Black hole formation in high-energy particle collisions

Hirotaka Yoshino
(University of Alberta)

with V.S. Rychkov

Black hole production at the LHC?

August 25-29, 2008: Short talk @ Bremen
CONTENTS

- Introduction
- High-energy two-particle system
- Finding the apparent horizon
- Numerical results
- Summary and discussion
CONTENTS

- Introduction
- High-energy two-particle system
- Finding the apparent horizon
- Numerical results
- Summary and discussion
TeV gravity scenarios

Arkani-Hamed et al. (1998)
In TeV gravity scenarios, our space is a 3-brane in large extra dimensions and Planck energy could be $O(\text{TeV})$. 

Arkani-Hamed et al. (1998)
In TeV gravity scenarios, our space is a 3-brane in large extra dimensions and Planck energy could be $O(\text{TeV})$.

If this is the case, trans-Planckian collision of particles would occur, and quantum gravity phenomena would be observed at the LHC.
TeV gravity scenarios

In TeV gravity scenarios, our space is a 3-brane in large extra dimensions and Planck energy could be $O(\text{TeV})$.

If this is the case, trans-Planckian collision of particles would occur, and quantum gravity phenomena would be observed at the LHC.

Theoretical studies of trans-Planckian collisions:
TeV gravity scenarios

Arkani-Hamed et al. (1998)

In TeV gravity scenarios, our space is a 3-brane in large extra dimensions and Planck energy could be $O$(TeV).

If this is the case, trans-Planckian collision of particles would occur, and quantum gravity phenomena would be observed at the LHC.

Theoretical studies of trans-Planckian collisions:

- string theory

  e.g., Veneziano, JHEP 0411, 011 (2004)
TeV gravity scenarios

Arkani-Hamed et al. (1998)

In TeV gravity scenarios, our space is a 3-brane in large extra dimensions and Planck energy could be O(TeV).

If this is the case, trans-Planckian collision of particles would occur, and quantum gravity phenomena would be observed at the LHC.

Theoretical studies of trans-Planckian collisions:

- string theory
  - e.g., Veneziano, JHEP 0411, 011 (2004)

- (semi-)classical approximation
  - mini BH production and subsequent decay.
Estimate of production rate
Estimate of production rate

Giddings & Thomas, Dimopoulos & Landsberg (2002)

\[
\sigma_{pp\rightarrow bh}(\tau_m, s) = \sum_{ij} \int_{\tau_m}^{1} d\tau \int_{\tau}^{1} \frac{dx}{x} f_i(x) f_j(\tau/x) \sigma_{ij\rightarrow bh}(\tau s)
\]

\[
\sigma_{ij\rightarrow bh}(\tau s) \simeq \pi [r_h(\sqrt{s})]^2
\]
Estimate of production rate

Giddings & Thomas, Dimopoulos & Landsberg (2002)

\[ \sigma_{pp\rightarrow bh}(\tau_m, s) = \sum_{ij} \int_{\tau_m}^1 d\tau \int_{\tau}^1 \frac{dx}{x} f_i(x)f_j(\tau/x)\sigma_{ij\rightarrow bh}(\tau s) \]

\[ \sigma_{ij\rightarrow bh}(\tau s) \simeq \pi[r_h(\sqrt{\tau s})]^2 \]

\[ \sim 1 \text{ BH} / 1 \text{ s} \]
Estimate of production rate

Giddings & Thomas, Dimopoulos & Landsberg (2002)

\[
\sigma_{pp\rightarrow bh}(\tau_m, s) = \sum_{ij} \int_{\tau_m}^{1} d\tau \int_{\tau}^{1} \frac{dx}{x} f_i(x) f_j(\tau/x) \sigma_{ij\rightarrow bh}(\tau s)
\]

\[
\sigma_{ij\rightarrow bh}(\tau s) \simeq \pi [r_h(\sqrt{\tau s})]^2
\]

\[\sim 1 \text{ BH} / 1 \text{ s}\]

Basic assumption:

\[
b \lesssim r_h(2\mu) \Rightarrow \text{BH production}
\]

Cross section:

\[
\sigma_{B\mu} \sim \pi [r_h(2\mu)]^2
\]
Estimate of production rate

Giddings & Thomas, Dimopoulos & Landsberg (2002)

\[
\sigma_{pp\to bh}(\tau_m, s) = \sum_{ij} \int_{\tau_m}^{1} d\tau \int_{1}^{\tau_m} \frac{dx}{x} \ f_i(x) f_j(\tau/x) \sigma_{ij\to bh}(\tau s)
\]

\[
\sigma_{ij\to bh}(\tau s) \simeq \pi [r_h(\sqrt{\tau s})]^2
\]

\[\sim 1 \text{ BH / 1 s}\]

Basic assumption:

\[
b \lesssim r_h(2\mu) \Rightarrow \text{BH production}
\]

Cross section:

\[
\sigma_{BH} \sim \pi [r_h(2\mu)]^2
\]

Accurate value of \(\sigma_{BH}\) ??

