The Meissner Effect for Weakly Isolated Horizons

Martin Scholtz and Norman Gürlebeck

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CENTER OF APPLIED SPACE TECHNOLOGY AND MICROGRAVITY
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Motivation

The Meissner Effect

Jet creation

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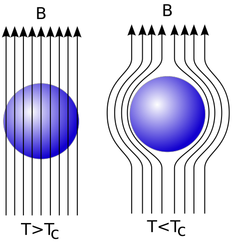
The Meissner effect

- Meissner Ochsenfeld effect for superconductors
  - expulsion of the magnetic field from the interior of the superconductor for $T < T_C$
- Meissner effect for black holes
  - black hole thermodynamics: surface gravity $\kappa \propto T$
  - expulsion of the magnetic field from *extremal horizons*



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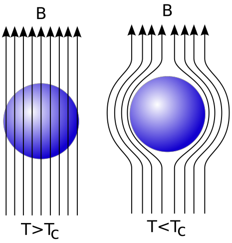
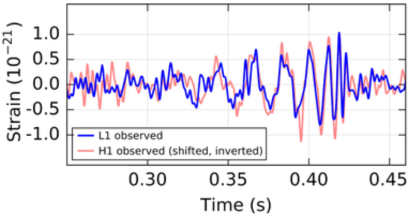


Observation of black holes

- gravitational wave signals of binaries
- motion of stars
- accretion disks
- gravitational lensing, shadows

⇒ additional sources of the gravitational field required



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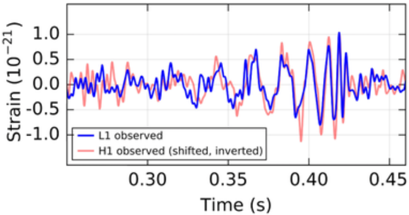

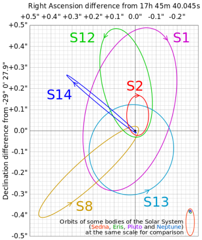

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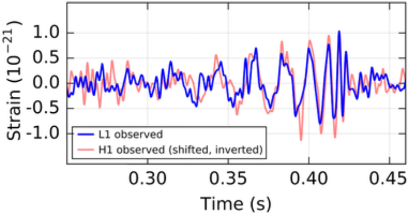

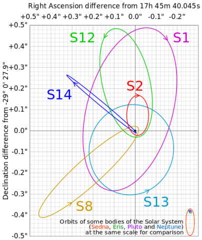

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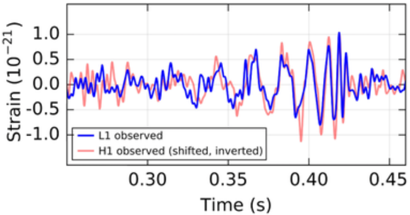

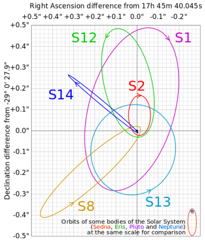

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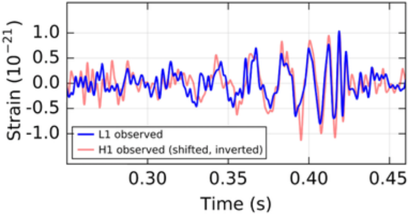

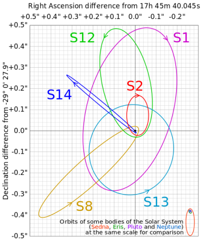

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Observation of black holes

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$\Rightarrow$ additional sources of the gravitational field required

Could deformations of black holes yield observational signatures?

Properties of isolated and deformed black holes

- no-hair theorem $\rightarrow$ Kerr black holes
- laws of black hole mechanics
- hidden symmetries
- Meissner effect
- ...

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Which properties are universal for black holes and which are particular to Kerr?

Properties of isolated and deformed black holes

- ✓ no-hair theorem $\rightarrow$ tidal Love number vanish
- ✓ laws of black hole mechanics
- × hidden symmetries
- ? **Meissner effect**

Which properties are universal for black holes and which are particular to Kerr?

