Classical aspects of Black Holes

Dynamical Black holes

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Do Black Holes exist? - The physics and philosophy of Black Holes.
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2. Singularity Theorems
3. Astrophysical black holes: dynamical solutions
4. Cosmological contexts
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Dynamic Black holes

In the dynamical case, when the gravitational field of a collapsing star is strong enough to form closed trapped surfaces, no matter how anisotropic or inhomogeneous the collapse is, in the classical case singularities will occur provided some reasonable energy inequalities and global conditions hold. Provided the Cosmic Censorship Hypothesis holds, an event horizon will come into being at some time (in contrast to the static case where event horizons exist at all time). Inner and outer marginally trapped surfaces (IMOTS and OMOTS) will limit the trapped domain. The OMOTs surface is not the same as the event horizon, in dynamic cases; it is locally defined and lies inside the event horizon. There are no bifurcating Killing horizons in this non-static case, although the exterior domain will be static if the collapse is non-rotating and spherically symmetric (by Birkhoff's Theorem) and hence will be Schwarzschild.

Collapsing spherically symmetric fluid object

The unique spherically symmetric exterior solution of the Einstein Field Equations is the Schwarzschild solution

- 1. Schwarzschild gets its astrophysical power from Birkhoff's theorem: it is the exterior domain of any spherical star whatever, no matter what its radial dynamical behaviour. Hence no spherical gravitational waves are possible: stars cannot radiate away mass and energy by radial oscillations.

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The unique spherically symmetric exterior solution of the Einstein Field Equations is the Schwarzschild solution

- 1. Schwarzschild gets its astrophysical power from Birkhoff's theorem: it is the exterior domain of any spherical star whatever, no matter what its radial dynamical behaviour. Hence no spherical gravitational waves are possible: stars cannot radiate away mass and energy by radial oscillations.
- 2. Birkhoff's theorem remains approximately true when the solution is perturbed geometrically or by inclusion of matter (arXiv:1304.3253). Thus it applies to physically realistic cases.

- 1: The solution crosses the horizon at $r = 2m$. One can follow it in Eddington-Finkelstein coordinates, or Lemaître coordinates:

$$ds^2 = d\tau^2 - \frac{r_g}{r} d\rho^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{1}$$

based on infalling timelike geodesics, where

$$r = \left[\frac{3}{2}(\rho - \tau)\right]^{2/3} r_g^{1/3} \ . \tag{2}$$

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Sperical collapse to a black hole

Crossing the horizon

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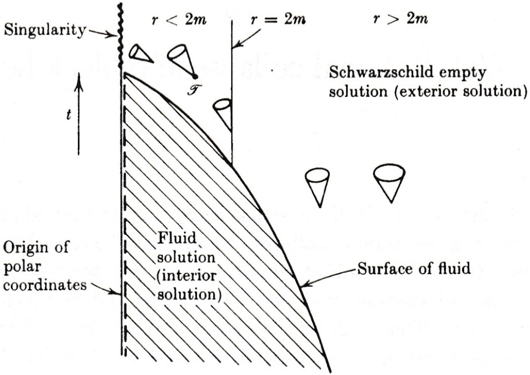
- 2.Nothing locally significant happens at the time of horizon crossing.
- 3: However the collapsing star is now doomed to fall into a future singularity, because the light cones tilt over. Everything inside the Event Horizon at $r = 2M$ is trapped. No matter or light can escape from here.

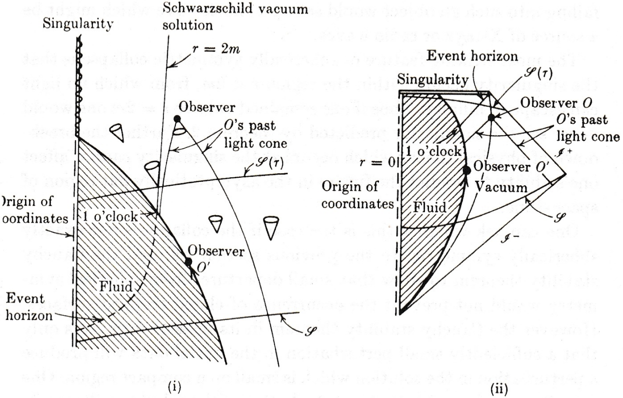
Sperical collapse to a black hole

Crossing the horizon



Spherical collapse, showing the tilting of the light cones, formation of event horizon at $r = 2m$, and formation of singularity at $r = 0$. The diagram does not correctly represent the spatial nature of the central singularity.

