

Black Holes in Higher Dimensions

Jutta Kunz



641. WE-Heraeus-Seminar

“Do Black Holes Exist? The Physics and Philosophy of Black Holes”

Bad Honnef

April 2017

Outline

1 Introduction

2 Vacuum Black Objects

- Black Holes and Black Rings in $D = 5$
- Black Holes and Black Rings in $D \geq 6$
- Black Holes and Black Ringoids in $D \geq 7$

3 Adding Charge

4 Conclusions and Outlook

Outline

1 Introduction

2 Vacuum Black Objects

- Black Holes and Black Rings in $D = 5$
- Black Holes and Black Rings in $D \geq 6$
- Black Holes and Black Ringoids in $D \geq 7$

3 Adding Charge

4 Conclusions and Outlook

Outline

1 Introduction

2 Vacuum Black Objects

- Black Holes and Black Rings in $D = 5$
- Black Holes and Black Rings in $D \geq 6$
- Black Holes and Black Ringoids in $D \geq 7$

3 Adding Charge

4 Conclusions and Outlook

Outline

1 Introduction

2 Vacuum Black Objects

- Black Holes and Black Rings in $D = 5$
- Black Holes and Black Rings in $D \geq 6$
- Black Holes and Black Ringoids in $D \geq 7$

3 Adding Charge

4 Conclusions and Outlook

Outline

1 Introduction

2 Vacuum Black Objects

- Black Holes and Black Rings in $D = 5$
- Black Holes and Black Rings in $D \geq 6$
- Black Holes and Black Ringoids in $D \geq 7$

3 Adding Charge

4 Conclusions and Outlook

Einstein–Maxwell Black Holes: $D = 4$

static spherically symmetric	Schwarzschild Reissner-Nordström	M M, Q
rotating axially symmetric	Kerr Kerr–Newman	M, J M, J, Q

- uniqueness

black holes are uniquely determined by their mass M , angular momentum J , charges Q and P

- horizon topology

black hole horizons have spherical topology

- staticity

stationary black holes with non-rotating horizon are static

- ...

Generalization of $D = 4$ Black Holes: $D > 4$

Myers and Perry, Ann. Phys. (N.Y.) 172 (1986) 304

	static	rotating
$D = 4$	Schwarzschild (M)	Kerr (M, J)
$D > 4$	Tangherlini (M)	Myers-Perry (M, J_1, \dots, J_N)



Myers-Perry Black Holes

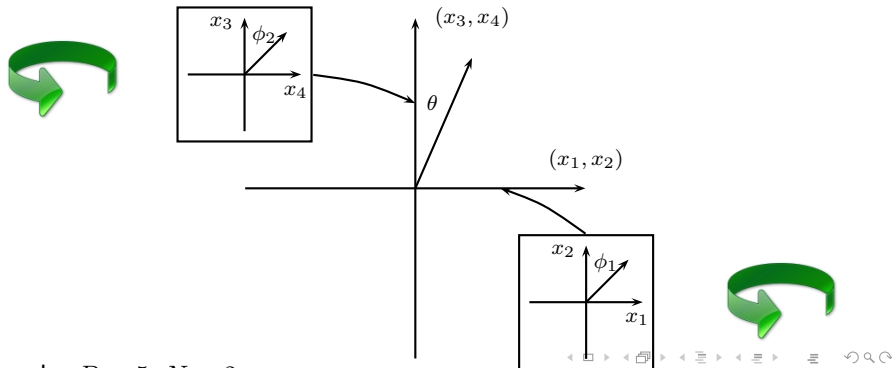
Myers and Perry 1986

D : dimension of space-time

N : number of independent angular momenta J_i :

$$N \equiv \left\lfloor \frac{D-1}{2} \right\rfloor$$

N : number of independent planes



Myers-Perry Black Holes

metric

$$ds_{D,MP}^2 = -dt^2 + \frac{\Pi F}{\Pi - mr^{2-\varepsilon}} dr^2 + \sum_{i=1}^N (r^2 + a_i^2) (d\mu_i^2 + \mu_i^2 d\varphi_i^2) + \frac{mr^{2-\varepsilon}}{\Pi F} \left(dt - \sum_{i=1}^N a_i \mu_i^2 d\varphi_i \right)^2 + \varepsilon r^2 d\nu^2$$

$$F \equiv 1 - \sum_{i=1}^N \frac{a_i^2 \mu_i^2}{r^2 + a_i^2}, \quad \Pi = \prod_{i=1}^N (r^2 + a_i^2)$$

constraint

$$\sum_{i=1}^N \mu_i^2 + \varepsilon \nu^2 = 1$$

coordinate ν enters only in even dimensions:

$$\text{odd } D: \quad \varepsilon = 0$$

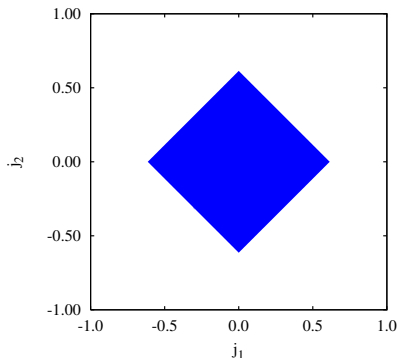
$$\text{even } D: \quad \varepsilon = 1$$

mass M and angular momenta J_i :

$$M = m (1 + (D - 3)) A(S^{D-2})$$

$$J_i = 2m a_i A(S^{D-2}), \quad i = 1, \dots, N$$

Myers-Perry Black Holes: Domain of Existence



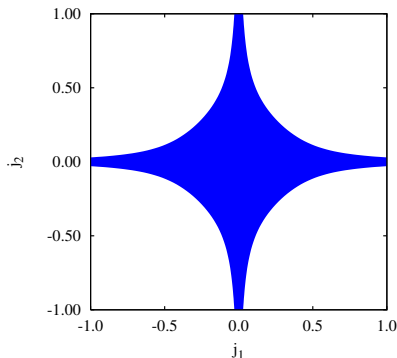
scaled angular momenta

$$j_1 = J_1/M^{(D-2)/(D-3)}$$

$$j_2 = J_2/M^{(D-2)/(D-3)}$$

- $D = 5$:
 - domain of existence is **bounded**

Myers-Perry Black Holes: Domain of Existence



scaled angular momenta

$$j_1 = J_1 / M^{(D-2)/(D-3)}$$

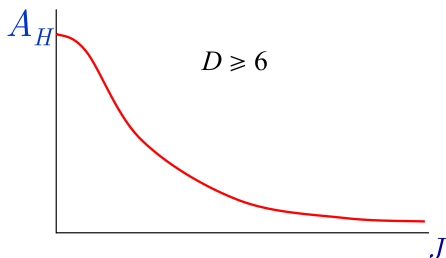
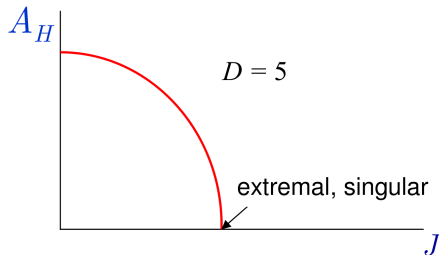
$$j_2 = J_2 / M^{(D-2)/(D-3)}$$

- $D = 5$:
 - domain of existence is **bounded**

- $D = 6$:
 - domain of existence is **unbounded on axes**:
for $J_1 = J, J_2 = 0$
for $J_1 = 0, J_2 = J$

Myers-Perry Black Holes: Domain of Existence

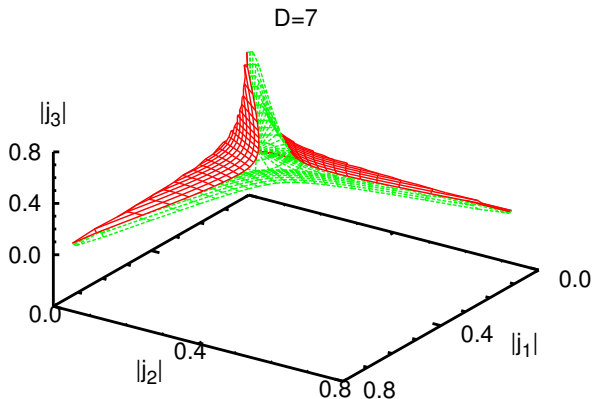
single angular momentum $J_1 = J$ ($J_i = 0, i > 1$)



scaled horizon area A_H versus scaled angular momentum J

- $D = 5$: $J_1 = J, J_2 = 0$: domain bounded, singular extremal limit
- $D \geq 6$: $J_1 = J, J_i = 0, i > 1$: domain unbounded

