Classical Black Holes Are Hot

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Outline

Black Holes

- region of "no escape" (event horizon)
- "no hair": characterized by mass, angular momentum, electric charge
- stationary: "in equilibrium", nothing changes over time

Thermodynamical Objects?

late 1960s Wheeler poses fundamental puzzles about black holes and the Second Law of thermodynamics

1970 Penrose shows how to extract energy; Hawking proves Area Theorem; Christodoulou characterizes irreducible mass, defines reversible and irreversible processes; Geroch's infamous *Gedankenexperiment*

1971–1973 Bekenstein proposes entropy proportional to surface area, formulates Generalized Second Law

1973 Bardeen, Carter and Hawking prove four Laws of Black-Hole Mechanics; most think analogy with thermodynamics purely formal

1974–1975 using quantum effects, Hawking shows black holes radiate like perfect black bodies: after brief resistance, most now think black holes are "true thermodynamical objects"

Philosophical Puzzles

- black hole surface gravity and area "real" thermodynamical quantities? (concept expansion, inter-theory relations, nature of physical analogies. . .)
- deep connection between general relativity (or: gravity, spacetime) and thermodynamics?
- quantum effects (Hawking radiation) only physical justification for taking black-holes as truly thermodynamical objects? (revisions to spatiotemporal concepts, possible insight into Second Law of thermodynamics. . .)
- relation to ordinary Second Law of Thermodynamics and cosmological arrow of time?
- relation to quantum gravity?
- status of claims, grounds for belief? (NO experimental evidence)

The Current State of Play in Physics

focus has been on quantum properties and effects; nothing of thermodynamical interest in a purely classical treatment of black holes

My Take on the Matter

black holes considered classically have a rich thermodynamical theory; there is a deep relation between gravitation and thermodynamics already, without taking quantum mechanics into account

Zeroth Law

Thermodynamics

The temperature T is constant throughout a body in thermal equilibrium.

Black Holes

The surface gravity κ is constant over the event horizon of a stationary black hole.

First Law (Energy Conservation)

Thermodynamics

change in energy = heat [temperature × change in entropy] + work done

$$(\mathrm{d}E = T\mathrm{d}S + p\mathrm{d}V + \Omega \mathrm{d}J)$$

Black Holes

change in mass = (surface gravity × change in area) + "rotational work"

$$(\delta M = \frac{1}{8\pi}\kappa\delta A + \Omega_{\mathsf{H}}\delta J)$$

Second Law

Thermodynamics

entropy never decreases ($\delta S \geq 0$) in any physical process

Black Holes

area of event horizon never decreases ($\delta A \geq 0$) in any physical process

Third Law (Nernst Theorem)

Thermodynamics

$T = 0$ is not achievable by any physical process

Black Holes

$\kappa = 0$ is not achievable by any physical process

“Minus First Law” (Brown and Uffink)

Thermodynamics

isolated thermodynamical systems tend to approach a unique equilibrium state

Black Holes

isolated, non-stationary black holes tend to settle down to a unique stationary state (Kerr spacetime)

The Status of the Analogy?

A purely formal analogy? Or are black holes truly thermodynamical objects? Are the black-hole laws the laws of thermodynamics extended to cover black holes?

The Standard Argument: No!

Classical black holes are perfect absorbers. Thus, by the laws of black-body radiation, they must have physical temperature absolute zero. (Carter, Geroch, Hawking, Israel, Jacobson, Penrose, Unruh, Wald, Wheeler, . . .)

A little more precisely: put a Kerr black hole immersed in thermal radiation in a box; the black hole will perfectly absorb all the radiation and emit none; thus, temperature absolute zero.

Orthodoxy: we need quantum mechanics!