Dynamical collapse



Spherically symmetric collapse to form a black hole and Weyl/Ricci singularities. The final stage of collapse is invisible from the outside. The star fades away as redshift (seen from outside) diverges as it crosses the event horizon ('frozen stars'). The star sees nothing unusual at this event.

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At first, it was suspected that the strange features of the black hole solutions were artifacts from the symmetry conditions imposed, and that the singularities would not appear in generic situations.

Causal Methods

Roger Penrose introduced completely new methods of proving existence of singularities in 1965 ("Gravitational Collapse and Space-Time Singularities" *Phys. Rev. Lett.* 14, 57) proving that singularities are a generic feature of gravitational collapse from a regular initial state

- Singularity characterised by geodesic incompleteness
- Proof by contradiction, using global causal properties: caustics in null geodesics imply they are inside the causal future
- Inequalities not equalities: consequence of closed trapped surfaces
- Energy conditions

Penrose Singularity Theorem

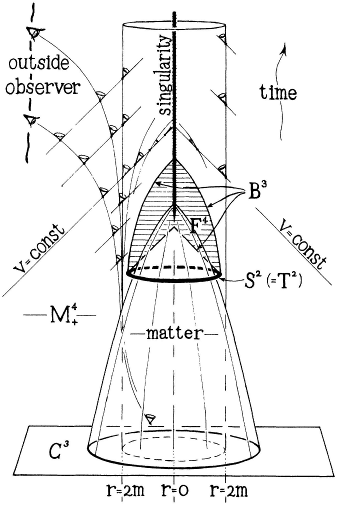


FIG. 1. Spherically symmetrical collapse (one space dimension surpressed). The diagram essentially also serves for the discussion of the asymmetrical case.

Basic points:

- If a closed trapped surface exists, then because of the null Raychaudhuri equation

$$\dot{\widehat{\theta}} = -\frac{1}{2}\widehat{\theta}^2 - 2\widehat{\sigma}^2 + 2\widehat{\omega}^2 - T_{\mu\nu}U^\mu U^\nu \tag{3}$$

  null geodesics will focus at conjugate points.
- They will then lie in the causal interior of the closed trapped surface, whose future is therefore compact
- This is incompatible with existence of a non-compact Cauchy surface

One should trust a solution of the EFE evolving from regular initial data up to the point where it, or any perturbations obtained by general perturbations of the initial data, becomes singular (Israel 1987)

S Hawking and R Penrose *Proc Roy Soc* A 314 529-548 (1970)

Theorem

A spacetime (M, g) cannot be time-like and null geodesically complete if the following are satisfied:

1. Energy condition: $R_{ij}K^iK^j \geq 0$ for all non-spacelike vectors K^i
2. The genericity condition is satisfied: every non-spacelike geodesic with tangent vector k^i has a point where $K_{[i]}R_{j]el[m]}K_{n]}K^cK^l \neq 0$
3. The chronology condition holds: there are no closed timelike curves in the spacetime M
4. There exists in M a closed trapped surface or a point p such that for either all past or all future directed geodesics from p, eventually $\theta := k^a_{;a}$ becomes negative

The latter condition will in all likelihood eventually hold for a sufficiently large collapsing star

Some issues arise:

- Weyl singularity or Ricci singularity? Or both?
- The energy condition need not hold for timelike vectors, because scalar fields can violate $\rho_{grav} := \rho + 3p \geq 0$.
- The genericity condition will be satisfied in all realistic spacetimes (no symmetries)
- The chronology condition is a sensible criterion for realistic solutions
- If the conditions are satisfied, the real conclusion is that General Relativity breaks down, either because it is in fact not the right theory of gravity, or because quantum gravity effects of unknown character come into play.
- These might lead to a regular bounce and expansion into a new spacetime domain hidden behind the event horizon.

Naked singularities

Closed trapped surfaces generically imply a singularity, but do they imply an event horizon? If so, is the singularity hidden behind the event horizon? If not we have a naked singularity: a gravitational singularity without an event horizon, or outside an event horizon.
In a black hole, the singularity is completely enclosed by an event horizon, inside which the gravitational force of the singularity is so strong that light cannot escape. Hence, objects inside the event horizon—including the singularity itself—cannot be directly observed.
A naked singularity, by contrast, is observable from the outside. This occurs for example in Kerr solutions with large enough angular momentum.