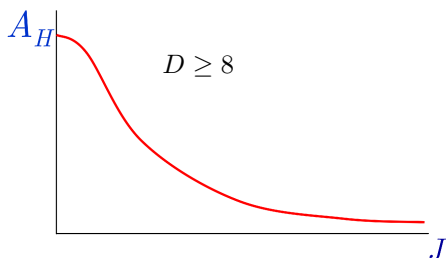
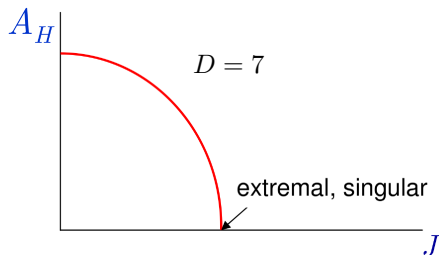
Myers-Perry Black Holes: Domain of Existence



- $J_1 = J_2 = J, J_3 = 0$: domain bounded, singular extremal limit
- $J_1 = J_2 = 0, J_3 = J$: domain unbounded

Myers-Perry Black Holes: Domain of Existence

two equal magnitude angular momenta $J_1 = J_2 = J$ ($J_i = 0, i > 2$)



scaled horizon area A_H versus scaled angular momentum J

- $D = 7$: $J_1 = J_2 = J, J_3 = 0$: domain bounded, singular extremal limit
- $D \geq 8$: $J_1 = J_2 = J, J_i = 0, i > 2$: domain unbounded

Black Rings and Further Beasts?

Myers and Perry, *Ann. Phys. (N.Y.)* 172 (1986) 304

'construction' of black rings:

1. take a piece of black string



2. bend it



3. glue endpoints



Black Rings and Further Beasts?



e.g. Black Ringoids

Outline

1 Introduction

2 Vacuum Black Objects

- Black Holes and Black Rings in $D = 5$
- Black Holes and Black Rings in $D \geq 6$
- Black Holes and Black Ringoids in $D \geq 7$

3 Adding Charge

4 Conclusions and Outlook

Outline

1 Introduction

2 Vacuum Black Objects

- Black Holes and Black Rings in $D = 5$
- Black Holes and Black Rings in $D \geq 6$
- Black Holes and Black Ringoids in $D \geq 7$

3 Adding Charge

4 Conclusions and Outlook

Black Rings in $D = 5$: Physical Picture

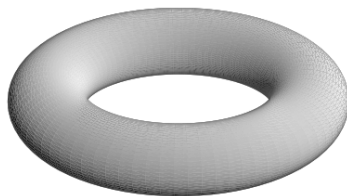
Emparan and Reall, PRD65 (2002) 084025, PRL 88 (2002) 101101

black rings

horizon topology $S^1 \times S^2$

static black rings

- attraction:
 - gravity/string tension
 - shrink rings
- repulsion:
 - conical singularity
 - inside: push
 - outside: pull
- unbalanced black rings

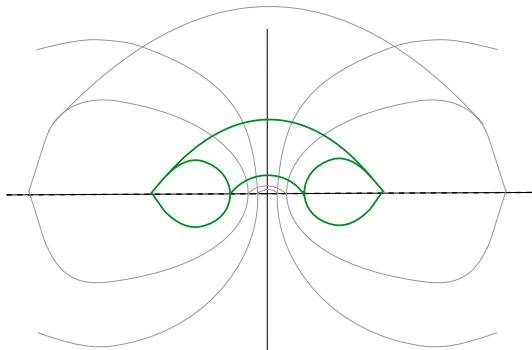


- attraction:
 - gravity/string tension
- repulsion:
 - rotation along S^1
 - centrifugal force
- balanced black rings

Black Rings in $D = 5$: Physical Picture

Emparan and Reall, PRD65 (2002) 084025, PRL 88 (2002) 101101

static black ring: string pulling from outside the ring (shown: deficit)

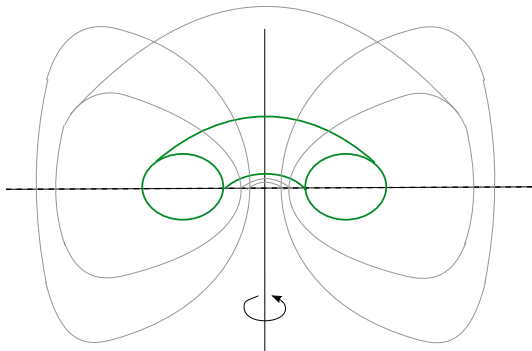


static black ring: strut pushing from inside the ring (not shown: excess)

Black Rings in $D = 5$: Physical Picture

Emparan and Reall, PRD65 (2002) 084025, PRL 88 (2002) 101101

rotating black ring:

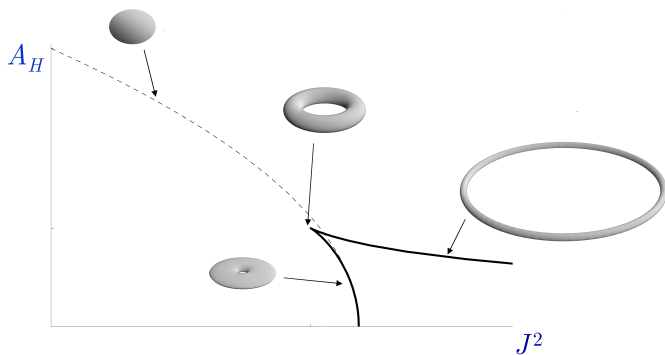


no conical singularity: appropriate horizon velocity

Black Rings in $D = 5$: Physical Picture

Emparan and Reall, PRD65 (2002) 084025, PRL 88 (2002) 101101

phase diagram: domain of existence



horizon area A_H vs. angular momentum J^2 at fixed mass

- black holes S^3
maximal J
- black rings $S^1 \times S^2$
minimal J
- **nonuniqueness**
region with
 - MP black holes
 - fat black rings
 - thin black rings

Black Rings in $D = 5$: A Coordinate Choice

a parametrization of 4-dimensional Euclidean space

$$ds^2 = d\rho^2 + \rho^2(d\Theta^2 + \cos^2 \Theta d\phi^2 + \sin^2 \Theta d\psi^2)$$

coordinate transformation

$$\rho = r\sqrt{U}, \quad \tan \Theta = \left(\frac{r^2 + \rho^2 + R^2}{r^2 + \rho^2 - R^2} \right) \tan \theta,$$

$$U = \sqrt{1 + \frac{R^4}{r^4} - \frac{2R^2}{r^2} \cos 2\theta}$$

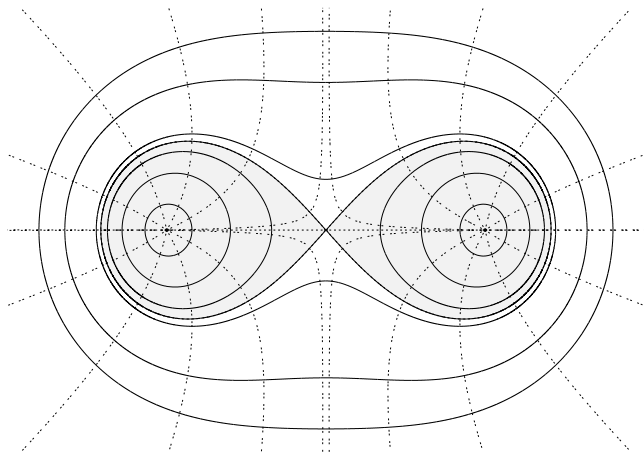
new metric

$$ds^2 = V_1(dr^2 + r^2 d\theta^2) + V_2 d\phi^2 + V_3 d\psi^2,$$

$$V_1 = \frac{1}{U}, \quad V_2 = r^2 \left[\cos^2 \theta - \frac{1}{2} \left(1 + \frac{R^2}{r^2} - U \right) \right], \quad V_3 = r^2 \left[\sin^2 \theta - \frac{1}{2} \left(1 - \frac{R^2}{r^2} - U \right) \right],$$

$$0 \leq r < \infty, \quad 0 \leq \theta \leq \pi/2, \quad 0 \leq \phi \leq 2\pi, \quad 0 \leq \psi \leq 2\pi, \quad R > 0$$