- Hawking radiation justifies the claim that κ is a physical temperature
- the validity of the Generalized Second Law justifies the claim that A is a physical entropy

standard argument misses real physics

A Kerr black hole immersed in thermal radiation in a box *will* couple with the radiation and emit energy: in-falling thermal radiation perturbs it, yielding gravitation radiation, which physically couples with the exterior thermal radiation.

classical black holes are not
perfect absorbers

(electromagnetic energy alone not proper medium
for "gravitational thermal coupling")

standard argument begs the question

black-body radiation is fundamentally quantum and statistical, not appropriate basis to characterize temperature for a classical thermodynamical system—we don't use it to define temperature in Navier-Stokes theory

inconsistent to model a black hole using a non-quantum theory and at the same time to try to attribute a temperature to it using criteria that make sense only in the context of a quantum theory

more poignantly. . .

Hawking radiation is not blackbody radiation

- standard blackbody radiation generated solely by internal degrees of freedom of the body itself (vibrational modes of atoms and electrons of the black body), and the *type* of radiation emitted (electromagnetic) is governed by the nature of the micro-constituents of the body
- Hawking radiation comes from scattering of an external field hitting the black hole, and the radiation scattered is always and only of that type.
- $\Rightarrow$ Hawking radiation *not* generated by internal degrees of freedom of black hole

arguments that black-hole thermodynamics need to save Second Law do not essentially rely on quantum phenomena

evidentiary and epistemic role

Without an understanding of black-hole entropy as a truly thermodynamical entropy, in the sense of phenomenological thermodynamics, we have no real evidence in the first place that black holes have a micro-structure appropriate for a statistical treatment of its dynamics that would yield a physically significant accounting of its entropy.

thus, motivation and grounding for:

- Einstein field equation as "equation of state" (Jacobson) and "emergent gravity" (Padmanabhan)
- generic attributions of entropy to purely classical "gravitational degrees of freedom" (cosmology, Penrose's Conformal Curvature Hypothesis, some quantum gravity programs)
- signature of quantum gravity, in particular the traces of statistical quantities needed to account for thermodynamical phenomena, show up already in purely classical theory—clues to what kind of underlying statistics may be needed (*cf.* Planck and blackbody radiation, and Einstein and Brownian motion)

philosophical payoffs?

- when physical quantities and principles extended into new regimes, then conceptual and formal modifications needed for understanding lead to deeper, more finely nuanced and multi-faceted understanding in original context (*e.g.*, 19th century extension of thermodynamics to electromagnetic field)
- may suggest new avenues of attack on old, well known, and deeply recalcitrant problems:
  - the nature of spacetime as an entity in its own right
  - the nature of entropy as physical quantity
  - the character and status of the Second Law
  - the relation among the different arrows of time

Some Counterfactual History

Imagine we knew only general relativity and classical thermodynamics—how would we decide?

1. Can κ be introduced in the same ways and does it play the same formal roles as T?
2. Does κ couple to ordinary thermodynamical systems in the same way as T?
3. Does A play the same role in the Gibbs relation ("equilibrium First Law") as S?
4. Do changes in A systematically save the validity of the Second Law?

This is just how classical thermodynamics was extended to cover black-body radiation at the end of the 19th Century.

Three Ways Temperature Enters into Classical Thermodynamics

1. indicator of heat flow
2. measure of capacity to do work
3. measure of possible efficiency of reversible engine

Three Ways Entropy Enters into Classical Thermodynamics

1. measure of energy needed to transform heat into work
2. that perfect differential dS into which temperature, as integrating factor, transforms exchanges of heat δQ (not perfect differential)
3. determinant of efficiency of reversible engine

The Clausius Postulate

A transformation whose only final result is to transfer heat from a body at a given temperature to a body of a higher temperature is impossible.

captures:

- temperature as indicator of direction of heat flow
- entropy equal to heat integrated using temperature

The Kelvin Postulate

A transformation whose only final result is to transform into work heat extracted from a source that is at the same temperature throughout is impossible.

captures:

- temperature as measure of capacity to do work
- entropy as measure of energy needed to transform heat into work

Carnot Cycles

Analysis of the efficiency of reversible thermal engines yields absolute temperature scale *à la* Kelvin, based on entropy changes, by computing ratios of transferred quantities of heat.