Black hole

\[\mu \]

\[b \]

\[\mu \]
Estimate of production rate

Giddings & Thomas, Dimopoulos & Landsberg (2002)

\[
\sigma_{pp\rightarrow bh}(\tau_m, s) = \sum_{ij} \int_{\tau_m}^{1} d\tau \int_{\tau}^{1} \frac{dx}{x} f_i(x)f_j(\tau/x)\sigma_{ij\rightarrow bh}(\tau s)
\]

\[
\sigma_{ij\rightarrow bh}(\tau s) \simeq \pi [r_h(\sqrt{\tau s})]^2
\]

\[\sim 1 \text{ BH} / 1 \text{ s}\]

Basic assumption:

accurate value of \( \sigma_{\text{BH}} \) ?? \(\Rightarrow\) AH is useful.
Apparent horizon
Apparent horizon

\[ u \quad \text{IV} \quad v \]

\[ \text{II} \quad \text{III} \quad \text{I} \]

\text{shock1} \quad \text{shock2}
AH is a (D-2)-dimensional closed surface whose outgoing null geodesic congruence has zero expansion.
AH is a (D-2)-dimensional closed surface whose outgoing null geodesic congruence has zero expansion.

AH existence is the sufficient condition for the BH formation.
AH is a \((D-2)\)-dimensional closed surface whose outgoing null geodesic congruence has zero expansion.

AH existence is the sufficient condition for the BH formation.
AH is a (D-2)-dimensional closed surface whose outgoing null geodesic congruence has zero expansion.

AH existence is the sufficient condition for the BH formation.

\[ b^{(AH)}_{max} < b^{(BH)}_{max} \]
AH is a (D-2)-dimensional closed surface whose outgoing null geodesic congruence has zero expansion.

AH existence is the sufficient condition for the BH formation.

\[ b_{max}^{(AH)} < b_{max}^{(BH)} \]

\[ \sigma^{(AH)} < \sigma^{(BH)} \]
Definition of the problem
Definition of the problem

We use the four- and higher-dimensional general relativity and study the AH formation in the particle collisions.
Definition of the problem

We use the four- and higher-dimensional general relativity and study the AH formation in the particle collisions.

We adopt the model of a high-energy particle by Aichelburg and Sexl (AS).
Definition of the problem

We use the four- and higher-dimensional general relativity and study the AH formation in the particle collisions.

We adopt the model of a high-energy particle by Aichelburg and Sexl (AS).

We ignore

charge, color charge, spin of incoming particle
the effect of the brane tension
the structure of extra dimensions
Definition of the problem

We use the four- and higher-dimensional general relativity and study the AH formation in the particle collisions.

We adopt the model of a high-energy particle by Aichelburg and Sexl (AS).

We ignore

- charge, color charge, spin of incoming particle
- the effect of the brane tension
- the structure of extra dimensions

We will find the lower bound on $\sigma_{\text{BH}}$. 
Studies on AHs in AS particle collision
Studies on AHs in AS particle collision

Studies on AHs in AS particle collision


4D, headon
Studies on AHs in AS particle collision


4D, headon
Studies on AHs in AS particle collision

  4D, headon
Studies on AHs in AS particle collision

  4D, head-on

  4D, non-head-on (analytic)
Studies on AHs in AS particle collision

  4D, head-on
  4D, non-head-on (analytic)
Studies on AHs in AS particle collision

  4D, headon

  4D, non-head-on (analytic)

  high-D, non-head-on (numerical)
Studies on AHs in AS particle collision

  - 4D, headon

  - 4D, non-head-on (analytic)

  - high-D, non-head-on (numerical)

\[
\frac{b_{\text{max}}/r_h(2\mu)}{\sigma_{AH}/\pi [r_h(2\mu)]^2} \leq 1.88
\]
Studies on AHs in AS particle collision

  - 4D, head-on

  - 4D, non-head-on (analytic)

  - high-D, non-head-on (numerical)

\[ \frac{b_{\text{max}}}{r_h(2\mu)} \]

\[ 1.08 \leq \frac{\sigma_{\text{AH}}}{\pi} [r_h(2\mu)]^2 \leq 1.88 \]
CONTENTS

- Introduction
- High-energy two-particle system
- Finding the apparent horizon
- Numerical results
- Summary and discussion
Aichelburg-Sexl particle
Aichelburg-Sexl particle