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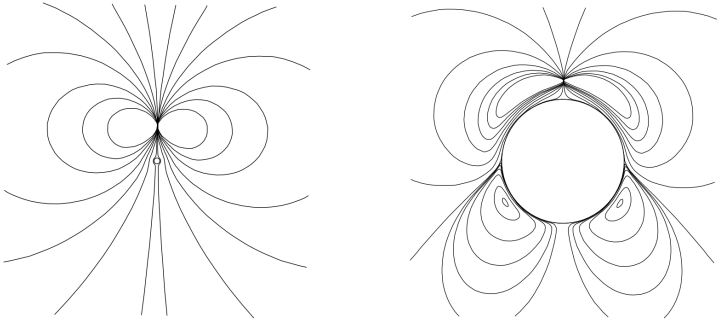

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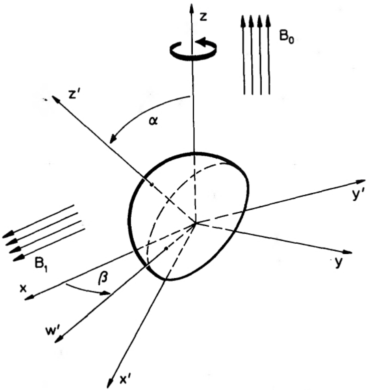

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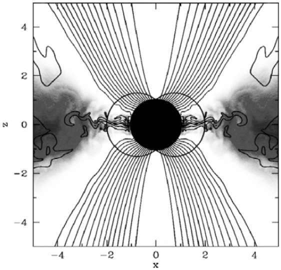

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here:

- Arbitrarily strong electromagnetic field
- generic uncharged black hole in equilibrium (deformations allowed)

Generic black holes: Non-Expanding Horizons

Idea: Quasi-local description of black holes *[Ashtekar, Lewandowski 2002]*.

- Non-expanding horizons $\mathcal{H}$:
  - null hypersurface $\mathcal{H} = \mathbb{R} \times S^2$
  - null normal ℓ^a with vanishing expansion
  - outgoing null vector n^a has negative expansion
  - energy conditions: $T_{ab}k^b$ is future pointing
  - Einstein equation holds

$\Rightarrow$
  - ℓ^a non-shearing, non-twisting
  - $\exists$ preferred intrinsic connection
  - $\ell^a \mathcal{D}_a \ell^b = \ell^a \omega_a \ell^b$
  - ℓ^a not unique (no zeroth law of thermodynamics)

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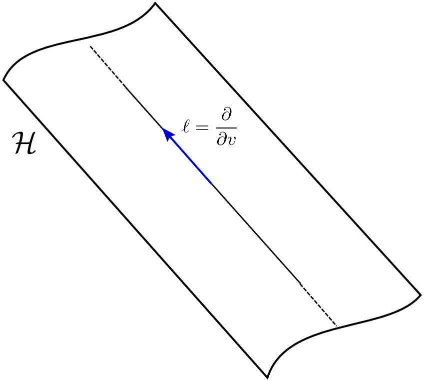
Generic black holes: Weakly Isolated Horizons

- $(\mathcal{H}, [\ell^a])$ *weakly isolated horizon* if:
  - $\mathcal{H}$ is non-expanding horizon
  - $[\ell^a]$ equivalence class of null normals under rescaling
  - $[\pounds_\ell, \mathcal{D}_a]\, \ell^b \doteq 0$

$\Rightarrow$ surface gravity $\kappa_{(\ell)} = \ell^a \omega_a$ is constant
  - but: depends on scaling: $\kappa_{(\ell)} = c\kappa_{(c\ell)}$
  - extremal horizons: $\kappa_{(\ell)} = 0$

Tetrad & Bondi-like coordinates near a WIH



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- $v = \text{const.} \cong S^2$ $(\ell^a \nabla_a v = 1)$
- $m^a, \bar{m}^a$ – basis of TS_0
- Propagation by $\pounds_\ell m^a \doteq 0$
- Choose n^a, $n^a n_a = 0, n^a \ell_a = 1$
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$$n = \frac{\partial}{\partial r}$$

- Propagate tetrad along n^a

- spherical coordinates on $\mathcal{S}_v$: ${}^{(2)}ds^2 = R(\theta, \varphi)^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right)$ *[Krishnan 2012]*

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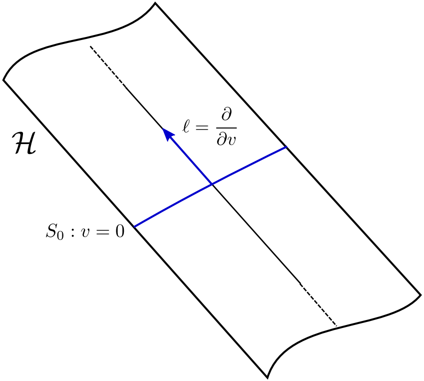

