On Rotating Regular Black Holes

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641WEH/Bad Honnef 2017

OUTLOOK

- REGULAR BLACK HOLES
- ALGORITHM TO MAKE THEM ROTATING
- de SITTER CENTER AND ENERGY CONDITIONS
- IS THERE A SIGNATURE OF THE REGULAR CENTER OUTSIDE THE HORIZON?
- A PARTICULAR EXAMPLE: THE ROTATING HAYWARD B.H.
  ITS PHOTOSPHERE AND QNM
- CONCLUSIONS

REGULAR BLACK HOLES

To have a regular black hole some kind of matter must be introduced.
Sakharov (1966), Gliner(1966) suggested a de Sitter core with EoS $p = -\rho$, hence $T_{\mu\nu} = \Lambda g_{\mu\nu}$
Several ways to regularize: introducing an internal structure, exotic matter.
Static Spherically Symmetric solutions of the EFE,

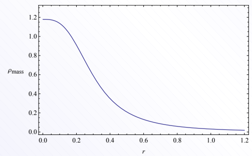
$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2[d\theta^2 + \sin^2\theta d\phi^2],$$

$$f(r) = 1 - \frac{2m(r)}{r}, \quad m(r) = \frac{M_0}{\left[1 + \left(\frac{r_0}{r}\right)^q\right]^{p/q}}$$

M_0, r_0 mass and length parameters;
$p, q > 0$ guarantee asymptotic flatness: $m(r) \approx M_0 \left(1 - \frac{p}{q}\left(\frac{r_0}{r}\right)^q\right)$
As $r \to 0$, $m(r) \approx M_0 \left(\frac{r}{r_0}\right)^p$
Markov, Brandenberger, Mukhanov in the 1990s: Spacetimes in the highly dense central region of a black hole would be de Sitter-like, $f(r) = 1 - \left(\frac{r}{l}\right)^2$, that requires $p = 3$.

Several SSS solutions have been derived with horizons, a regular center, asymptotically flat, with finite curvature quantities, R, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

- Bardeen (1968), magnetic monopole,
- Hayward (2006)
  $f(r) = 1 - \frac{2m}{r}$, as $r \to \infty$,
  $f(r) = 1 - \frac{r^2}{l^2}$, as $r \to 0$,
  an effective cc $\Lambda = 3/l^2$ at small distances with Hubble length l



- $f(r) = 1 - \frac{2mr^2}{r^3+2ml^2}$
  recovering Schwarzschild when $l = 0$ and flat for $m = 0$
  $m > m_\star = \frac{3\sqrt{3}}{4}l$ for having a two horizons BH, with $r_- > l$

NLED-REGULAR BLACK HOLES

- Sourced by NLED coupled to gravity, with $\mathcal{L}(F)$,
  $F = F_{\mu\nu}F^{\mu\nu}$, with $\mathcal{L}(F) \to F$ in the weak field limit
  Regular black holes have been determined by demanding a regular center

$$f(r) = 1 - 2\frac{m(r)}{r}, \quad m(r) = \frac{1}{2}\int T_0^0 r^2 dr,$$

$$8\pi G T_\mu^\nu = -2\mathcal{L}_F F_{\mu\alpha}F^{\alpha\nu} + \frac{1}{2}g_\mu^\nu \mathcal{L},$$

- NLED Magnetically charged can have a Maxwell center
  Electrically charged should have a de Sitter center
- Most of them satisfy WEC
- Examples: Ayon-Beato-Garcia (1999), Dymnikova (2000), Bronnikov (2001)

SET THEM SPINNING

- Algorithm by Trautman (1962), Newman-Janis (1965), Gurses-Gursey (1975)
- Kerr can be derived from Schwarzschild

  Kerr-Newman can be derived from Reissner-Nordstrom
- Starting with a static spherically symmetric metric,

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - h(r)(\sin^2\theta d\varphi^2 + d\theta^2),$$

- Transform to null coordinates:

$$du = dt - \frac{dr}{f(r)}, \quad g^{\mu\nu} = l^\mu n^\nu + l^\nu n^\mu - m^\mu \bar{m}^\nu - m^\nu \bar{m}^\mu$$

- Introduce rotation by hand (there is no a unique way to do it)

$$r \to r' = r + ia\cos\theta, \quad u \to u' - ia\cos\theta,$$

- Recover the metric in Boyer-Lindquist coordinates.