Black Rings in $D = 5$: A Coordinate Choice



for $0 < r < R$: a surface of constant r has ring-like topology

Black Rings in $D = 5$: A Coordinate Choice

Euclidean metric

$$ds^2 = V_1(dr^2 + r^2d\theta^2) + V_2d\phi^2 + V_3d\psi^2,$$

new metric ansatz for black rings

$$ds^2 = f_1(r, \theta)(dr^2 + r^2d\theta^2) + f_2(r, \theta)d\phi^2 + f_3(r, \theta)(d\psi - W(r, \theta)dt)^2 - f_0(r, \theta)dt^2$$

- Emparan Reall solution can be expressed explicitly in these coordinates
- metric is well suited for numerical calculations
- new metric is easily generalized to more than 5 dimensions

Black Rings in $D = 5$: A Coordinate Choice

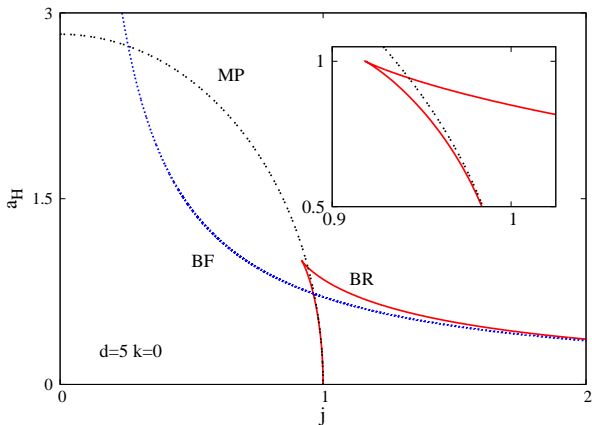
numerical schemes

- finite difference solver
- spectral solver

R	Ω_H	$M(num)$	$M(ex)$	$ J(num) $	$ J(ex) $	$A_H(num)$	$A_H(ex)$
1.61803	0.182574	24.0003	24.0000	109.547	109.545	773.616	773.605
1.93186	0.204124	16.0000	16.0000	58.7880	58.7878	446.647	446.645
2.18890	0.207020	13.3332	13.3333	45.0836	45.0843	332.911	332.909
2.41421	0.204124	12.0001	12.0000	39.1922	39.1918	273.514	273.518
2.80588	0.193649	10.6671	10.6667	34.4289	34.4265	210.552	210.563
3.14626	0.182574	9.99982	10.0000	32.8624	32.8634	176.553	176.555
3.45197	0.172516	9.59981	9.60000	32.4596	32.4607	154.723	154.726
3.99215	0.155902	9.14274	9.14286	32.9869	32.9877	127.614	127.616
4.46653	0.143019	8.88879	8.88889	34.1828	34.1834	110.970	110.973

accuracy: 10^{-5}

Black Rings in $D = 5$: A Coordinate Choice



MP: Myers-Perry, BR: black ring, BF: blackfold

Black Rings in $D = 5$: A Coordinate Choice

black ring metric in these coordinates:

$$ds^2 = f_1(r, \theta)(dr^2 + r^2 d\theta^2) + f_2(r, \theta)d\phi^2 + f_3(r, \theta)(d\psi - W(r, \theta)dt)^2 - f_0(r, \theta)dt^2$$

with functions:

$$f_0(r, \theta) = \frac{Q_2(r, \theta)}{Q_1(r, \theta)} U_1(r, \theta) U_2(r, \theta), \quad f_1(r, \theta) = \frac{r_H^2 R^4}{(R^4 - r_H^4)^2} \frac{U_1(r, \theta) Q_3(r, \theta)}{S(r, \theta)},$$

$$f_2(r, \theta) = \left(1 + \frac{r_H^2}{r^2}\right)^2 \frac{r^2 \sin^2 2\theta}{2U_2(r, \theta)}, \quad f_3(r, \theta) = \frac{r^2 \left(1 - \frac{r_H^2}{r^2}\right)^2}{2 \left(1 + \frac{r_H^2}{r^2}\right)^2} \frac{Q_1(r, \theta)}{Q_2(r, \theta) U_1(r, \theta)},$$

$$W(r, \theta) = \frac{4\sqrt{2}(r_H^2 + R^2)\sqrt{R^4 + r_H^4}}{R(R^2 - r_H^2)} \frac{\left(1 - \frac{r_H^2}{r^2}\right)^2}{r^2 \left(1 + \frac{r_H^2}{r^2}\right)^2} \frac{Q_2(r, \theta) Q_4(r, \theta)}{Q_1(r, \theta)}.$$

Black Rings in $D = 5$: A Coordinate Choice

$$U_1(r, \theta) = \frac{(r_H^2 + R^4)}{r^2 R^2} \left(1 + \frac{4r_H^2}{r^2} + \frac{r_H^4}{r^4} \right) + \frac{4r_H^2}{r^2} \cos 2\theta - 2S(r, \theta),$$

$$U_2(r, \theta) = \frac{r_H^2 + R^4}{r^2 R^2} - \left(1 + \frac{r_H^4}{r^4} \right) \cos 2\theta + S(r, \theta),$$

$$Q_1(r, \theta) = U_1^2(r, \theta) U_2(r, \theta) - \frac{4(r_H^2 + R^2)^2 (r_H^4 + R^4)}{r^2 R^2 (R^2 - r_H^2)^2} \frac{\left(1 + \frac{r_H^2}{r^2} \right)^2}{\left(1 - \frac{r_H^2}{r^2} \right)^2} \times$$

$$\left[U_1(r, \theta) - \left(\frac{(r_H^2 - R^2)^2}{r_H^2 R^2} + \frac{r_H^2 (r_H^2 - R^2)^2}{r^4 R^2} + \frac{2(r_H^2 + R^2)^2}{r^2 R^2} + \frac{4(r_H^4 + R^4)}{r^2 R^2} \right) \right]^2,$$

$$Q_2(r, \theta) = U_1(r, \theta) - \frac{8(r_H^4 + R^4)}{r^2 R^2},$$

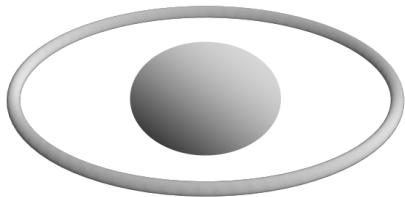
$$Q_3(r, \theta) = -U_1(r, \theta) + \left(1 + \frac{r_H^2}{r^2} \right)^2 \frac{2(r_H^4 + R^4)}{r_H^2 R^2},$$

$$Q_4(r, \theta) = U_2(r, \theta) - 2 \left(1 - \frac{r_H^2}{r^2} \right)^2 \sin^2 \theta,$$

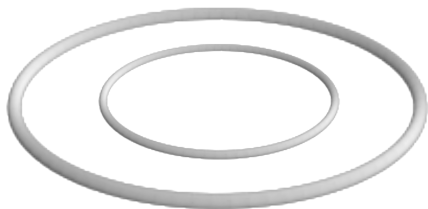
$$S(r, \theta) = \sqrt{\left(1 + \frac{R^4}{r^4} - \frac{2R^2}{r^2} \cos 2\theta \right) \left(1 + \frac{r_H^8}{r^4 R^4} - \frac{2r_H^4}{r^2 R^2} \cos 2\theta \right)}$$

A Zoo of Composite Species in $D = 5$

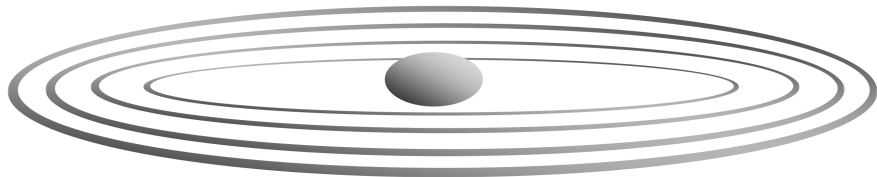
black Saturn



black dirings



etc.