- Does a horizon in fact form during collapse?
- Assumption: yes. This is Cosmic Censorship
- There are some counter-examples
- But they are assumed to be non-physical

Singularities that arise in the solutions of Einstein's equations in the spherical case are hidden within event horizons, and therefore cannot be seen from the rest of spacetime. Is this true more generally?

- The cosmic censorship hypothesis asserts there can be no singularity visible from future null infinity arising from gravitational collapse of an isolated star in the generic case: no 'naked singularities' will occur during such gravitational collapse.
- Any singularities that occur will be hidden from an observer at infinity by an event horizon, hence a black hole will come into being.
- This means that what happens in the exterior can be predicted from maximal initial data at the start of the collapse.
- If a naked singularity exists, this is not the case: there is no control over what data may enter the universe from the singularity.

The Kerr metric, corresponding to a black hole of mass M and angular momentum J, can be used to derive the effective potential V for particle orbits restricted to the equator:

$$V_{\text{eff}}(r, e, l) = -\frac{M}{r} + \frac{l^2 - a^2(e^2 - 1)}{2r^2} - \frac{M(l - ae)^2}{r^3}, \qquad (4)$$

where r is the coordinate radius, e and l are the test-particle's conserved energy and angular momentum respectively (constructed from the Killing vectors), and $a \equiv \frac{J}{M}$ is the black hole angular momentum. For there to exist an event horizon around the singularity, and hence black hole formation, $a < 1$ must be satisfied.
Thus to preserve cosmic censorship, the solution must be restricted to the case $a < 1$ (Carter: electron black hole would have a naked singularity).

Anisotropies, inhomogeneities, and scalar fields can prevent existence of a horizon, e.g. the singularity can sometimes form before the horizon (Joshi).

Cosmic censorship

Overall

- Hoop conjecture (Thorne): Black holes with horizons form when and only when a mass m gets compacted into a region whose circumsference in *every* direction is $C \leq 4\pi m$

Wald (1999): "*Although the question of whether weak cosmic censorship holds remains very far from being settled, there appears to be growing evidence in support of its validity. This evidence consists primarily of*

- *The stability of black holes*
- *The proof of the failure of certain classes of counter-examples: collapsing shells of null dust*
- *Proof of a cosmic censorship theorem for the spherically symmetric Einstein-Klein-Gordon system*"

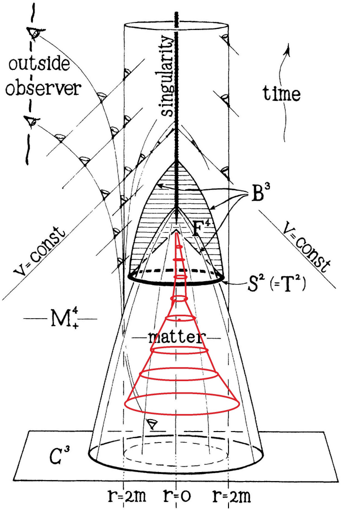
Key issue: stability of black holes, developing from Vishveshswara's stability theorem and work by Price and by Kay and Wald. But there do indeed seem to be many counter examples! (Joshi et al)

"Why do naked singularities form in gravitational collapse?"
Pankaj S. Joshi, Naresh Dadhich, and Roy Maartens. *Phys. Rev.* D 65:101501 (2002) - [arxiv:gr-qc/0109051]

- *We investigate what are the key physical features that cause the development of a naked singularity, rather than a black hole, as the end-state of spherical gravitational collapse. We show that sufficiently strong shearing effects near the singularity delay the formation of the apparent horizon. This exposes the singularity to an external observer, in contrast to a black hole, which is hidden behind an event horizon due to the early formation of an apparent horizon.*

For spherical gravitational collapse with homogeneous density (and arbitrary pressures), the final outcome is necessarily a black hole, but more generally naked singularities will form: so Oppenheimer-Schneider is highly misleading [Joshi and Malafarina: arXiv:1405.1146].

Naked Singularity



The singularity may form as a ring ($r > 0$) outside the event horizon.

Roger Penrose *J. Astrophys. Astr.* (1999) 20, 233–248:
“Two familiar mathematical criteria for ‘unstoppable collapse’ are the existence of a trapped surface or of a point whose future light cone begins to reconverge in every direction along the cone. In either of these situations, in the presence of some other mild and physically reasonable assumptions, like the nonnegativity of energy (plus the sum of pressures), the nonexistence of closed timelike curves, and some condition of genericity (like the assumption that every causal geodesic contains at least one point at which the Riemann curvature is not lined up in a particular way with the geodesic), it follows (by results in Hawking and Penrose 1970) that a space-time singularity of some kind must occur. (Technically: the space-time manifold must be geodesically incomplete in some timelike direction.)”