NJ algorithm in general does not preserve field equations.
Improvements to the method: Azreg-Ainou (2014),

WEAK, DOMINANT, STRONG ENERGY CONDITIONS

- SSS can be written in Kerr-Schild form, $g_{\mu\nu} = \eta_{\mu\nu} + \frac{2\tilde{f}(r)}{\Sigma} k_\mu k_\nu$, k^μ are null and geodesics directions; $\Sigma = r^2 + a^2 \cos^2\theta$.
- The energy-momentum tensor can be considered as

$$
\begin{aligned}
8\pi T_{\mu\nu} &= (D + 2G) g_{\mu\nu} - (D + 4G)(l_\mu l_\nu - u_\mu u_\nu), \\
G &= \frac{\tilde{f}' r - \tilde{f}}{\Sigma^2}; \quad D = -\frac{\tilde{f}''}{\Sigma}, \quad 8\pi\rho = 2G, \quad p_{\tan} = \rho + \frac{D}{8\pi},
\end{aligned}
$$

- WEC is satisfied if $\quad T^{\mu\nu} V_\mu V_\nu \geq 0, \quad \rho \geq 0, \quad \rho' \leq 0.$
  i.e. energy density should be a decreasing function,
- DEC is satisfied if $\quad T^{00} \geq T^{ij}$, for all i, j
- SEC is satisfied if $\quad R^{\mu\nu} V_\mu V_\nu \geq 0, \quad p_{\tan} \geq 0, \quad \rho' \leq 0.$

Violation of WEC

For instance in the core of a non-rotating regular BH:
$\rho = 8\pi\Lambda, \quad p_{\tan} = -8\pi\Lambda, \quad \rho' = 0$
it depends on the sign of Λ, in de Sitter case, $\Lambda > 0$, WEC is fulfilled and SEC is violated.
Note that since $\tilde{f}$ does not depend on a, ρ has the same sign in the static and its rotating counterpart, Note that if $\tilde{f}'' \neq 0$ the fluid may not be perfect when set rotating

Violation of WEC

Introducing rotation WEC may be violated:
A generic solution with $m(r)$, at the poles,
$T^{00} = \frac{2r^2 m'(r)}{8\pi(r^2+a^2)^2} = -T^{11}, \quad T^{22} = -\frac{2a^2 m' + r(r^2+a^2)m''}{8\pi(r^2+a^2)^2} = T^{33},$
WEC: $T^{00} \geq 0, \quad T^{00} + T^{ii} \geq 0,$
Considering the case $m(r) \propto r^3$,
As $r \approx 0, \quad T^{00} + T^{22} = T^{00} + T^{33} \propto -\frac{12r^2 a^2}{(r^2+a^2)^2}$
Then in the case of a de Sitter core, violation of WEC cannot be prevented irrespective of the details of $m(r)$.
In the NLED case more freedom exists to adjust the fulfilment of WEC:
$8\pi(p_{\tan} + \rho) = -F\mathcal{L}_F$
F is non positive (purely electrical), $\mathcal{L}_F$ plays the role of the electric permeability, then $\mathcal{L}_F > 0$

IS THERE A SIGNATURE OF THE REGULAR CENTER OUTSIDE THE HORIZON?