Outline

1 Introduction

2 Vacuum Black Objects

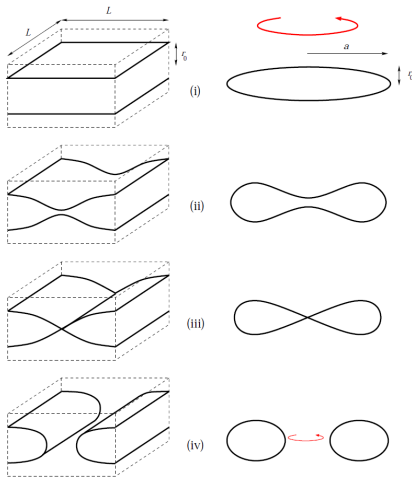
- Black Holes and Black Rings in $D = 5$
- **Black Holes and Black Rings in $D \geq 6$**
- Black Holes and Black Ringoids in $D \geq 7$

3 Adding Charge

4 Conclusions and Outlook

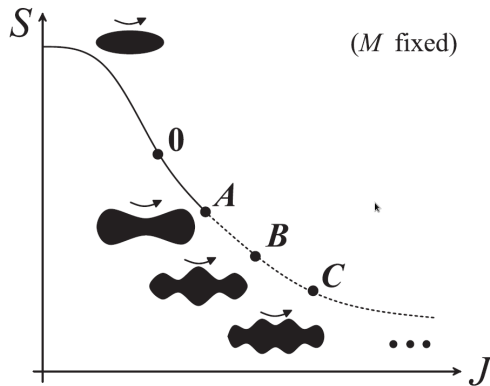
Myers-Perry Black Holes in $D \geq 6$

Emparan, Harmark, Niarchos, Obers, Rodriguez, JHEP 2007



Myers-Perry Black Holes in $D \geq 6$

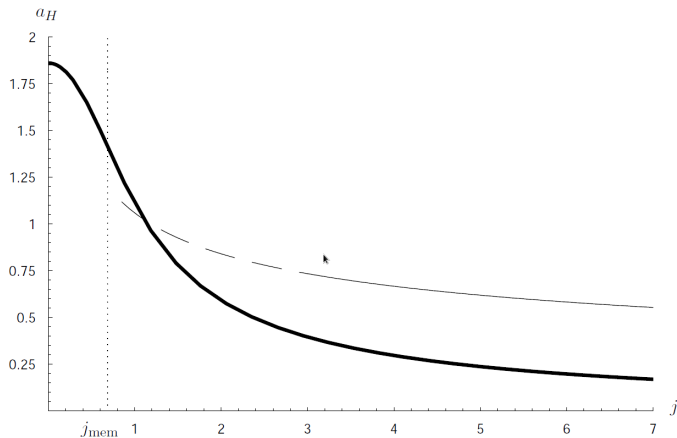
Dias, Figueras, Monteiro, Santos, Emparan, PRD 2009



unstable modes of Myers-Perry black holes: $D \geq 6$

Black Rings in $D > 5$: Perturbative Approach

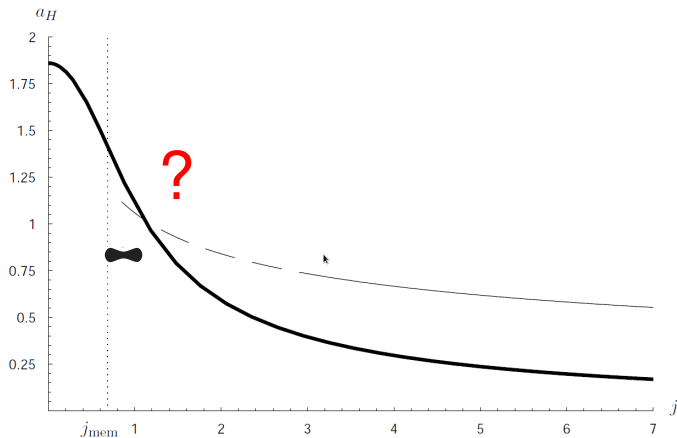
Emparan, Harmark, Niarchos, Obers, Rodriguez, JHEP 2007



perturbative approach: solutions for thin black rings in $D = 7$

Black Rings in $D > 5$: Perturbative Approach

Emparan, Harmark, Niarchos, Obers, Rodriguez, JHEP 2007



perturbative approach: solutions for thin black rings in $D = 7$

Black Rings in $D \geq 6$: New Coordinates

Kleihaus, Kunz, Radu, PLB 2013

a parametrization of d -dimensional Euclidean space (with $d = D - 1$)

$$ds^2 = d\rho^2 + \rho^2(d\Theta^2 + \cos^2 \Theta d\Omega_{d-3}^2 + \sin^2 \Theta d\psi^2)$$

coordinate transformation

$$\rho = r\sqrt{U}, \quad \tan \Theta = \left(\frac{r^2 + \rho^2 + R^2}{r^2 + \rho^2 - R^2} \right) \tan \theta,$$

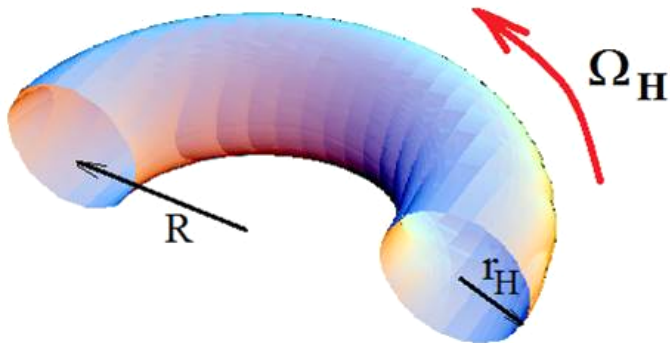
metric

$$ds_d^2 = V_1(dr^2 + r^2 d\theta^2) + V_2 d\Omega_{d-3}^2 + V_3 d\psi^2,$$

Ansatz for the D -dimensional black ring metric

$$ds^2 = f_1(r, \theta)(dr^2 + r^2 d\theta^2) + f_2(r, \theta) d\Omega_{D-4}^2 + f_3(r, \theta)(d\psi - W(r, \theta) dt)^2 - f_0(r, \theta) dt^2$$

Black Rings in $D \geq 6$: New Coordinates



regular solutions: adjust Ω_H for fixed r_H and R

in practice: $r_H = 1$, $R > r_H$

Black Rings in $D \geq 6$: New Coordinates

event horizon area

$$A_H = 2\pi r_H V_{D-4} \int_0^{\pi/2} d\theta \sqrt{f_1 f_2^{D-4} f_3} \Big|_{r=r_H}$$

temperature

$$T_H = \frac{1}{2\pi} \lim_{r \rightarrow r_H} \frac{1}{r - r_H} \sqrt{\frac{f_0}{f_1}}$$

mass and angular momentum

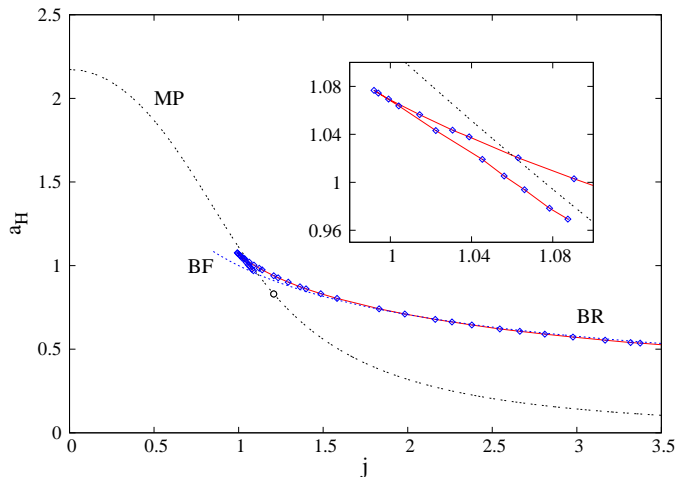
$$M = \frac{(D-2)V_{D-2}}{16\pi G} c_t, \quad J = \frac{V_{D-2}}{16\pi G} c_\psi$$

$$g_{tt} \rightarrow -1 + \frac{c_t}{r^{D-3}} + \dots, \quad g_{\psi t} \rightarrow \sin^2 \theta \frac{c_\psi}{r^{D-3}} + \dots$$

Smarr formula

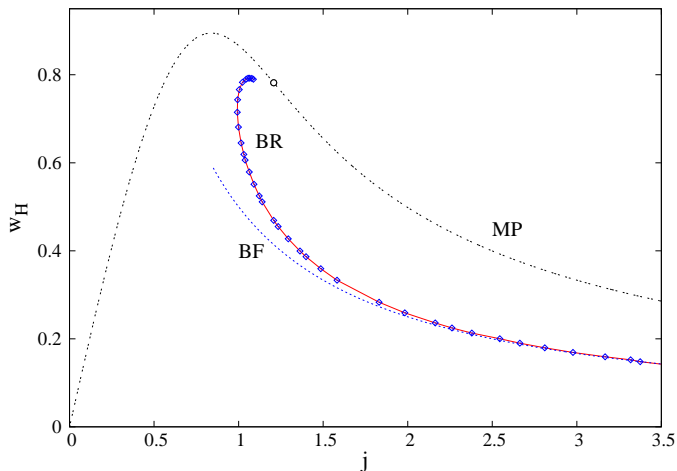
$$\frac{D-3}{D-2} M = \frac{T_H A_H}{4G} + \Omega_H J$$

