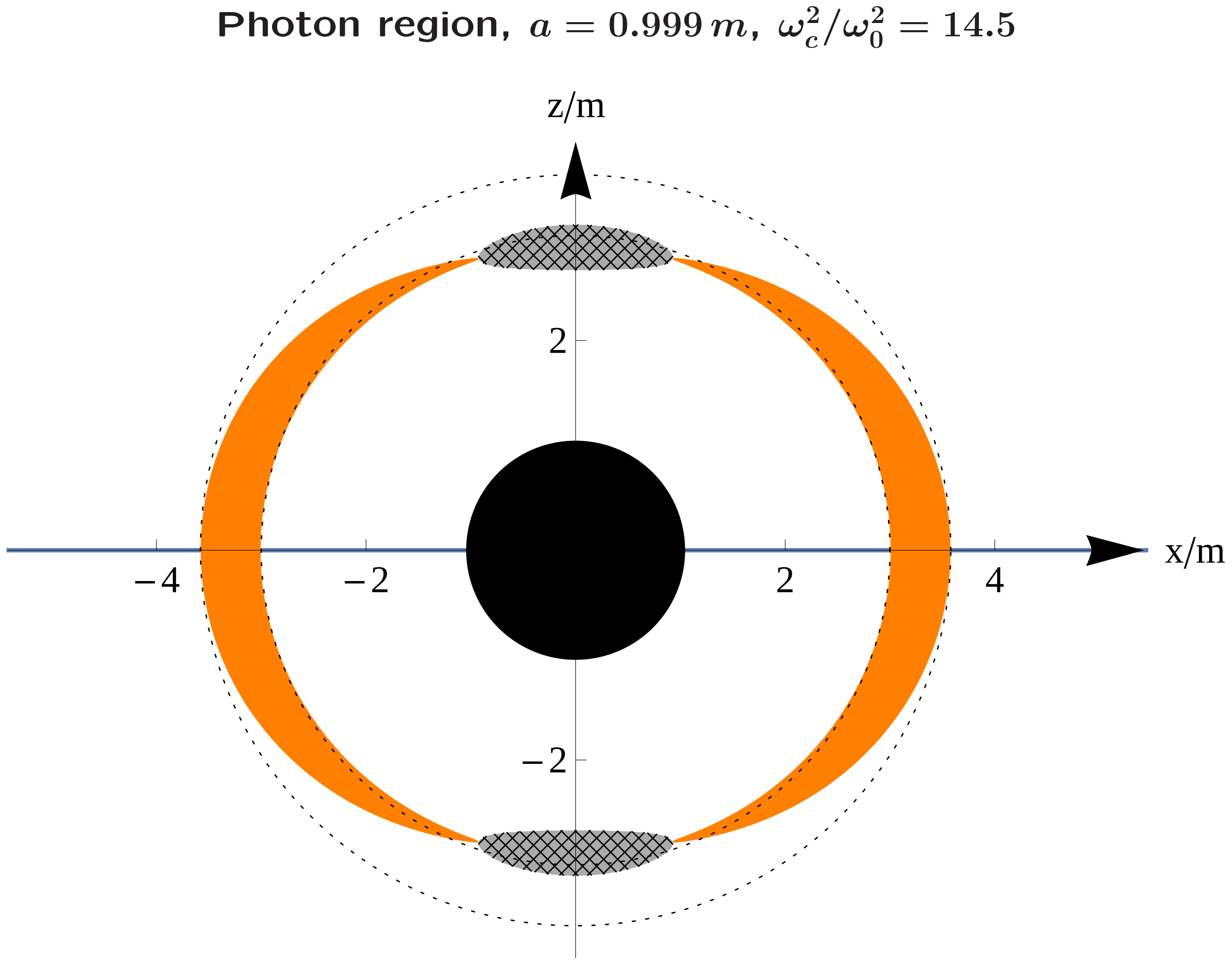
Otherwise a Carter constant does not exist

Example: $\omega_p(r,\vartheta)^2 = \frac{\omega_c^2\sqrt{m^3 r}}{r^2+a^2\cos^2\vartheta}$