Schwarzschild-BH metric in isotropic coordinate

\[
d s^2 = - \left( \frac{1 - M/2 \bar{R}^{D-3}}{1 + M/2 \bar{R}^{D-3}} \right)^2 d \bar{T}^2 + \left( 1 + \frac{M}{2 \bar{R}^{D-3}} \right)^{4/(D-4)} \left( d \bar{Z}^2 + \sum_{i=1}^{D-2} d \bar{X}_i^2 \right)
\]

\[
M = \frac{8\pi G m}{(D - 2) \Omega_{D-2}}.
\]
Aichelburg-Sexl particle

Schwarzschild-BH metric in isotropic coordinate

\[ ds^2 = - \left( \frac{1 - M/2\bar{R}^{D-3}}{1 + M/2\bar{R}^{D-3}} \right)^2 d\bar{T}^2 + \left( 1 + \frac{M}{2\bar{R}^{D-3}} \right)^{4/(D-4)} \left( d\bar{Z}^2 + \sum_{i=1}^{D-2} d\bar{X}_i^2 \right) \]

\[ M = \frac{8\pi Gm}{(D - 2)\Omega_{D-2}}. \]

Lorentz transformation: \[ \bar{T} = \gamma(t - vz), \]
\[ \bar{Z} = \gamma(-vt + z), \]
\[ \bar{X}_i = x_i. \]
Aichelburg-Sexl particle

Schwarzschild-BH metric in isotropic coordinate

\[ ds^2 = - \left( \frac{1 - M/2\bar{R}^{D-3}}{1 + M/2\bar{R}^{D-3}} \right)^2 d\tilde{T}^2 + \left( 1 + \frac{M}{2\bar{R}^{D-3}} \right)^{4/(D-4)} \left( d\tilde{Z}^2 + \sum_{i=1}^{D-2} d\tilde{X}_i^2 \right) \]

\[ M = \frac{8\pi G m}{(D-2)\Omega_{D-2}}. \]

Lorentz transformation:

\[ \gamma \to \infty \]

\[ \tilde{T} = \gamma(t - vz), \]

\[ \tilde{Z} = \gamma(-vt + z), \]

\[ \tilde{X}_i = x_i. \]
Aichelburg-Sexl particle

Schwarzschild-BH metric in isotropic coordinate

\[ ds^2 = -\left(\frac{1 - M/2\bar{R}^{D-3}}{1 + M/2\bar{R}^{D-3}}\right)^2 d\bar{T}^2 + \left(1 + \frac{M}{2\bar{R}^{D-3}}\right)^{4/(D-4)} \left( d\bar{Z}^2 + \sum_{i=1}^{D-2} d\bar{X}_i^2 \right) \]

\[ M = \frac{8\pi Gm}{(D-2)\Omega_{D-2}}. \]

Lorentz transformation:

\[ \gamma \to \infty \]

\[ \bar{T} = \gamma(t - vz), \]

\[ \bar{Z} = \gamma(-vt + z), \]

\[ \bar{X}_i = x_i. \]

\[ \mu = m\gamma \]
Aichelburg-Sexl particle

Schwarzschild-BH metric in isotropic coordinate

\[ ds^2 = -\left(\frac{1 - M/2\bar{R}^{D-3}}{1 + M/2\bar{R}^{D-3}}\right)^2 d\bar{T}^2 + \left(1 + \frac{M}{2\bar{R}^{D-3}}\right)^{4/(D-4)} \left(d\bar{Z}^2 + \sum_{i=1}^{D-2} d\bar{X}_i^2\right) \]

\[ M = \frac{8\pi Gm}{(D-2)\Omega_{D-2}}. \]

Lorentz transformation:

\[ \gamma \rightarrow \infty \]

\[ \bar{T} = \gamma(t - vz), \]

\[ \bar{Z} = \gamma(-vt + z), \]

\[ \bar{X}_i = x_i. \]

\[ \mu = m\gamma \]

\[ ds^2 = -d\bar{u}d\bar{v} + d\bar{r}^2 + \bar{r}^2 d\Omega_{D-3}^2 + \Phi(\bar{r})\delta(\bar{u})d\bar{u}^2, \]

\[ \Phi(\bar{r}) = \begin{cases} 2\log\bar{r} & (D = 4), \\ 2/(D-4)\bar{r}^{D-4} & (D \geq 5). \end{cases} \]
Aichelburg-Sexl particle

Schwarzschild-BH metric in isotropic coordinate

\[ ds^2 = - \left( \frac{1 - M/2 \bar{R}^{D-3}}{1 + M/2 \bar{R}^{D-3}} \right)^2 d\bar{T}^2 + \left( 1 + \frac{M}{2 \bar{R}^{D-3}} \right)^{4/(D-4)} \left( d\bar{Z}^2 + \sum_{i=1}^{D-2} d\bar{X}_i^2 \right) \]

\[ M = \frac{8\pi Gm}{(D-2)\Omega_{D-2}}. \]