Three last stages of black hole perturbations

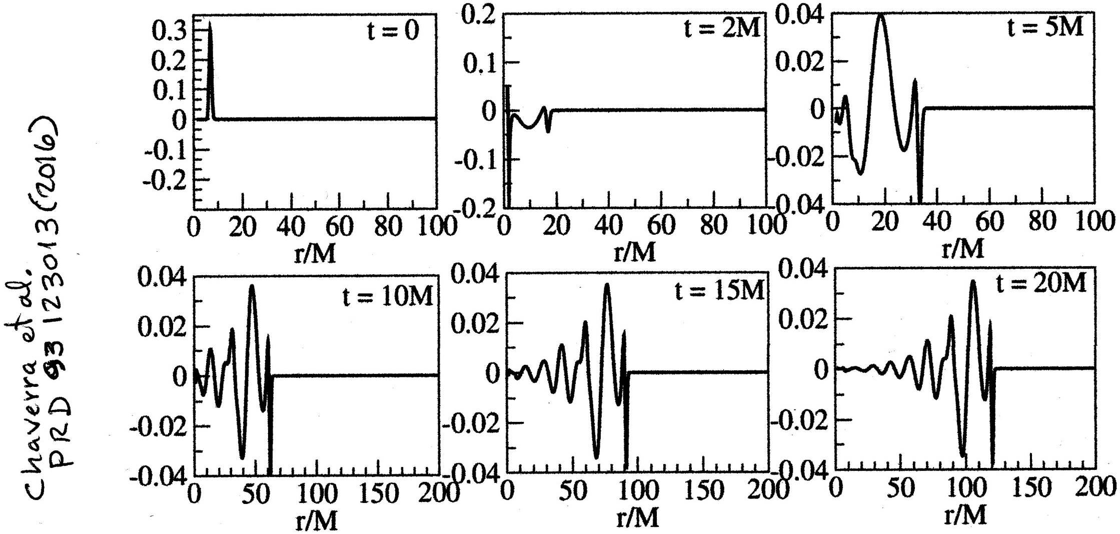


FIG. 1: Time evolution of $R_{\pm}^{(1)}$ for $\ell = 2$ for initial data of the form specified in Eq. (46). Most of the initial pulse falls into the black hole (note the different scales in the vertical axis when passing from $t = 0$ to $t = 2M$ to $t = 5M$) after which the spacetime reacts with an oscillating characteristic signal which contains the QN modes.

Three last stages of black hole perturbations

Chaverra et al. PRD 93 123013 (2016)

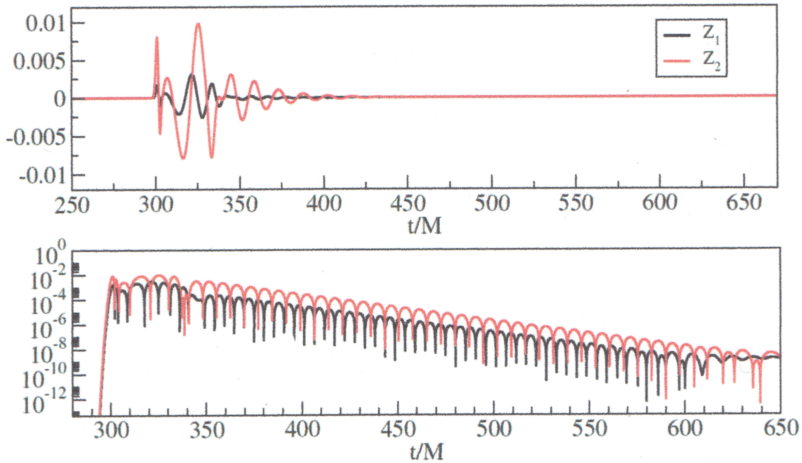


FIG. 2: A plot of the signal as a function of time for the ring-down part of the waveform (upper panel). The exponential decay rate and the constant frequency are visible in the semi-logarithmic representation (bottom panel).

QNM are particular solutions of the pulsation equations

$$\{\partial_t^2 - \partial_r^2 + \psi(r)V_{l\pm}(r)\}\begin{pmatrix}\Psi_{\mathrm{lm}\pm}\\ \Phi_{\mathrm{lm}\pm}\end{pmatrix} = 0,$$