Black Rings in $D = 6$: Results



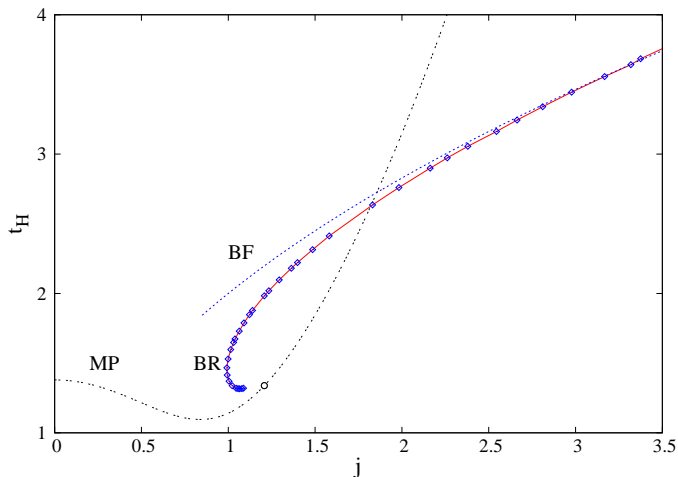
horizon area A_H vs. angular momentum J for fixed mass

Black Rings in $D = 6$: Results



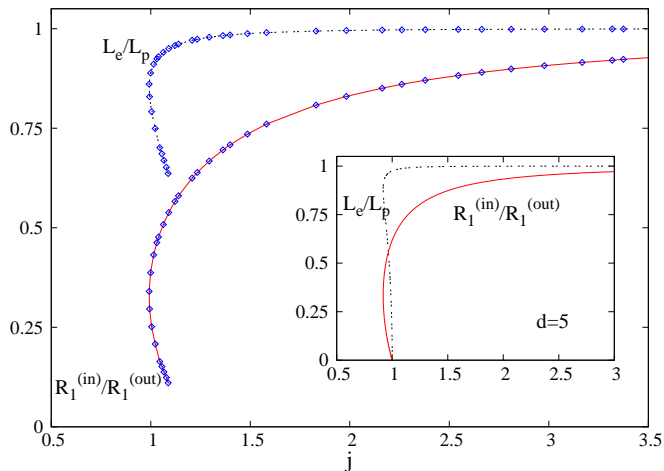
horizon angular velocity w_H vs. angular momentum J for fixed mass

Black Rings in $D = 6$: Results



temperature T_H vs. angular momentum J for fixed mass

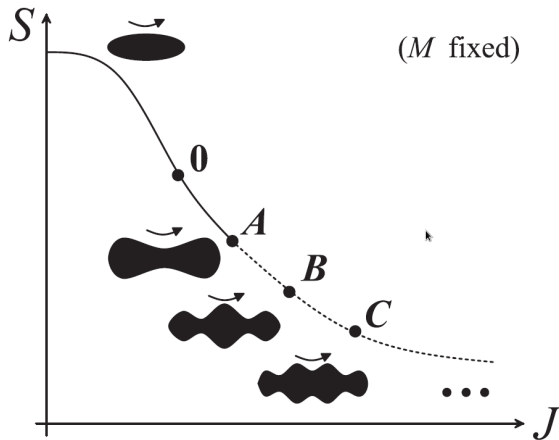
Black Rings in $D = 6$: Results



horizon geometry vs. angular momentum J for fixed mass

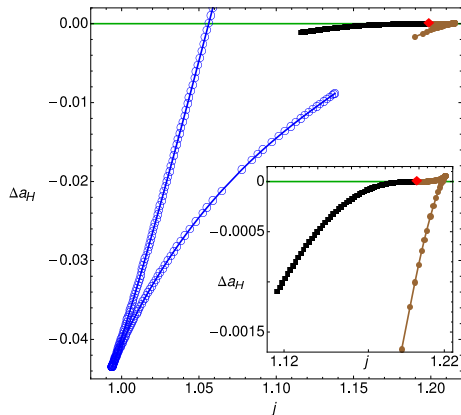
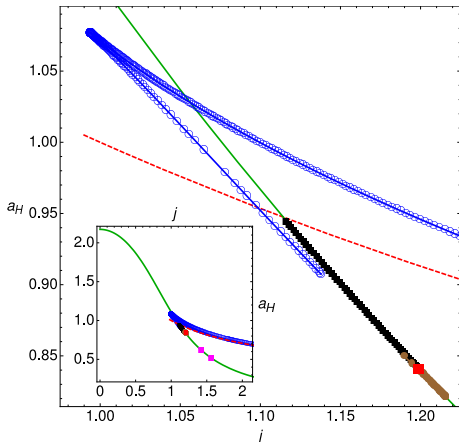
Black Rings and Pinched Black Holes

Dias, Figueras, Monteiro, Santos, Emparan, PRD 2009



Black Rings and Pinched Black Holes

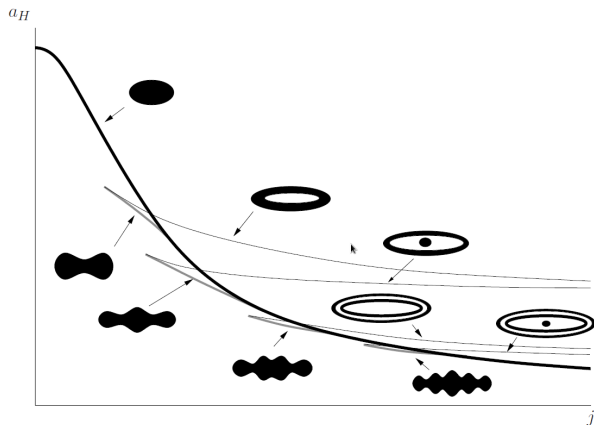
O. J. C. Dias, J. E. Santos and B. Way, JHEP 2014



- black rings and pinched black holes in $D = 6$
- extension to $D = 7$

A Phase Diagram in $D \geq 6$

Empanan, Figueras, JHEP 1011 (2010) 022



$D \geq 6$ horizon area vs. angular momentum at fixed mass

Outline

1 Introduction

2 Vacuum Black Objects

- Black Holes and Black Rings in $D = 5$
- Black Holes and Black Rings in $D \geq 6$
- Black Holes and Black Ringoids in $D \geq 7$

3 Adding Charge

4 Conclusions and Outlook

Black Ringoids: The Scheme

Kleihaus, Kunz, Radu, JHEP 2015

Balanced black objects with $S^{n+1} \times S^{2k+1}$ horizon topology

	<i>spherical horizon</i>	<i>black rings</i>	<i>black ringoids</i>		
	MP/'pinched'	$k = 0$	$k = 1$	$k = 2$	$k = 3$
$d = 5$	S^3	$\mathbf{S}^2 \times \mathbf{S}^1$			
$d = 6$	S^4	$S^3 \times S^1$			
$d = 7$	S^5	$S^4 \times S^1$	$\mathbf{S}^2 \times \mathbf{S}^3$		
$d = 8$	S^6	$S^5 \times S^1$	$S^3 \times S^3$		
$d = 9$	S^7	$S^6 \times S^1$	$S^4 \times S^3$	$\mathbf{S}^2 \times \mathbf{S}^5$	
$d = 10$	S^8	$S^7 \times S^1$	$S^5 \times S^3$	$S^3 \times S^5$	
$d = 11$	S^9	$S^8 \times S^1$	$S^6 \times S^3$	$S^4 \times S^5$	$\mathbf{S}^2 \times \mathbf{S}^7$

etc.