Lorentz transformation:
\[ \bar{T} = \gamma(t - vz), \]
\[ \bar{Z} = \gamma(-vt + z), \]
\[ \bar{X}_i = x_i. \]

\[ ds^2 = -d\bar{u}d\bar{v} + d\bar{r}^2 + \bar{r}^2 d\bar{\Omega}_{D-3}^2 + \Phi(\bar{r})\delta(\bar{u})d\bar{u}^2, \]

\[ \Phi(\bar{r}) = \begin{cases} 
2 \log \bar{r} & (D = 4), \\
2/(D - 4)\bar{r}^{D-4} & (D \geq 5). 
\end{cases} \]

hereafter \[ r_0 = \left( \frac{8\pi G_D \mu}{\Omega_{D-3}} \right)^{1/(D-3)} \] is the unit of length.
Aichelburg-Sexl particle

Schwarzschild-BH metric in isotropic coordinate

\[
 ds^2 = -\left(\frac{1 - M/2\bar{R}^{D-3}}{1 + M/2\bar{R}^{D-3}}\right)^2 d\bar{T}^2 + \left(1 + \frac{M}{2\bar{R}^{D-3}}\right)^{4/(D-4)} \left(d\bar{Z}^2 + \sum_{i=1}^{D-2} d\bar{X}_i^2\right)
\]

\[
 M = \frac{8\pi Gm}{(D-2)\Omega_{D-2}}.
\]

Lorentz transformation:

\[
 \bar{T} = \gamma(t - vz),
\]

\[
 \bar{Z} = \gamma(-vt + z),
\]

\[
 \bar{X}_i = x_i.
\]

\[
 ds^2 = -d\bar{u}d\bar{v} + d\bar{r}^2 + \bar{r}^2 d\bar{\Omega}_{D-3}^2 + \Phi(\bar{r})\delta(\bar{u})d\bar{u}^2,
\]

\[
 \Phi(\bar{r}) = \begin{cases} 
 2\log\bar{r} & (D = 4), \\
 2/(D - 4)\bar{r}^{D-4} & (D \geq 5).
\end{cases}
\]

hereafter \( r_0 = \left(\frac{8\pi G_D\mu}{\Omega_{D-3}}\right)^{1/(D-3)} \) is the unit of length.
Aichelburg-Sexl particle

Flat coordinates

\[ ds^2 = -d\bar{u}d\bar{v} + d\bar{r}^2 + \bar{r}^2d\Omega_{D-3}^2 + \Phi(\bar{r})\delta(\bar{u})d\bar{u}^2, \]

\[ \Phi(\bar{r}) = \begin{cases} 
-2 \ln \bar{r} & (D = 4) \\
\frac{2}{(D - 4)\bar{r}^{D-4}} & (D \geq 5) 
\end{cases} \]

length unit: \( r_0 = \left( \frac{8\pi G_D\mu}{\Omega_{D-3}} \right)^{1/(D-3)} \)
Aichelburg-Sexl particle

Flat coordinates

$$ds^2 = -d\tilde{u}d\tilde{v} + d\tilde{r}^2 + \tilde{r}^2 d\Omega_{D-3}^2 + \Phi(\tilde{r})\delta(\tilde{u})d\tilde{u}^2,$$

$$\Phi(\tilde{r}) = \begin{cases} -2 \ln \tilde{r} & (D = 4) \\ \frac{2}{(D - 4)\tilde{r}^{D-4}} & (D \geq 5) \end{cases}$$

Null geodesic coordinates

$$\tilde{u} = u$$

$$\tilde{v} = \begin{cases} v - 2 \log r\theta(u) + u\theta(u)/r^2 & (D = 4), \\ v + 2\theta(u)/(D - 4)r^{D-4} + u\theta(u)/r^{2D-6} & (D \geq 5), \end{cases}$$

$$\tilde{r} = r \left(1 - \frac{u}{r^{D-2}\theta(u)}\right),$$

length unit: $$r_0 = \left(\frac{8\pi G_D \mu}{\Omega_{D-3}}\right)^{1/(D-3)}$$
Aichelburg-Sexl particle

Flat coordinates

\[ ds^2 = -d\bar{u}d\bar{v} + d\bar{r}^2 + \bar{r}^2d\bar{\Omega}_{D-3}^2 + \Phi(\bar{r})\delta(\bar{u})d\bar{u}^2, \]

\[ \Phi(\bar{r}) = \begin{cases} 
-2 \ln \bar{r} & (D = 4) \\
\frac{2}{(D-4)\bar{r}^{D-4}} & (D \geq 5) 
\end{cases} \]

Null geodesic coordinates

\[ \bar{u} = u \]

\[ \bar{v} = \begin{cases} 
v - 2 \log r\theta(u) + u\theta(u)/r^2 & (D = 4), \\
v + 2\theta(u)/(D-4)r^{D-4} + u\theta(u)/r^{2D-6} & (D \geq 5), \end{cases} \]

\[ \bar{r} = r \left(1 - \frac{u}{r^{D-2}\theta(u)}\right), \]

\[ ds^2 = -dudv + \left[1 + (D-3)\frac{u\theta(u)}{r^{D-2}}\right]^2 dr^2 + r^2 \left[1 - \frac{u\theta(u)}{r^{D-2}}\right]^2 d\Omega_{D-3}^2 \]
Aichelburg-Sexl particle