$-\mapsto$ odd parity sector $\mapsto$ axial perturbations (Regge-Wheeler eq. 1957)
$+\mapsto$ even parity sector $\mapsto$ polar perturbations (Zerilli eq. 1970)
QNM solutions are of the form $e^{\omega t}\psi(r), \quad \omega = \omega_R + i\omega_I$
with $\psi(r)$ solution of

$$\left[w^2 - f(r)\partial_r f(r)\partial_r + f(r)V_{l\pm}(r)\right]\psi(r) = 0,$$

where $f(r)$ is the metric component of a SSS solution to EE

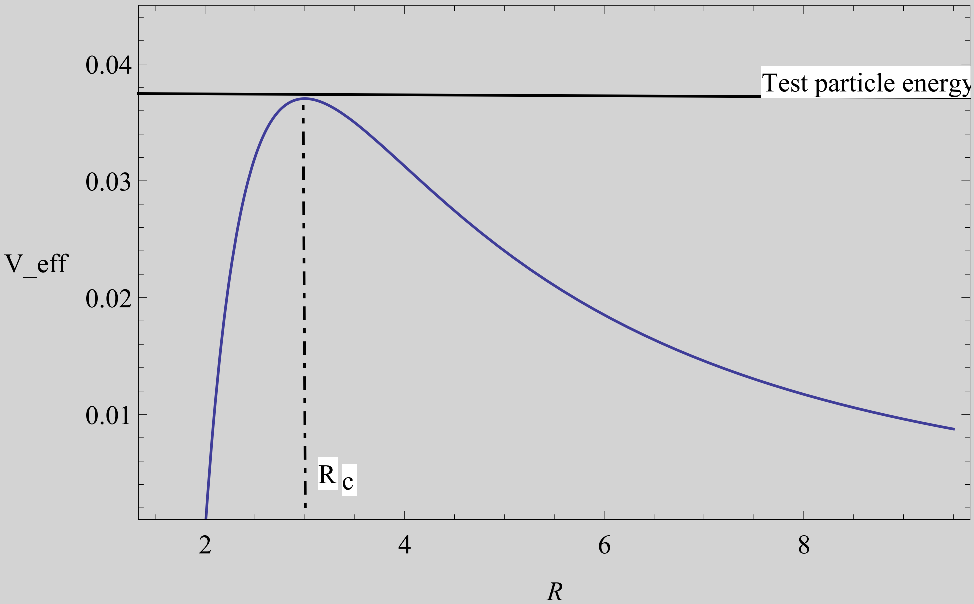
$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2 d\Omega^2,$$

BOUNDARY CONDITIONS:

- at ∞ purely outgoing waves,
- at the horizon purely ingoing waves.