Black Ringoids: The Scheme

Euclidean metric

$$ds^2 = V_1(dr^2 + r^2 d\theta^2) + V_2 d\Omega_n^2 + V_3 d\Omega_p^2$$

surface of constant r :

$$\begin{cases} 0 < r < R: & S^{n+1} \times S^p \text{ topology} \\ r > R: & S^{n+1+p} \text{ topology} \end{cases}$$

appropriate spatial coordinates for black objects with $S^{n+1} \times S^p$ topology

general case: difficult

simplification: $p = 2k + 1, \quad k \geq 0$

$k + 1$ equal magnitude angular momenta: $J_1 = J_2 = \dots = J_k = J_{k+1} = J$

Black Ringoids: The Scheme

odd-dimensional sphere: as an S^1 fibration over $\mathbb{C}\mathbb{P}^k$

$$d\Omega_{2k+1}^2 = (d\psi + \mathcal{A})^2 + d\Sigma_k^2$$

$\mathcal{A} = A_i dx^i$: Kähler form $d\Sigma_k^2$: metric on the unit $\mathbb{C}\mathbb{P}^k$

$$d\Omega_{2k+1}^2 = \sum_i dz_i d\bar{z}_i$$

$k + 1$ complex coordinates z_i with $\sum_i^{k+1} z_i \bar{z}_i = 1$, e.g.

$$z_i = e^{i(\psi + \phi_i)} \cos \theta_i \prod_{j < i} \sin \theta_j, \quad \text{for } i = 1, \dots, k, \quad \text{and} \quad z_{k+1} = e^{i\psi} \prod_{j=1}^k \sin \theta_j$$

$$\mathcal{A} = A_i dx^i = \sum_{i=1}^k \cos^2 \theta_i \left[\prod_{j < i} \sin^2 \theta_j \right] d\phi_i .$$

Black Ringoids: The Scheme

$S^{n+1} \times S^{2k+1}$ horizon topology

Ansatz for the metric of black ringoids: $k \geq 1, n \geq 1$

$$ds^2 = f_1(r, \theta) (dr^2 + r^2 d\theta^2) + f_2(r, \theta) d\Omega_n^2 - f_0(r, \theta) dt^2 \\ + f_3(r, \theta) (d\psi + \mathcal{A} - W(r, \theta) dt)^2 + f_4(r, \theta) d\Sigma_k^2,$$

special case $n = 1$: $S^2 \times S^{2k+1}$ horizon topology

$$ds^2 = f_1(r, \theta) (dr^2 + \Delta(r) d\theta^2) + f_2(r, \theta) d\phi^2 - f_0(r, \theta) dt^2 \\ + f_3(r, \theta) (d\psi + \mathcal{A} - W(r, \theta) dt)^2 + f_4(r, \theta) d\Sigma_k^2,$$

MP ($J_1 = \dots = J_{k+1} = J, J_{k+2} = 0$): domain bounded, singular limit

general case $n > 1$:

MP ($J_1 = \dots = J_{k+1} = J, J_i = 0, i > k + 1$): domain unbounded

Black Ringoids: The Scheme

event horizon area

$$A_H = r_H V_n V_{2k+1} \int_0^{\pi/2} d\theta \sqrt{f_1 f_2^n f_3 f_4^{2k}} \Big|_{r=r_H}$$

temperature

$$T_H = \frac{1}{2\pi} \lim_{r \rightarrow r_H} \frac{1}{r - r_H} \sqrt{\frac{f_0}{f_1}}$$

mass and angular momentum

$$M = \frac{(D-2)V_{D-2}}{16\pi G} c_t, \quad J = \frac{V_{D-2}}{8\pi G} c_\psi$$

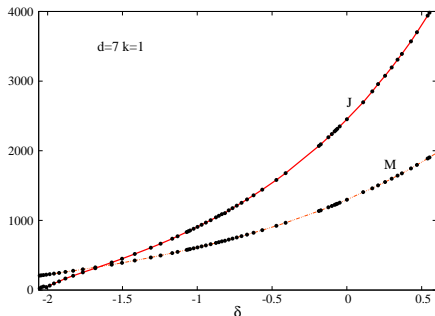
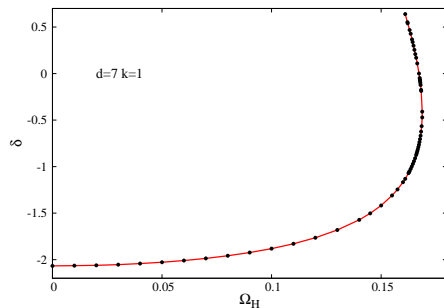
$$g_{tt} \rightarrow -1 + \frac{c_t}{r^{D-3}} + \dots, \quad g_{\psi t} \rightarrow \sin^2 \theta \frac{c_\psi}{r^{D-3}} + \dots$$

Smarr formula

$$\frac{D-3}{D-2} M = \frac{T_H A_H}{4G} + (k+1) \Omega_H J$$

Black Ringoids: Results

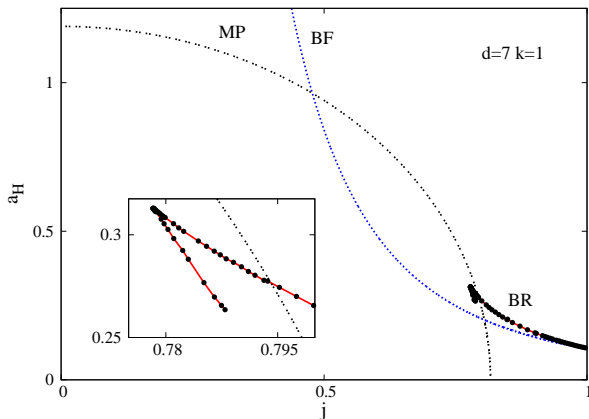
balancing the ringoids: variation of Ω_H



conical deficit/excess $\delta = 2\pi(1 - \lim_{\theta \rightarrow 0} \frac{f_2}{\theta^2 r^2 f_1})$

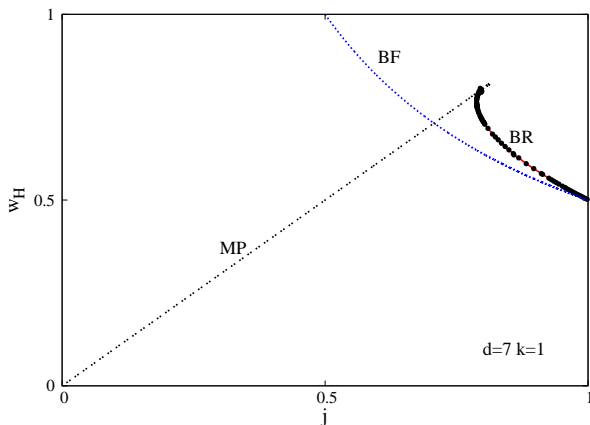
$d = 7, k = 1$ black ringoids ($r_H = 1, R = 4.6$)

Black Ringoids: Results



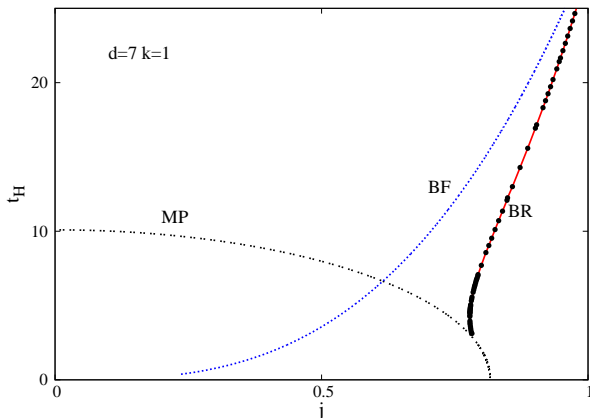
area of balanced $D = 7$ ringoids with $S^2 \times S^3$ horizon topology
 higher dimensional counterparts of $D = 5$ black rings

Black Ringoids: Results



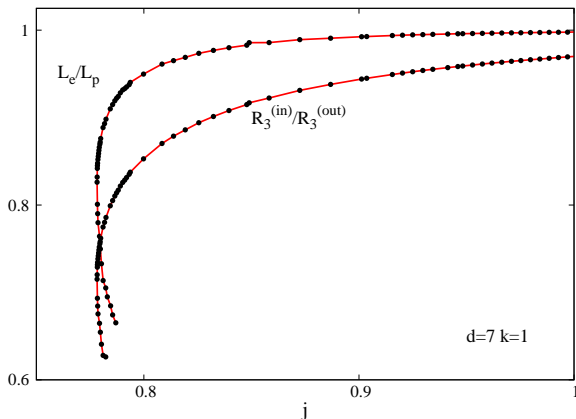
Ω_H of balanced $D = 7$ ringoids with $S^2 \times S^3$ horizon topology
higher dimensional counterparts of $D = 5$ black rings

Black Ringoids: Results



temperature of balanced $D = 7$ ringoids with $S^2 \times S^3$ horizon topology
 higher dimensional counterparts of $D = 5$ black rings

Black Ringoids: Results



geometry of balanced $D = 7$ ringoids with $S^2 \times S^3$ horizon topology

higher dimensional counterparts of $D = 5$ black rings

Black Ringoids: Results

Balanced black objects with $S^{n+1} \times S^{2k+1}$ horizon topology

	<i>spherical horizon</i>	<i>black rings</i>	<i>black ringoids</i>		
	MP/'pinched'	$k = 0$	$k = 1$	$k = 2$	$k = 3$
$d = 5$	S^3	$S^2 \times S^1$			
$d = 6$	S^4	$S^3 \times S^1$			
$d = 7$	S^5	$S^4 \times S^1$	$S^2 \times S^3$		
$d = 8$	S^6	$S^5 \times S^1$	$S^3 \times S^3$		
$d = 9$	S^7	$S^6 \times S^1$	$S^4 \times S^3$	$S^2 \times S^5$	
$d = 10$	S^8	$S^7 \times S^1$	$S^5 \times S^3$	$S^3 \times S^5$	
$d = 11$	S^9	$S^8 \times S^1$	$S^6 \times S^3$	$S^4 \times S^5$	$S^2 \times S^7$

etc.