Flat coordinates

\[
\begin{align*}
\text{length unit: } r_0 &= \left( \frac{8\pi G_D \mu}{\Omega_{D-3}} \right)^{1/(D-3)} \\
\begin{align*}
ds^2 &= -d\tilde{u}d\tilde{v} + d\tilde{r}^2 + \tilde{r}^2 d\tilde{\Omega}_{D-3}^2 + \Phi(\tilde{r})\delta(\tilde{u})d\tilde{u}^2, \\
\Phi(\tilde{r}) &= \begin{cases} 
-2 \ln \tilde{r} & (D = 4) \\
\frac{2}{(D - 4)\tilde{r}^{D-4}} & (D \geq 5)
\end{cases}
\end{align*}
\end{align*}
\]

Null geodesic coordinates

\[
\begin{align*}
\tilde{u} &= u \\
\tilde{v} &= \begin{cases} 
v - 2 \log r\theta(u) + u\theta(u)/r^2 & (D = 4), \\
v + 2\theta(u)/(D - 4)r^{D-4} + u\theta(u)/r^{2D-6} & (D \geq 5)
\end{cases} \\
\tilde{r} &= r \left(1 - \frac{u}{r^{D-2}\theta(u)}\right), \\
\begin{align*}
ds^2 &= -dudv + \left[1 + (D - 3)\frac{u\theta(u)}{r^{D-2}}\right]^2 dr^2 + r^2 \left[1 - \frac{u\theta(u)}{r^{D-2}}\right]^2 d\Omega_{D-3}^2
\end{align*}
\]

- \(v, r, \phi_i = \text{const.}\) is a null geodesic
- \(\tilde{u}\) is an affine parameter
### Aichelburg-Sexl particle

**Flat coordinates**

\[
 ds^2 = -d\bar{u}d\bar{v} + d\bar{r}^2 + \bar{r}^2d\bar{\Omega}_{D-3}^2 + \Phi(\bar{r})\delta(\bar{u})d\bar{u}^2,
\]

\[
 \Phi(\bar{r}) = \begin{cases} 
  -2 \ln \bar{r} & (D = 4) \\
  \frac{2}{(D - 4)\bar{r}^{D-4}} & (D \geq 5)
 \end{cases}
\]

**Null geodesic coordinates**

\[
 \bar{u} = u \\
 \bar{v} = \begin{cases} 
 v - 2 \log r\theta(u) + u\theta(u)/r^2 & (D = 4), \\
 v + 2\theta(u)/(D - 4)r^{D-4} + u\theta(u)/r^{2D-6} & (D \geq 5),
 \end{cases}
\]

\[
 \bar{r} = r \left(1 - \frac{u}{r^{D-2}\theta(u)}\right),
\]

\[
 ds^2 = -dudv + \left[1 + (D - 3)\frac{u\theta(u)}{r^{D-2}}\right]^2dr^2 + r^2\left[1 - \frac{u\theta(u)}{r^{D-2}}\right]^2d\Omega_{D-3}^2
\]

- $v, r, \phi_i = \text{const.}$ is a null geodesic
- $u$ is an affine parameter

**Length unit:**

\[
 r_0 = \left(\frac{8\pi G_D\mu}{\Omega_{D-3}}\right)^{1/(D-3)}
\]
**Aichelburg-Sexl particle**

**Flat coordinates**

\[
\begin{align*}
    ds^2 &= -d\bar{u}d\bar{v} + d\bar{r}^2 + \bar{r}^2 d\bar{\Omega}_{D-3}^2 + \Phi(\bar{r})\delta(\bar{u})d\bar{u}^2, \\
    \Phi(\bar{r}) &= \begin{cases} 
        -2 \ln \bar{r} & (D = 4) \\
        \frac{2}{(D - 4)\bar{r}^{D-4}} & (D \geq 5) 
    \end{cases}
\end{align*}
\]

**Null geodesic coordinates**

\[
\begin{align*}
    \bar{u} &= u \\
    \bar{v} &= \begin{cases} 
        v - 2 \log r\theta(u) + u\theta(u)/r^2 & (D = 4), \\
        v + 2\theta(u)/(D - 4)r^{D-4} + u\theta(u)/r^{2D-6} & (D \geq 5), 
    \end{cases} \\
    \bar{r} &= r \left(1 - \frac{u}{r^{D-2}\theta(u)}\right),
\end{align*}
\]

\[
\begin{align*}
    ds^2 &= -dudv + \left[1 + (D - 3)\frac{u\theta(u)}{r^{D-2}}\right]^2 dr^2 + r^2 \left[1 - \frac{u\theta(u)}{r^{D-2}}\right]^2 d\Omega_{D-3}^2.
\end{align*}
\]