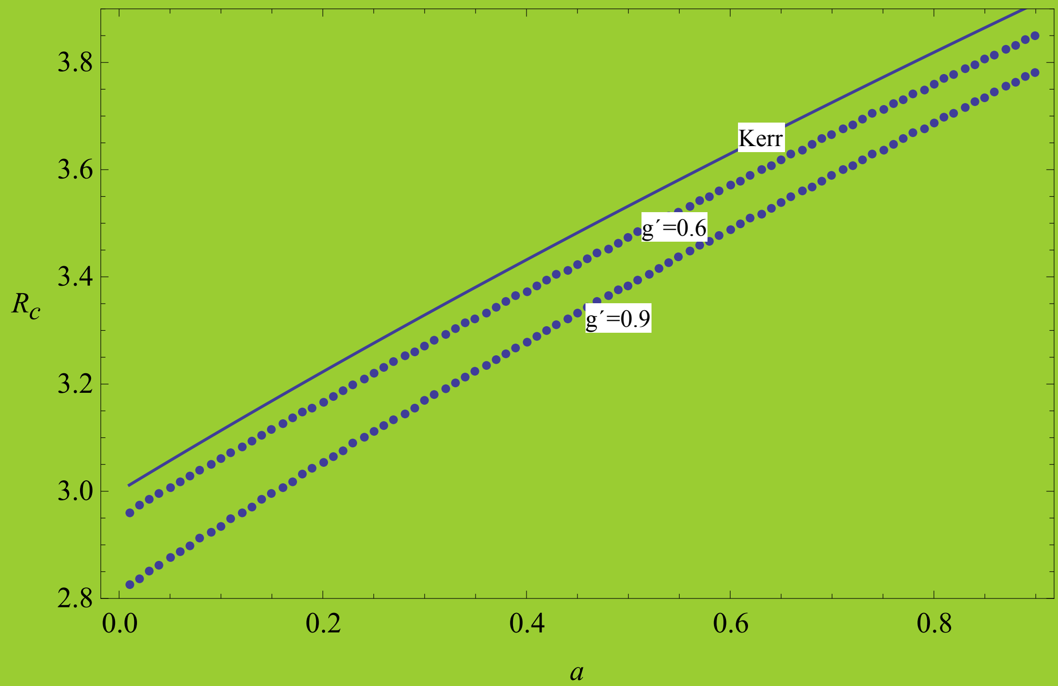
QNM and unstable null geodesics

In the high-frequency approx. QNF can be derived directly from the properties of the unstable circular NULL orbits [Mashhoon and Ferrari (1984), Cardoso (2009)]: $\omega_{QNM} = \Omega_c l - \imath(n + \frac{1}{2})|\lambda|$,
n is the overtone number, l is the angular momentum of the perturbation.
Ω_c is the angular velocity at the unstable null geodesic.
λ is the Lyapunov exponent, determining the instability time scale of the orbit.



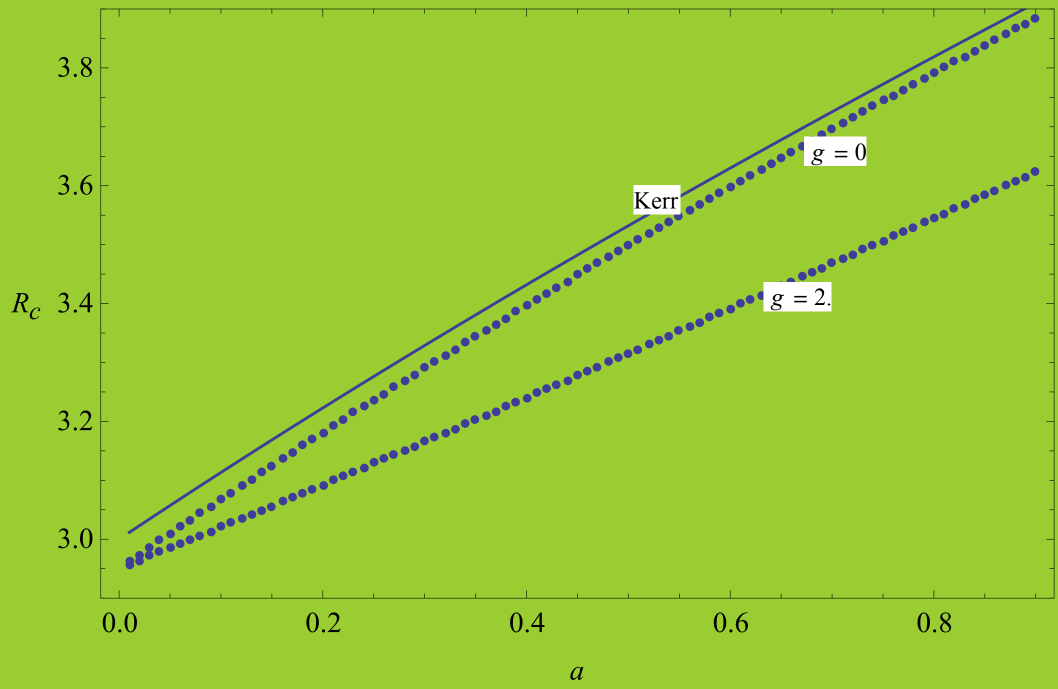
$M\omega >> 1, \quad R = r/M$

R_c the radius of the photosphere of the rotating regular Hayward BH



The radius of the photosphere of the rotating regular Hayward BH $M = 1, g' = 0{,}6; 0{,}9$ and $g = 1.$, as a function of a, g' g is the parameter associated to the regular center, a kind of de Sitter parameter, $m(r) = Mr^3/(r^3 + g'^3)$.