Irreducible Mass

M_{irr} a convex, positive-definite function of total mass and angular momentum; total mass cannot be reduced below initial value of M_{irr} by any physical process

$$M_{\text{irr}}^2 := \frac{1}{2}[M^2 + (M^4 - J^2)^{\frac{1}{2}}]$$

so

$$M^2 = M_{\text{irr}}^2 + \frac{J^2}{4M_{\text{irr}}^2} \geq M_{\text{irr}}^2$$

Reversible and Irreversible Processes

$$A = 16\pi M_{\text{irr}}^2$$
$$\Downarrow$$

Second Law implies M_{irr}^2 cannot decrease through any physical process

Reversible Transformation: $\Delta M_{irr} = 0$
Irreversible Transformation: $\Delta M_{irr} > 0$

Rotational Energy and "Heat" of Black Holes

- since M cannot be reduced below M_{irr}:

$$\text{“rotational energy”} := M - M_{\mathsf{irr}}$$

- define gravitational "quantity of heat" transferred in any process to be change in total black hole energy minus change in its rotational energy:

$$\Delta Q_{\mathsf{BH}} := \Delta M_{\mathsf{irr}} \, (> 0)$$

What is "thermal contact" for a black hole?

I claim: exchange of "non-rotational gravitational energy" ("heat"), *e.g.*, emission and absorption of gravitational radiation, monopole coupling

Clausius Postulate for Black Holes

A transformation whose only final result is that the change in irreducible mass of black hole at a given surface gravity is less than the change in irreducible mass of one with a higher surface gravity is impossible.

Kelvin Postulate for Black Holes

A transformation whose only final result is to decrease the angular momentum and increase the irreducible mass of a black hole while leaving its surface gravity unchanged is impossible.

(provable by same methods as Gao and Wald's proof of "physical process version" of First and Second Laws)

Coupling with Ordinary Thermodynamical Systems and Absolute Temperature

Carnot-Geroch Cycle Using a Black Hole as a Heat Sink

1. empty box "at infinity", one side a piston; "quasi-statically" fill box with fluid from heat bath at fixed temperature; when piston has withdrawn part of the way, seal box; box now has total energy depending on rest mass, temperature, entropy, and work done
2. "quasi-statically" lower box towards black hole, extracting work from its changing gravitational potential energy; an observer inside box sees nothing change; "at infinity" measured total energy (with respect to Killing field) has decreased by redshift factor (work done)

Carnot-Geroch Cycle Using a Black Hole as a Cold Sink, cont.

3. at fixed proper distance inside ergosphere, hold box stationary
4. "quasi-statically" draw piston back more, lowering temperature of fluid, keeping entropy unchanged, lowering total energy by work done (value of final temperature determined by condition that entire cycle is isentropic)
5. "quasi-statically" eject fluid; fluid falls into black hole delivering mass-energy (as heat), and positive entropy; piston returns to initial position
6. pull box back "to infinity" (takes no work), returning to its initial state

Assume:

1. black hole has physical entropy
2. black-hole entropy additive with ordinary entropy
3. First Law

now, calculate efficiency of process in standard way, using ratio of heats ejected and absorbed during cycle, so defining absolute temperature scale

Then:

the demand that total cycle be isentropic is consistent if and only if black hole temperature is proportional to κ and black hole entropy is proportional to A

And:

there necessarily exists a universal constant for black holes with proper physical dimension to give κ units of temperature and A units of entropy

Problem

why do black holes have such enormous entropy, at least 10^{10} times more than bodies that collapse to form them?

Possible Insight

reliance on classical notion of entropy ("measure of extractibility of energy") suggests that black holes *should* have much higher entropy: it's difficult to extract energy from them!

Problem

why are the cosmological and the ordinary thermodynamical arrows of time correlated?

Possible Insight

they both have the same thermodynamical root!

Problem

(*pace* Carlo) why does general relativity resist standard quantization?

Possible Insight

may suggest it's only an effective—thermodynamical—theory!