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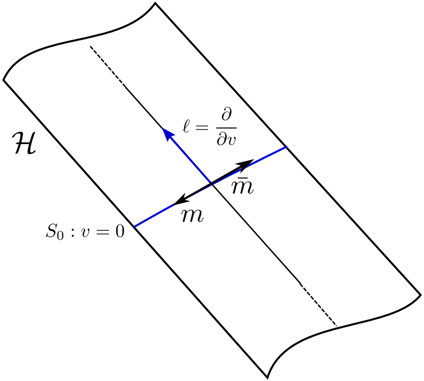

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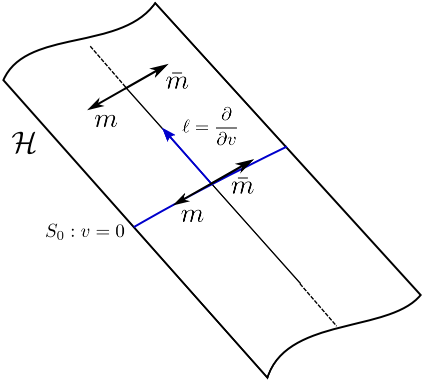

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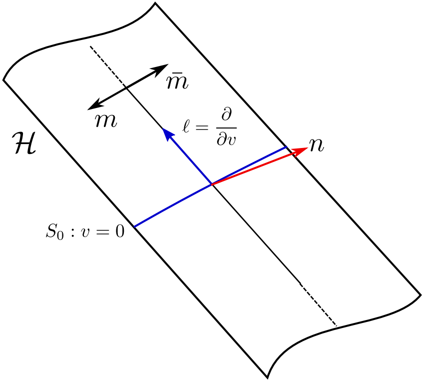

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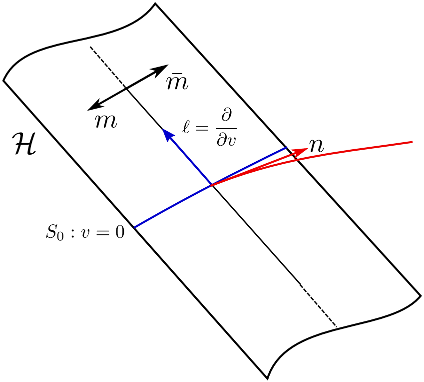

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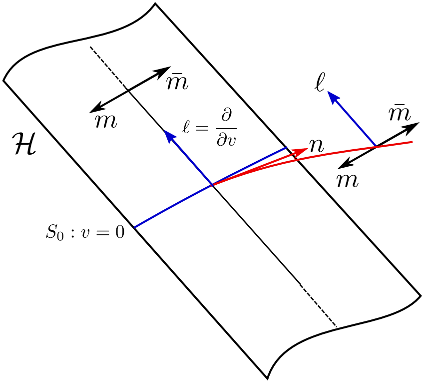

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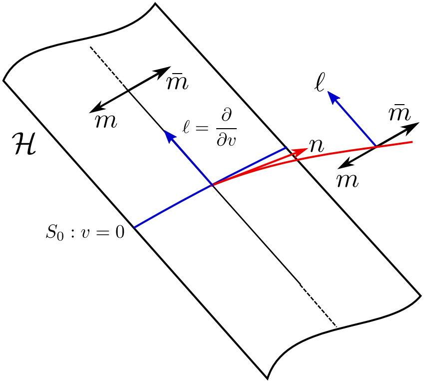

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The assumptions & constraints

- axial symmetry (η^a), stationarity (ξ^a) & electro-vacuum *near* the WIH
- electromagnetic field with the same symmetries
- expansion of all functions in r: $f = f^{(0)} + f^{(1)} r + \dots$

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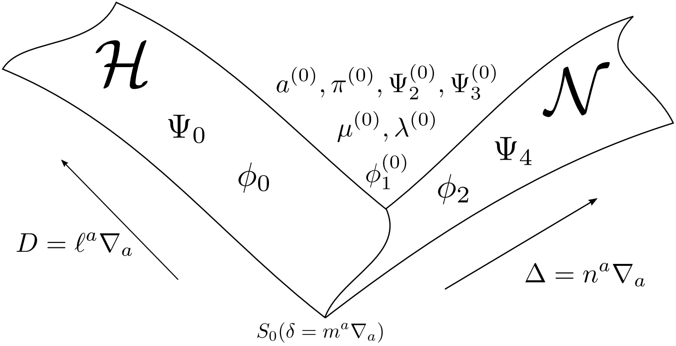