Outline

1 Introduction

2 Vacuum Black Objects

- Black Holes and Black Rings in $D = 5$
- Black Holes and Black Rings in $D \geq 6$
- Black Holes and Black Ringoids in $D \geq 7$

3 Adding Charge

4 Conclusions and Outlook

Einstein-Maxwell-Dilaton Black Holes

Kaluza-Klein construction:

- embedding of the D -dimensional MP metric in $(D + 1)$ spacetime with extra coordinate U

$$ds_{D+1}^2 = dU^2 + ds_{D,MP}^2$$

- boost in the $t - U$ plane with the 2×2 matrix

$$L = \begin{pmatrix} \cosh \alpha & \sinh \alpha \\ \sinh \alpha & \cosh \alpha \end{pmatrix}$$

- $(D + 1)$ -dimensional metric

$$ds_{D+1}^2 = e^{2\iota\Phi} g_{\rho\sigma} dx^\rho dx^\sigma + e^{-2(D-2)\iota\Phi} (dU + A_\rho dx^\rho)^2$$

charged rotating black holes for the KK dilaton coupling constant h

$$h = \frac{D - 1}{\sqrt{2(D - 1)(D - 2)}} = (D - 1)\iota$$

Einstein-Maxwell-Dilaton Black Holes

asymptotic expansion and global charges

$$g_{tt} = -1 + \frac{M}{(D-2)A} \frac{1}{r^{D-3}} + \dots$$

$$M = m (1 + (D-3) \cosh^2 \alpha) A$$

$$g_{t\varphi_i} = -\frac{J_i}{2A} \mu_i^2 \frac{1}{r^{D-3}} + \dots$$

$$J_i = 2m a_i \cosh \alpha A$$

$$A_t = \frac{Q}{(D-3)A} \frac{1}{r^{D-3}} + \dots$$

$$Q = (D-3)m \sinh \alpha \cosh \alpha A$$

$$A_{\varphi_i} = -\frac{\mathcal{M}_i}{(D-3)A} \mu_i^2 \frac{1}{r^{D-3}} + \dots$$

$$\mathcal{M}_i = (D-3)m a_i \sinh \alpha A$$

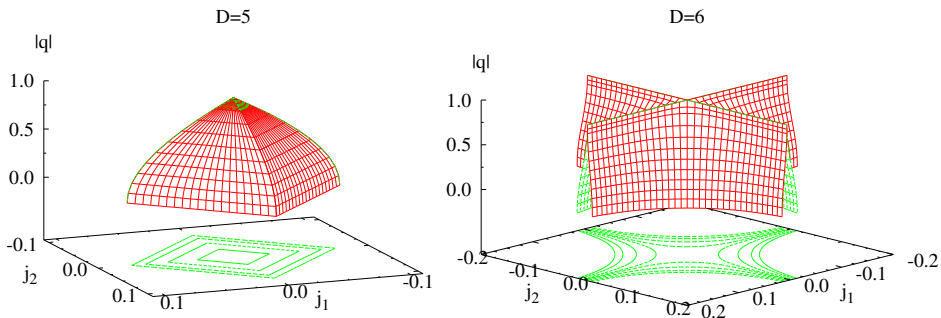
$$\Phi = \frac{\Sigma}{(D-3)A(S^{D-2})} \frac{1}{r^{D-3}} + \dots$$

$$\Sigma = -\frac{(D-3)m \sinh^2 \alpha}{2(D-2)\iota} A$$

where $A := A(S^{D-2})$

Einstein-Maxwell-Dilaton Black Holes

Myers-Perry black holes

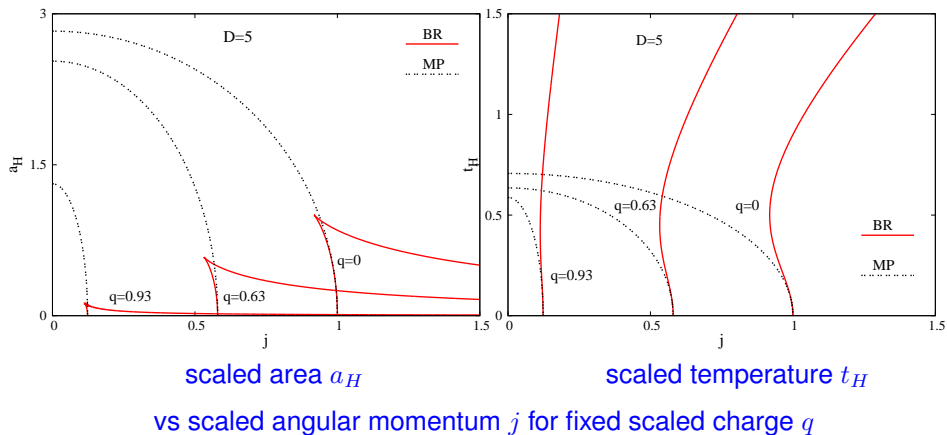


domain of existence of EMD MP black holes

scaled charge $|q| = \frac{|Q|}{M}$ vs scaled angular momenta $j_i = \frac{J_i}{M^{(D-2)/(D-3)}}$