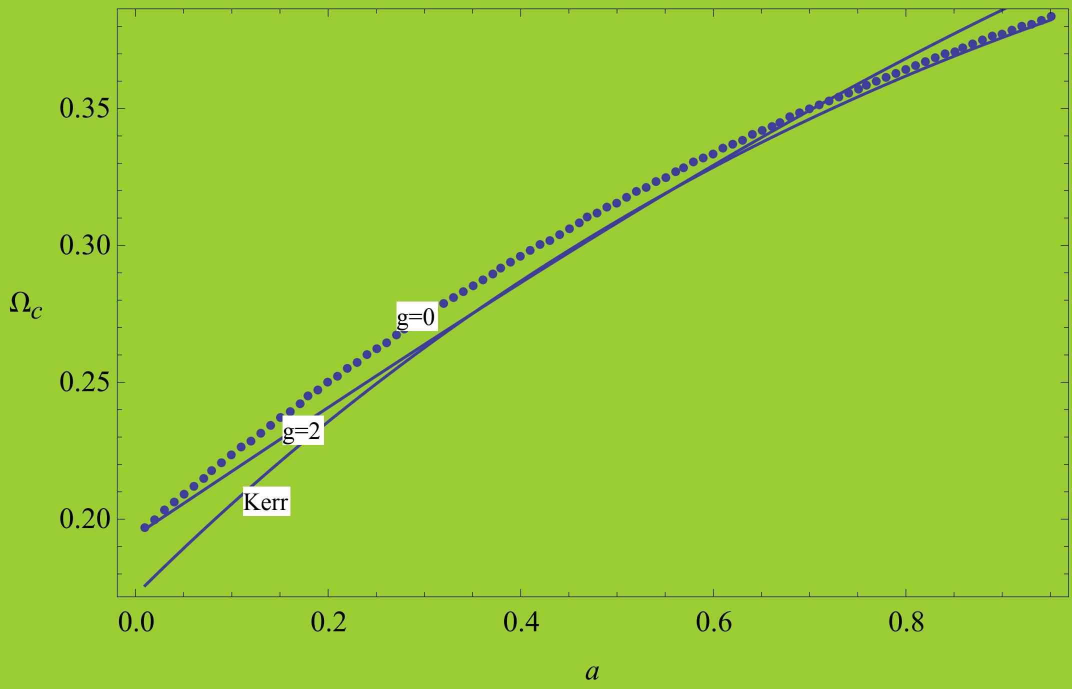
R_c the radius of the photosphere of the rotating regular Hayward BH



The radius of the photosphere of the rotating regular Hayward BH $M = 1, g' = 0{,}6$ and $g = 0, 2$, as a function of a; g is the parameter associated to a non-rigid rotation, $a \to ar^3/(r^3 + g^3)$

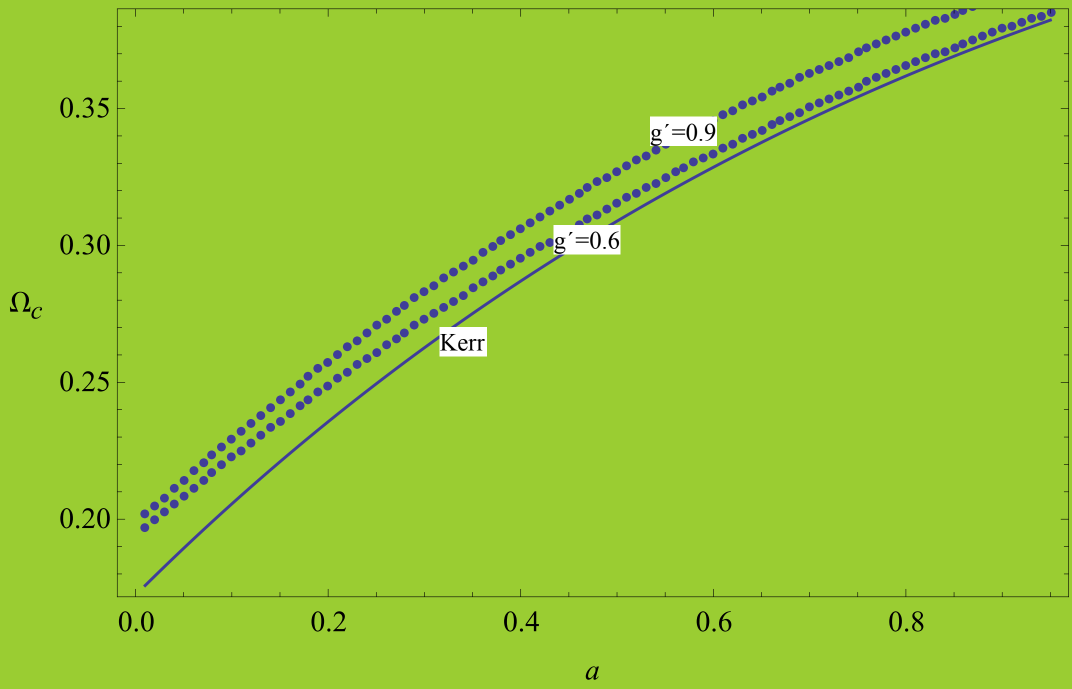
The photosphere of Hayward's BH is smaller than Schwarzschild and Kerr's.