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$$0 = \nabla_a \nabla_b \xi_c - R_{abcd} \xi^d$$

$$0 = \nabla_{[a} (F_{b]c} \eta^c + \mathrm{i} F^*_{b]c} \eta^c)$$

$$\Rightarrow \quad \kappa_{(\ell)} \lambda^{(0)} = \bar{\eth} \pi^{(0)} + \left(\pi^{(0)}\right)^2$$

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- $\lambda = n^a \bar{m}^b \nabla_b \bar{m}_a, \quad \pi = n^a l^b \nabla_b \bar{m}_a$
- electric and magnetic flux: $\phi_1 = \frac{1}{2} F_{ab} \left[l^a n^b - m^a \bar{m}^b \right]$
- $\phi_2 = F_{ab} \bar{m}^a n^b$
- $\eth \eta = \left(m^a \nabla_a + s \bar{m}^a m^b \nabla_b m_a \right) \eta$

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- vanishing charges: $Q_e + \mathrm{i} Q_m = 0 = \oint_{S_0} \phi_1 dS$

Extremal WIH

- explicit solution in the extremal case: $\kappa_{(\ell)} = 0$

$$\pi^{(0)}(\theta) = \frac{R(\theta)\sin\theta}{c_\pi + \sqrt{2}\int\limits_0^\theta R^2(\tilde{\theta})\,\sin\tilde{\theta}\,d\tilde{\theta}}$$

$$\phi_1^{(0)}(\theta) = \frac{c_\phi}{\left(c_\pi + \sqrt{2}\int\limits_0^\theta R^2(\tilde{\theta})\,\sin\tilde{\theta}\,d\tilde{\theta}\right)^2},$$

- $c_\phi,\ c_\pi$ integration constants

- charge of the WIH: $Q_e + \mathrm{i}Q_m = \dfrac{2\sqrt{2}\,\pi\, c_\phi}{c_\pi + \sqrt{2}\int\limits_\theta^\pi R^2(\tilde\theta)\,\sin\tilde\theta\, d\tilde\theta}$

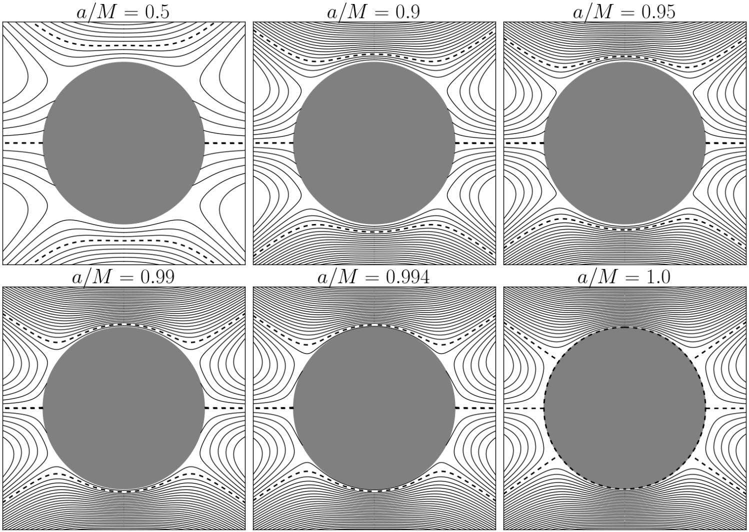
$\Rightarrow$ for uncharged WIH: $c_\phi = 0$

$\Rightarrow$ $\phi_1 \doteq 0$

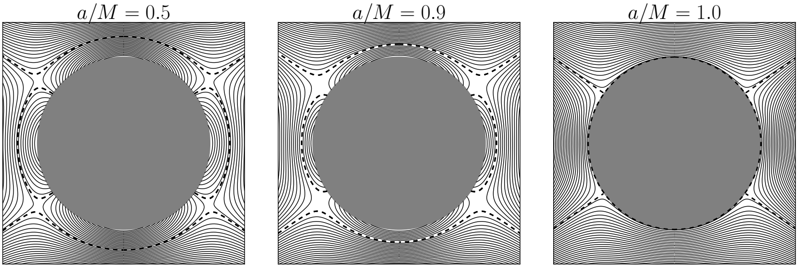
$\Rightarrow$ no magnetic and electric flux across any part of the horizon of an extremal black hole