Roger Penrose: *J. Astrophys. Astr.* (1999) 20, 233–248
"It appears to be a not uncommon impression among workers in the field that as soon as one of these conditions is satisfied – say the existence of a trapped surface - then a black hole will occur; and, conversely, that a naked singularity will be the result if not. However, it should be made clear that neither of these deductions is in fact valid. The deduction that a black hole comes about whenever a trapped surface is formed requires the assumption of cosmic censorship. Moreover, the deduction that some kind of space-time singularity comes about (in general situations), whether or not it is a naked one, requires some such assumption like that of the existence of a trapped surface. Thus, the presence of a trapped surface does not imply the absence of naked singularities; still less does the absence of a trapped surface imply the presence of a naked singularity,

Uniquenss of endpoint of collapse

Assuming cosmic censorship,

- Collapsing stars radiate away anisotropies (Israel, Penrose, Price) mostly downward through the horizon, and so are assumed to end up stationary and axisymmetric
- Then the black hole uniqueness theorems imply one ends up with a Kerr metric, which is a black hole as it has an event horizon
- It is described by only two parameters (m, a)
- Hence "Black holes have no hair" (Wheeler): they radiate away all information about material dropped in except total mass and angular momentum.

The inner horizon experiences infinite blue-shift that causes a singularity there and closes off the throat to the infinite further outer domains of th emximal extension (Israel). This probably saves the solution from a break down of predictability due to closed timelike curves.

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Spherical collapse to a black hole

Penrose Diagram and formation of trapped surfaces

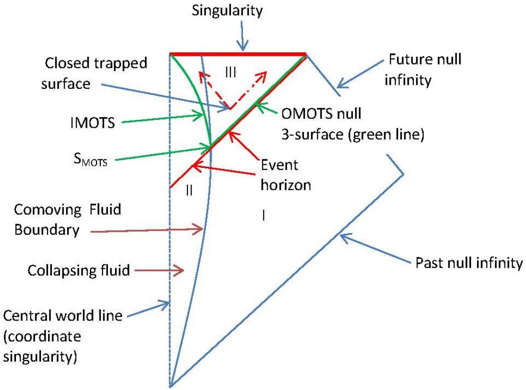

Figure: Penrose diagram of collapse to form a singularity inside a Schwarzschild spacetime. The horizon bifurcation no longer occurs. Closed trapped surfaces occur in region III. There is only one asymptotically flat domain (no wormhole).

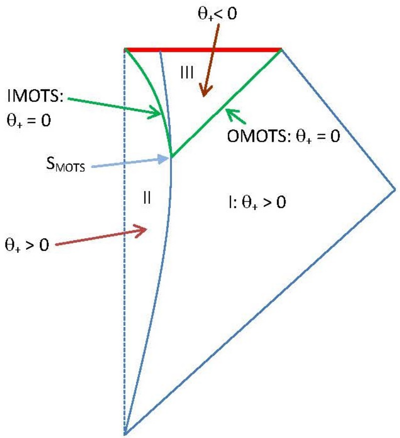

Figure: Penrose diagram of collapse to form singularity, showing trapping domains separated by MOTS surfaces

Event horizon and apparent horizon generically no longer the same:

- *Event horizon* is a globally defined null surface
- *Apparent horizon* is a locally defined Marginally Outer Trapped Surface (MOTS) which can be timelike or spacelike
- They coincide in the static case due to existence of timelike Killing vectors (Birkhoff if spherical), but are not the same in the case of dynamic solutions, specifically black holes being formed from stars and with infalling matter and tradiation
- Need to solve interior field equations to locate the apparent horizons. This is an interesting and complex task. The Oppenheimer-Snyder homogeneous case, easy to construct, is atypical and misleading.

Detailed study: “Causal Nature and Dynamics of Trapping Horizons in Black Hole Collapse” Alexis Helou, Ilia Musco, John C. Miller [arXiv:1601.05109v5]; Ilia’s talk

Sperical collapse to a black hole

Infalling matter and radiation: trapping domains

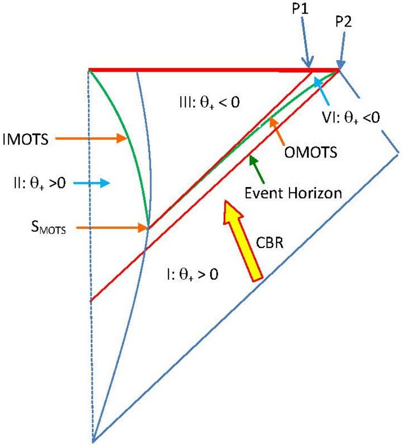

Figure: Black hole in a cosmological context: incoming radiation and matter. Event horizon (global) and apparent horizon (local) differ (not Schwarzschild).