The real part of the QNMs, Hayward BH



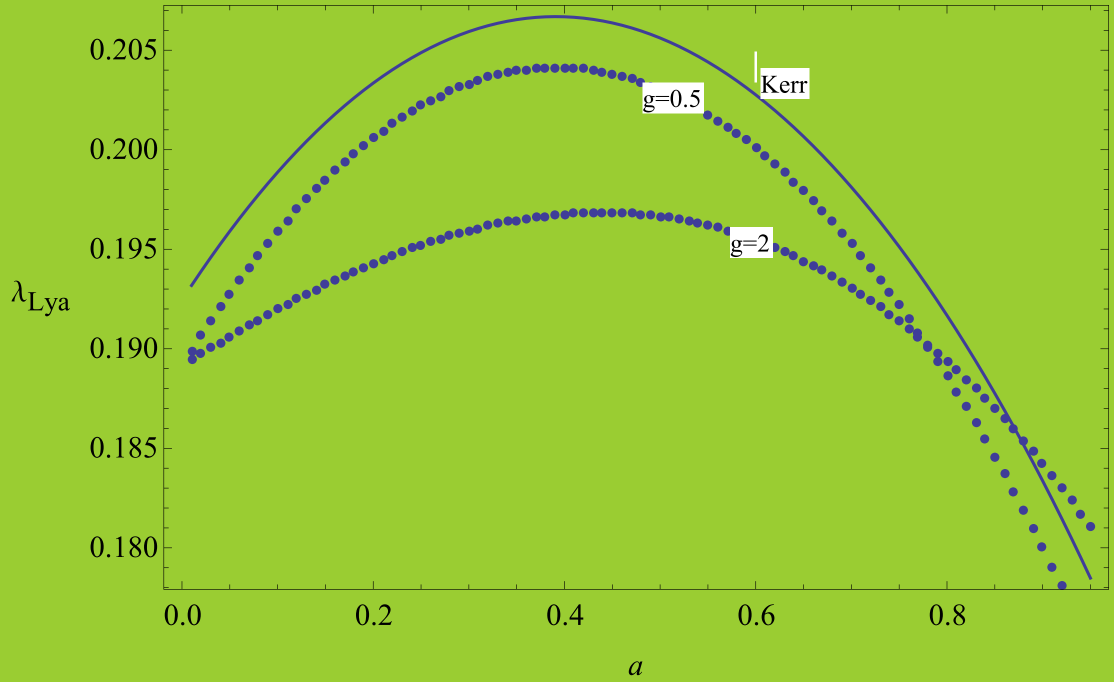
The real frequency of the QNMs of Hayward's BH, as a function of the rotation a. It is compared with Kerr's, and for $a = 0$ the Schwarzschild case is recovered. $M = 1$, $g' = 0{,}6$ and $g = 0; 2$; $a \to ar^3/(r^3 + g^3)$