$\Rightarrow$ **Meissner effect**

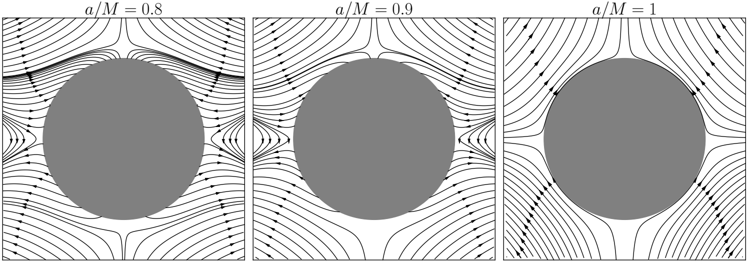
Magnetic Flux



Electric flux



Expulsion of the magnetic field



- Field lines of the magnetic field: B_a measured by an observer with the four-velocity $u^a = (\ell^a + n^a)/\sqrt{2}$

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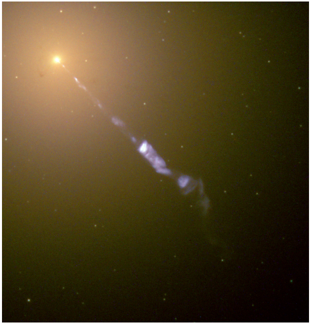
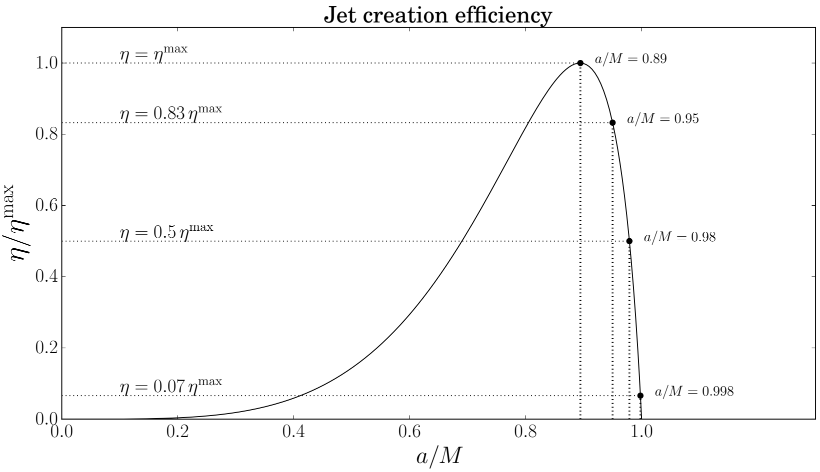
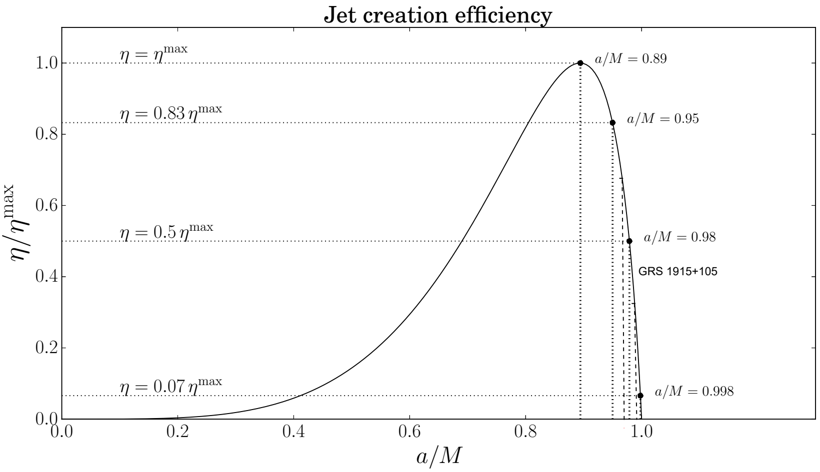
Jet creation





- $\eta = \frac{\varkappa}{4\pi} x^2 \left\langle \Phi_{\rm BH}^2 \, (\dot{M}\, M^2)^{-1/2} \right\rangle (1 + 1.38\, x^2 - 9.2\, x^4)$

  $x = \frac{a/M}{2(1+\sqrt{1-(a/M)^2})}$ *[Tchekhovskoy 2010, 2011]*



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Thank You!