Rotating astrophysical black holes

Dynamic collapse of rotating matter is far more complex

- Asymptotic Kerr geometry
- Ergosphere and energy extraction
- Linked to accretion discs (Straub), magnetic fields (Karas)
- Infalling matter and ring singularity
- Location of trapped regions and MOTS surfaces??
- Cosmic censorship probably holds if angular momentum is limited

The spherical models are a first step on the way

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Cosmological black holes

In realistic cases, the black hole is imbedded in an expanding universe rather than an asymptotically flat spacetime, and has infalling matter and radiation rather than being in a vacuum. In that case, many of the standard theorems do not apply unchanged.

In particular the event horizon cannot be defined as before, and the outer apparent horizon will be spacelike rather than null. The nature of the MOTS surfaces and the interior singularities will depend on the fluid equation of state and initial data.

If a cosmological constant exists, as seems to be the case, the future boundary of the universe is spacelike because the spacetime is asymptotically de Sitter. Definition of black hole event horizon is not obvious.

Cosmological contexts

Asymptotically de Sitter

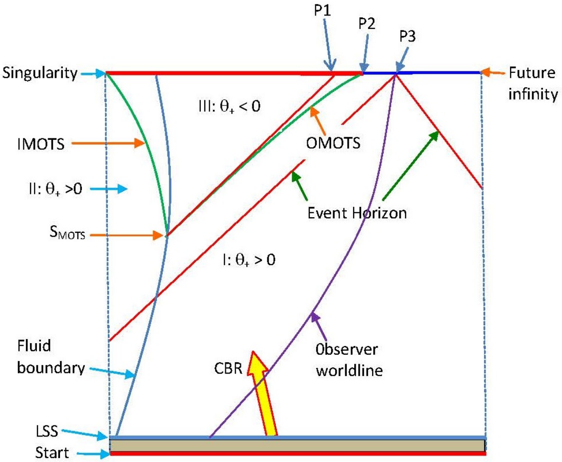

Figure: Black hole in a cosmological context with a cosmological constant

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Black Hole Thermodynamics

Temperature $T_{\mathrm{H}} = \frac{1}{8\pi m}$. and entropy $S = \pi R^2_{Sch} = \frac{A}{4}$.

- comprehensive talk by Erik Curiel.

Nb: implications of black hole entropy for starting conditions for inflation: need a Weyl Curvature Hypothesis (Penrose)

Gravitational waves

Colliding black holes generate gravitational waves which have been detected.

Combine post-Newtonian methods (perturbation series) and numerical methods (highly nonlinear).

- How to handle singularities combining?
- How to handle event horizons combining?

Conclusion

Black holes are plausibly observed to occur in many contexts: stellar mass black holes, intermediate mass black holes (*Nature* 542:203, 2017), massive black holes in galaxies, as engines of QSO's, through black hole collisions producing gravitational waves

However we do not in fact observe them: we deduce they are there by their effect on surrounding matter and radiation.

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The significance of black holes

Black holes started off as a purely mathematical construct. They are now central to much of high energy astrophysics. We cannot do without them.

The defining feature of a black hole is existence of an event horizon — a boundary in spacetime through which matter and light can only pass inward towards the centre of the black hole. Nothing, not even light, can escape from inside the event horizon. No information from events inside reach an outside observer. Information is lost in black holes.

Real collapse will not be spherically symmetric, and may be very anisotropic, as for example in Zeldovich collapse. The final exterior result of collapse will still be a Schwarzschild or Kerr solution – provided a cosmic censorship condition holds. But cosmic censorship is still not proven: it remains a physically plausible hypothesis that it holds in physically relevant cases. It may be that singularities form before an event horizon comes into being, for example if the singularity is very anisotropic and hence falls outside the Hoop Conjecture, thus leading to naked singularities.
By the no-hair theorem, a black hole can only have three fundamental properties: mass, electric charge and angular momentum (spin). The spin of a stellar black hole is due to the conservation of angular momentum of the star that produced it. The charge is zero in astrophysical cases.