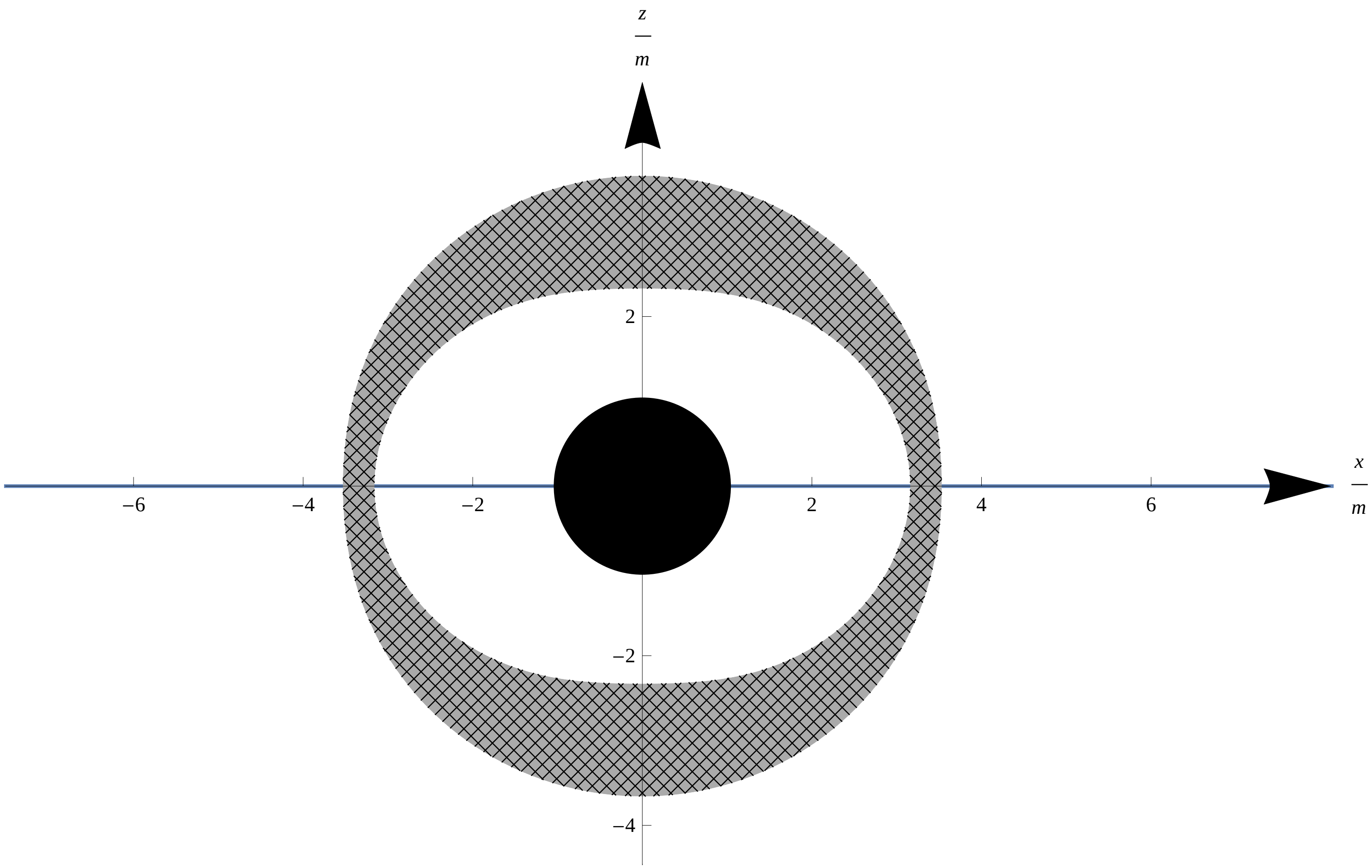


Photon region, $a = 0.999\,m$, $\omega_c^2/\omega_0^2 = 0$

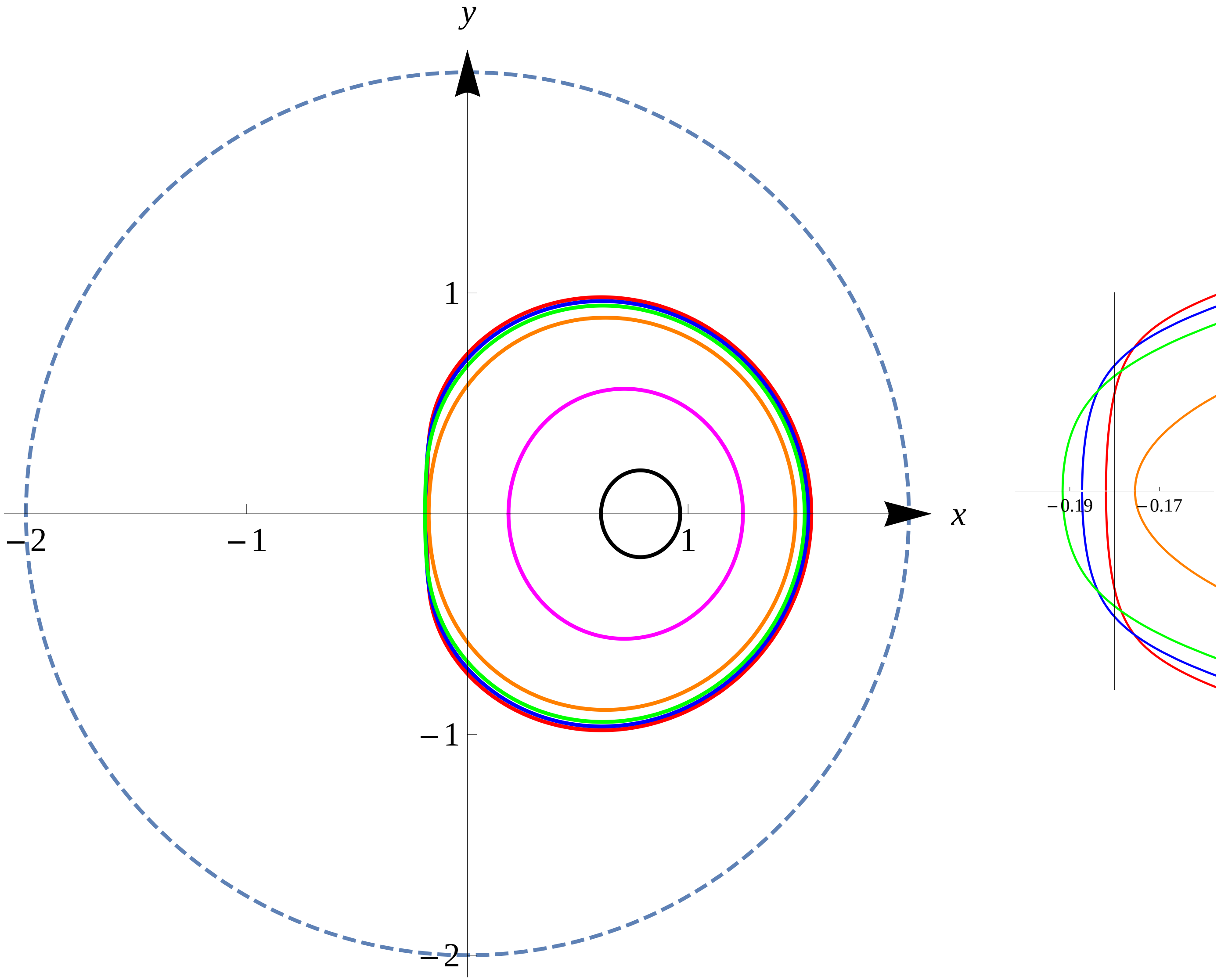



Photon region, $a = 0.999\,m$, $\omega_c^2/\omega_0^2 = 14.5$
z/m
x/m
2
−2
−4
−2
2
4

Photon region, $a = 0.999\,m$, $\omega_c^2/\omega_0^2 = 15.3$



Shadow, $a = 0.999\,m, r_O = 5\,m\,,\ \vartheta_O = \pi/2$



Shadow shrinks with increasing ω_c^2/ω_0^2

Other example: $\omega_p(r,\vartheta) = \omega_c = \mathbf{constant}$

Photon region, $a = 0.999\,m$, $\omega_c^2/\omega_0^2 = 1.085$