- \(v, r, \phi_i = \text{const.}\) is a null geodesic
- \(u\) is an affine parameter

**length unit:** \(r_0 = \left(\frac{8\pi G_D\mu}{\Omega_{D-3}}\right)^{1/(D-3)}\)
Aichelburg-Sexl particle

Flat coordinates

\[ ds^2 = -d\bar{u}d\bar{v} + d\bar{r}^2 + \bar{r}^2d\bar{\Omega}_D^2 + \Phi(\bar{r})\delta(\bar{u})d\bar{u}^2, \]

\[ \Phi(\bar{r}) = \begin{cases} 
-2 \ln \bar{r} & (D = 4) \\
\frac{2}{(D - 4)\bar{r}^{D-4}} & (D \geq 5) 
\end{cases} \]

Null geodesic coordinates

\[ \bar{u} = u \]

\[ \bar{v} = \begin{cases} 
v - 2\log r\theta(u) + u\theta(u)/r^2 & (D = 4), \\
v + 2\theta(u)/(D - 4)r^{D-4} + u\theta(u)/r^{2D-6} & (D \geq 5), 
\end{cases} \]

\[ \bar{r} = r \left( 1 - \frac{u}{r^{D-2}}\theta(u) \right), \]

\[ ds^2 = -dudv + \left[ 1 + (D - 3)\frac{u\theta(u)}{r^{D-2}} \right]^2dr^2 + r^2\left[ 1 - \frac{u\theta(u)}{r^{D-2}} \right]^2d\Omega_D^2 \]

\[ v, r, \phi_i = \text{const. is a null geodesic} \]

\[ u \text{ is an affine parameter} \]

length unit: \[ r_0 = \left( \frac{8\pi G_D\mu}{\Omega_{D-3}} \right)^{1/(D-3)} \]
Aichelburg-Sexl particle

Flat coordinates

\[
\begin{align*}
ds^2 &= -d\tilde{u}d\tilde{v} + d\tilde{r}^2 + \tilde{r}^2 d\Omega^2_{D-3} + \Phi(\tilde{r})\delta(\tilde{u})d\tilde{u}^2, \\
\Phi(\tilde{r}) &= \begin{cases} 
-2 \ln \tilde{r} & (D = 4) \\
\frac{2}{(D - 4)\tilde{r}^{D-4}} & (D \geq 5)
\end{cases}
\end{align*}
\]

Null geodesic coordinates

\[
\begin{align*}
\tilde{u} &= u \\
\tilde{v} &= \begin{cases} 
v - 2 \log r\theta(u) + u\theta(u)/r^2 & (D = 4), \\
v + 2\theta(u)/(D - 4)r^{D-4} + u\theta(u)/r^{2D-6} & (D \geq 5),
\end{cases} \\
\tilde{r} &= r \left(1 - \frac{u}{r^{D-2}\theta(u)}\right), \\
ds^2 &= -dudv + \left[1 + (D - 3)\frac{u\theta(u)}{r^{D-2}}\right]^2 dr^2 + r^2 \left[1 - \frac{u\theta(u)}{r^{D-2}}\right]^2 d\Omega^2_{D-3}
\end{align*}
\]

coordinate singularity
\[u = r^{D-2}\]

\[v, r, \phi_i = \text{const.}\] is a null geodesic
\[u\] is an affine parameter

length unit: 
\[r_0 = \left(\frac{8\pi G_D \mu}{\Omega_D^{-3}}\right)^{1/(D-3)}\]
Aichelburg-Sexl particle

**Flat coordinates**

\[
 ds^2 = -d\bar{u}d\bar{v} + d\bar{r}^2 + \bar{r}^2 d\bar{\Omega}_{D-3}^2 + \Phi(\bar{r})\delta(\bar{u})d\bar{u}^2, 
\]

\[
 \Phi(\bar{r}) = \begin{cases} 
 -2 \ln \bar{r} & (D = 4) \\
 \frac{2}{(D - 4)\bar{r}^{D-4}} & (D \geq 5) 
\end{cases}
\]

**Null geodesic coordinates**

\[
 \bar{u} = u
\]

\[
 \bar{v} = \begin{cases} 
 v - 2 \log r\theta(u) + u\theta(u)/r^2 & (D = 4), \\
 v + 2\theta(u)/(D - 4)r^{D-4} + u\theta(u)/r^{2D-6} & (D \geq 5), 
\end{cases}
\]

\[
 \bar{r} = r \left(1 - \frac{u}{r^{D-2}}\theta(u)\right),
\]

**Coordinate singularity**

\[
 u = r^{D-2}
\]

- \( v, r, \phi_i = \text{const.} \) is a null geodesic
- \( u \) is an affine parameter