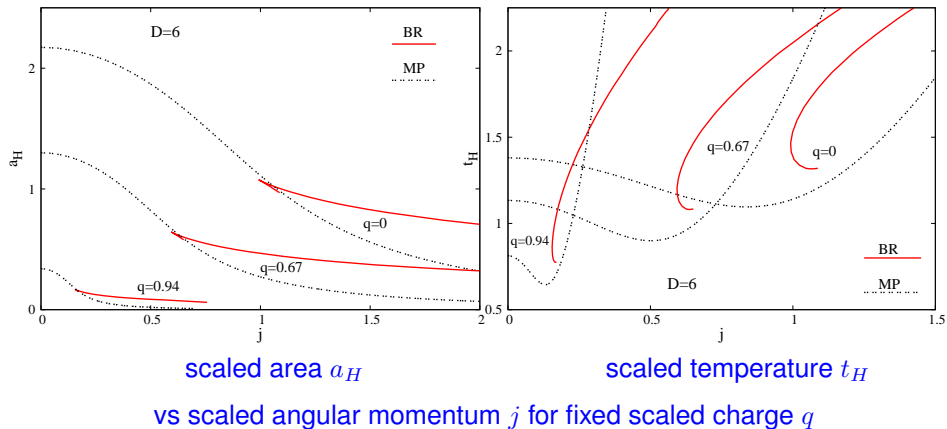
Charged Black Rings and Ringoids

$D = 5$ black rings



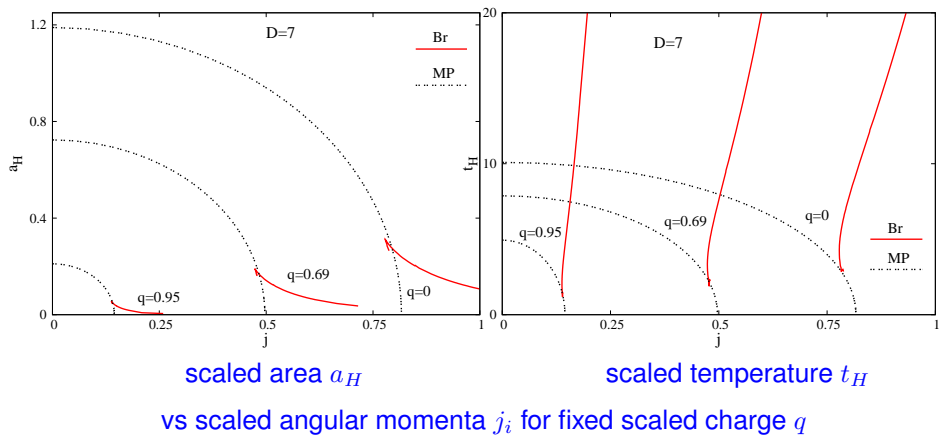
Charged Black Rings and Ringoids

$D = 6$ black rings



Charged Black Rings and Ringoids

$D = 7$ black ringoids



Outline

1 Introduction

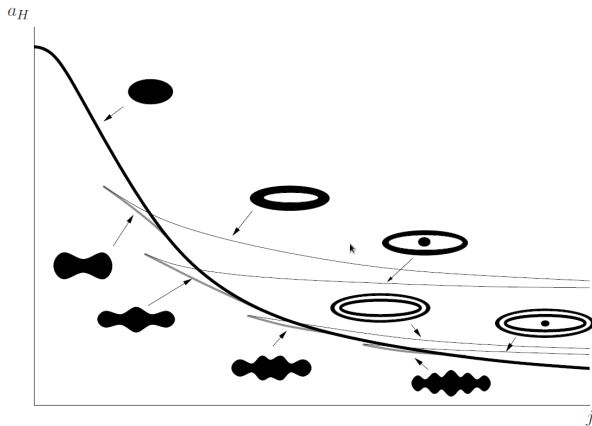
2 Vacuum Black Objects

- Black Holes and Black Rings in $D = 5$
- Black Holes and Black Rings in $D \geq 6$
- Black Holes and Black Ringoids in $D \geq 7$

3 Adding Charge

4 Conclusions and Outlook

Conclusions



$D = 5$: rings

- well-known

$D = 6$: rings

- horizon topology change?
- pinched black holes

$D = 7$: ringoids

- analogous to $D = 5$

$D \geq 8$

- systematic study to be done

A Black Hole Zoo

I'm scratching the tip
of the iceberg.



A Black Hole Zoo

PERIODIC TABLE OF THE ELEMENTS

<http://www.periodni.com>

GROUP	PERIODIC TABLE OF THE ELEMENTS																18	
1	IA												VIIA		VIIIA			
1	1.0079 H HYDROGEN																2	4.0026 He HELIUM
2	3 6.941 Li LITHIUM	4 9.0122 Be BERYLLIUM															10	20.180 Ne NEON
3	11 22.990 Na SODIUM	12 24.305 Mg MAGNESIUM															18	39.948 Ar ARGON
4	19 39.098 K POTASSIUM	20 40.078 Ca CALCIUM	21 44.956 Sc SCANDIUM	22 47.867 Ti TITANIUM	23 50.942 V VANADIUM	24 51.996 Cr CHROMIUM	25 54.938 Mn MANGANESE	26 55.845 Fe IRON	27 58.933 Co COBALT	28 58.693 Ni NICKEL	29 63.546 Cu COPPER	30 65.38 Zn ZINC	31 69.723 Ga GALLIUM	32 72.64 Ge GERMANIUM	33 74.922 As ARSENIC	34 78.96 Se SELENIUM	35 79.904 Br BROMINE	36 83.798 Kr KRYPTON
5	37 85.468 Rb RUBIDIUM	38 87.62 Sr STRONTIUM	39 88.906 Y YTRIUM	40 91.224 Zr ZIRCONIUM	41 92.906 Nb NIOBIUM	42 95.96 Mo MOLYBDENUM	43 (98) Tc TECHNETIUM	44 101.07 Ru RUTHENIUM	45 102.91 Rh RHODIUM	46 106.42 Pd PALLADIUM	47 107.87 Ag SILVER	48 112.41 Cd CADMIUM	49 114.82 In INDIUM	50 118.71 Sn TIN	51 121.76 Sb ANTIMONY	52 127.60 Te TELLURIUM	53 126.90 I IODINE	54 131.29 Xe XENON
6	55 132.91 Cs CAESIUM	56 137.33 Ba BARIUM	57-71 La-Lu Lanthanide	72 178.49 Hf HAFNIUM	73 180.95 Ta TANTALUM	74 183.84 W TUNGSTEN	75 186.21 Re RHENIUM	76 190.23 Os OSMIUM	77 192.22 Ir IRIDIUM	78 195.08 Pt PLATINUM	79 196.97 Au GOLD	80 200.59 Hg MERCURY	81 204.38 Tl THALLIUM	82 207.2 Pb LEAD	83 208.98 Bi BISMUTH	84 (209) Po POLONIUM	85 (210) At ASTATINE	86 (222) Rn RADON
7	87 (223) Fr FRANCIUM	88 (226) Ra RADIUM	89-103 Ac-Lr Actinide	104 (267) Rf RUTHERFORDIUM	105 (268) Db DUBNIUM	106 (271) Sg SEABORGIUM	107 (272) Bh BOHRNIUM	108 (277) Hs HASSIUM	109 (276) Mt MEITNERIUM	110 (281) Ds DARMSSTADTIUM	111 (280) Rg ROENTGENIUM	112 (285) Cn COPERNICIUM	113 (...) Uut UNUNTRIUM	114 (287) Fl FLEROVIUM	115 (...) Uup UNUNPENTIUM	116 (291) Lv LIVERMORIUM	117 (...) Uus UNUNSEPTIUM	118 (...) Uuo UNUNOCTIUM

RELATIVE ATOMIC MASS (A)

GROUP IUPAC

GROUP CAS

ATOMIC NUMBER

SYMBOL

ELEMENT NAME

■ Metal ■ Semimetal ■ Nonmetal
■ Alkali metal ■ Chalcogens element
■ Alkaline earth metal ■ Halogens element
■ Transition metals ■ Noble gas
■ Lanthanide
■ Actinide

STANDARD STATE (25 °C; 101 kPa)

■ Ne - gas ■ Fe - solid
■ Hg - liquid ■ Te - synthetic

LANTHANIDE

57 138.91 La LANTHANUM	58 140.12 Ce CERIUM	59 140.91 Pr PRASEODYMIUM	60 144.24 Nd NEODYMIUM	61 (145) Pm PROMETHIUM	62 150.36 Sm SAMARIUM	63 151.96 Eu EUROPIUM	64 157.25 Gd GADOLINIUM	65 158.93 Tb TERBIUM	66 162.50 Dy DYSPROSIUM	67 164.93 Ho HOLMIUM	68 167.26 Er ERBIUM	69 168.93 Tm THULIUM	70 173.05 Yb YTTERIUM	71 174.97 Lu LUTETIUM
-------------------------------------	----------------------------------	--	-------------------------------------	-------------------------------------	------------------------------------	------------------------------------	--------------------------------------	-----------------------------------	--------------------------------------	-----------------------------------	----------------------------------	-----------------------------------	------------------------------------	------------------------------------

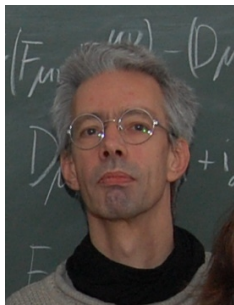
ACTINIDE

89 (227) Ac	90 232.04 Th	91 231.04 Pa	92 238.03 U	93 (237) Np	94 (244) Pu	95 (243) Am	96 (247) Cm	97 (247) Bk	98 (251) Cf	99 (252) Es	100 (257) Fm	101 (258) Md	102 (259) No	103 (262) Lr
-----------------------	------------------------	------------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	-----------------------	------------------------	------------------------	------------------------	------------------------

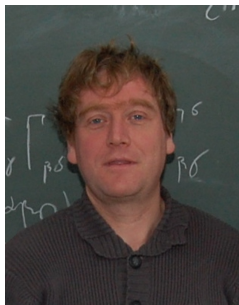
Pure Appl. Chem., 81, No. 11, 2131-2156 (2009)
 Relative atomic masses are expressed with five significant figures. For elements that have no stable nuclides, the value enclosed in brackets indicates the mass number of the longest-lived isotope of the element. However three such elements (Th, Pa and U) do have a characteristic terrestrial isotopic composition, and for these an atomic weight is tabulated.

Copyright © 2012 IUPAC

THANKS



Burkhard Kleihaus



Eugen Radu

THANKS

*Thank you very much
for your attention*