The real part of the QNMs, Hayward BH



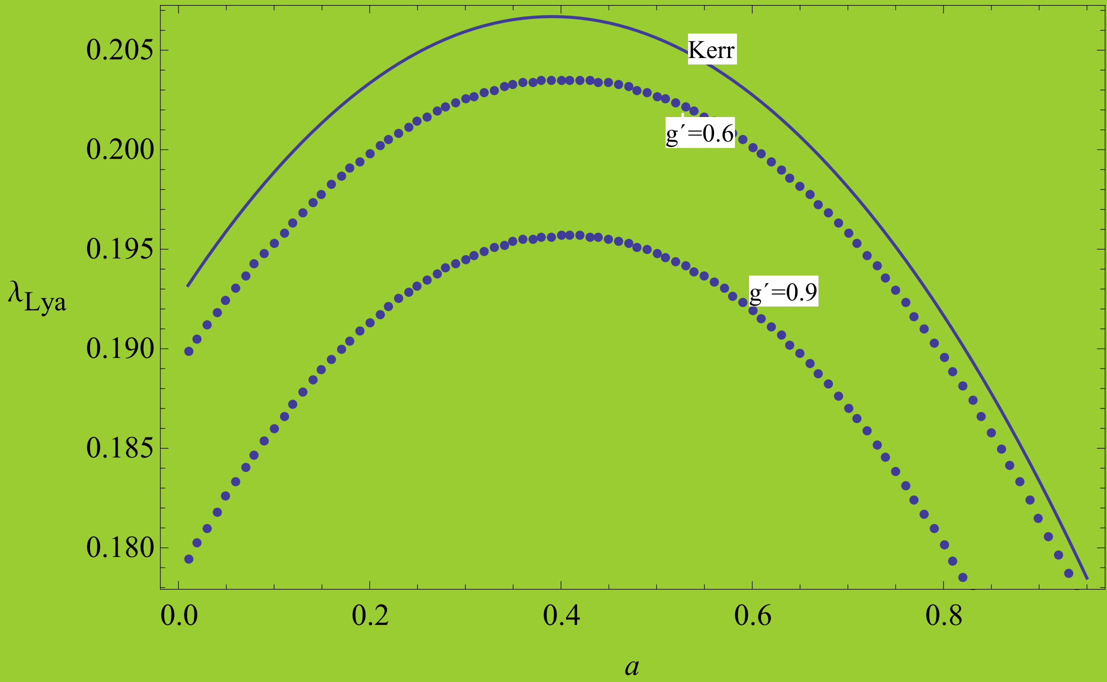
The real frequency of the QNMs of Hayward's BH, as a function of the rotation a. It is compared with Kerr's, for two values of g'; for $a = 0$ the Schwarzschild case is recovered. $M = 1$, $g' = 0{,}6; 0{,}9$ and $g = 1$. $m(r) = Mr^3/(r^3 + g'^3)$

Imaginary part of the QNMs for Hayward Black Hole



Comparison for varying acceleration between the ω_i of the Hayward black hole and Kerr's. In this plot $M = 1$, $g' = 0{,}6$, $g = 0; 2$ and $n = 0$; $a \to ar^3/(r^3 + g^3)$

Imaginary part of the QNMs for Hayward Black Hole



Comparison for varying acceleration between the ω_i of the Hayward black hole and Kerr's. In this plot $M = 1$, $g' = 0{,}6; 0{,}9$, $g = 1$ and $n = 0$; $m(r) = Mr^3/(r^3 + g'^3)$

CONCLUSIONS

- Precaution when dealing with rotating solutions generated via NJ
- WEC is violated in the de Sitter core; including NLED WEC may be fulfilled.
- The (in principle) observable signatures of the BH like shadow and QNM from perturbations would be different if coming from a regular or from a singular rotating body.
- Photosphere would be smaller for a regular BH than Kerr's
- The imaginary part of QNMs of the regular BH is smaller than Kerr's, $\Rightarrow$
- Regular rotating BH with a de Sitter core is more stable than Kerr