**length unit:** \( r_0 = \left(\frac{8\pi G_D \mu}{\Omega_{D-3}}\right)^{1/(D-3)} \)
Aichelburg-Sexl particle

Flat coordinates

\[ ds^2 = -d\bar{u}d\bar{v} + d\bar{r}^2 + \bar{r}^2 d\bar{\Omega}_{D-3}^2 + \Phi(\bar{r})\delta(\bar{u})d\bar{u}^2, \]

\[ \Phi(\bar{r}) = \begin{cases} 
-2 \ln \bar{r} & (D = 4) \\
\frac{2}{(D - 4)\bar{r}^{D-4}} & (D \geq 5)
\end{cases} \]

Null geodesic coordinates

\[ \bar{u} = u \]

\[ \bar{v} = \begin{cases} 
v - 2 \log r\theta(u) + u\theta(u)/r^2 & (D = 4), \\
v + 2\theta(u)/(D - 4)r^{D-4} + u\theta(u)/r^{2D-6} & (D \geq 5),
\end{cases} \]

\[ \bar{r} = r \left(1 - \frac{u}{r^{D-2}}\theta(u)\right), \]

\[ ds^2 = -du dv + \left[1 + (D - 3)\frac{u\theta(u)}{r^{D-2}}\right]^2 dr^2 + r^2 \left[1 - \frac{u\theta(u)}{r^{D-2}}\right]^2 d\Omega_{D-3}^2 \]

- \( v, r, \phi_i = \text{const.} \) is a null geodesic
- \( u \) is an affine parameter

length unit: \( r_0 = \left(\frac{8\pi G_D \mu}{\Omega_{D-3}}\right)^{1/(D-3)} \)
Aichelburg-Sexl particle

**Flat coordinates**

\[
\begin{align*}
 ds^2 &= -d\bar{u}d\bar{v} + d\bar{r}^2 + \bar{r}^2d\bar{\Omega}_{D-3}^2 + \Phi(\bar{r})\delta(\bar{u})d\bar{u}^2, \\
 \Phi(\bar{r}) &= \begin{cases} 
 -2 \ln \bar{r} & (D = 4) \\
 \frac{2}{(D - 4)\bar{r}^{D-4}} & (D \geq 5)
\end{cases}
\end{align*}
\]

**Null geodesic coordinates**

\[
\begin{align*}
 \bar{u} &= u \\
 \bar{v} &= \begin{cases} 
 v - 2 \log r \theta(u) + u \theta(u)/r^2 & (D = 4), \\
 v + 2 \theta(u)/(D - 4)r^{D-4} + u \theta(u)/r^{2D-6} & (D \geq 5),
\end{cases} \\
 \bar{r} &= r \left(1 - \frac{u}{r^{D-2}}\theta(u)\right)
\end{align*}
\]

\[
\begin{align*}
 ds^2 &= -dudv + \left[1 + (D - 3)\frac{u\theta(u)}{r^{D-2}}\right]^2 dr^2 + r^2 \left[1 - \frac{u\theta(u)}{r^{D-2}}\right]^2 d\Omega_{D-3}^2
\end{align*}
\]

- \(v, r, \phi_i = \text{const.}\) is a null geodesic
- \(u\) is an affine parameter

**length unit:** \(r_0 = \left(\frac{8\pi G_D \mu}{\Omega_{D-3}}\right)^{1/(D-3)}\)
Aichelburg-Sexl particle

Flat coordinates

\[
 ds^2 = -d\tilde{u}d\tilde{v} + d\tilde{r}^2 + \tilde{r}^2 d\tilde{\Omega}_{D-3}^2 + \Phi(\tilde{r})\delta(\tilde{u})d\tilde{u}^2,
\]

\[
 \Phi(\tilde{r}) = \begin{cases}
 -2 \ln \tilde{r} & (D = 4) \\
 \frac{2}{(D - 4)\tilde{r}^{D-4}} & (D \geq 5)
\end{cases}
\]

Null geodesic coordinates

\[
 \tilde{u} = u
\]

\[
 \tilde{v} = \begin{cases}
 v - 2 \log r \theta(u) + u \theta(u)/r^2 & (D = 4), \\
 v + 2 \theta(u)/(D - 4)r^{D-4} + u \theta(u)/r^{2D-6} & (D \geq 5),
\end{cases}
\]

\[
 \tilde{r} = r \left(1 - \frac{u}{r^{D-2}} \theta(u)\right),
\]

\[
 ds^2 = -dudv + \left[1 + (D - 3)\frac{u\theta(u)}{r^{D-2}}\right]^2 dr^2 + r^2 \left[1 - \frac{u\theta(u)}{r^{D-2}}\right]^2 d\Omega_{D-3}^2
\]

\[
 v, r, \phi_i = \text{const. is a null geodesic}
\]

\[
 u \text{ is an affine parameter}
\]

length unit: \( r_0 = \left(\frac{8\pi G_D \mu}{\Omega_{D-3}}\right)^{1/(D-3)} \)
Aichelburg-Sexl particle

Flat coordinates

\[
\begin{align*}
    ds^2 &= -d\bar{u}d\bar{v} + d\bar{r}^2 + \bar{r}^2 d\bar{\Omega}^2_{D-3} + \Phi(\bar{r}) \delta(\bar{u}) d\bar{u}^2, \\
    \Phi(\bar{r}) &= \begin{cases} 
        -2 \ln \bar{r} & (D = 4) \\
        \frac{2}{(D - 4)\bar{r}^{D-4}} & (D \geq 5)
    \end{cases}
\end{align*}
\]

Null geodesic coordinates

\[
\begin{align*}
    \bar{u} &= u \\
    \bar{v} &= \begin{cases} 
        v - 2 \log r\theta(u) + u\theta(u)/r^2 & (D = 4), \\
        v + 2\theta(u)/(D - 4)r^{D-4} + u\theta(u)/r^{2D-6} & (D \geq 5),
    \end{cases} \\
    \bar{r} &= r \left( 1 - \frac{u}{r^{D-2}} \theta(u) \right),
\end{align*}
\]

\[
\begin{align*}
    ds^2 &= -dudv + \left[ 1 + (D - 3)\frac{u\theta(u)}{r^{D-2}} \right]^2 dr^2 + r^2 \left[ 1 - \frac{u\theta(u)}{r^{D-2}} \right]^2 d\Omega^2_{D-3}
\end{align*}
\]

- \(v, r, \phi_i = \text{const.}\) is a null geodesic
- \(u\) is an affine parameter

length unit: \(r_0 = \left( \frac{8\pi G_D \mu}{\Omega_{D-3}} \right)^{1/(D-3)}\)
Setup of two particle system

shock 1

shock 2
Setup of two particle system

shock 1

shock 2
Setup of two particle system

collision!

shock 1

shock 2
Setup of two particle system

$b$ : Impact parameter

collision!
Setup of two particle system

\( b \) : Impact parameter

collision!
Setup of two particle system

\[ b \]: Impact parameter

\[ x, y_i \]

collision!
Setup of two particle system
Setup of two particle system
Setup of two particle system
Setup of two particle system
Setup of two particle system
Setup of two particle system
Setup of two particle system

\( b \) : Impact parameter
Setup of two particle system

$b$: Impact parameter
Setup of two particle system

\[ b \quad : \text{Impact parameter} \]
Setup of two particle system

\( b \) : Impact parameter
Setup of two particle system

\( b \): Impact parameter
Setup of two particle system

\[ b \] : Impact parameter
Setup of two particle system

\[ b \quad : \text{Impact parameter} \]
Setup of two particle system

\[ b \]: Impact parameter
Setup of two particle system

\( b \): Impact parameter
CONTENTS

- Introduction
- High-energy two-particle system
- Finding the apparent horizon
- Numerical results
- Summary and discussion
AH:
AH:
Inner boundary:

AH:
Inner boundary:

AH:
Outer boundary: $r = r_{max}$

Inner boundary:

AH:
Outer boundary: $r = r_{max}$

Inner boundary:

AH:
- Expansion is zero.
Outer boundary: \( r = r_{max} \)

Continuity of the surface;

Inner boundary:

Continuity of the surface;

AH:

Expansion is zero.
Outer boundary: \( r = r_{\text{max}} \)

- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:
- Continuity of the surface;
- Continuity of the null tangent vector.

AH:
- Expansion is zero.
Outer boundary: \( r = r_{\text{max}} \)

- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:

- Continuity of the surface;
- Continuity of the null tangent vector.

AH:

- Expansion is zero.
Outer boundary: \( r = r_{max} \)

- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:

- Continuity of the surface;
- Continuity of the null tangent vector.

AH:

- Expansion is zero.
Outer boundary: \( r = r_{max} \)

- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:

- Continuity of the surface;
- Continuity of the null tangent vector.

AH:

- Expansion is zero.
Outer boundary: \( r = r_{max} \)

- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:

- Continuity of the surface;
- Continuity of the null tangent vector.

AH:

- Expansion is zero.
Outer boundary: \( r = r_{max} \)
- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:
- Continuity of the surface;
- Continuity of the null tangent vector.

AH:
- Expansion is zero.
Outer boundary: \( r = r_{max} \)

- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:

- Continuity of the surface;
- Continuity of the null tangent vector.

AH:

- Expansion is zero.
Outer boundary: \( r = r_{\text{max}} \)

- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:

- Continuity of the surface;
- Continuity of the null tangent vector.

AH:

- Expansion is zero.
Outer boundary: \( r = r_{\text{max}} \)

- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:

- Continuity of the surface;
- Continuity of the null tangent vector.

AH:

- Expansion is zero.
Outer boundary:  \( r = r_{max} \)

- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:

- Continuity of the surface;
- Continuity of the null tangent vector.

AH:

- Expansion is zero.
Outer boundary:  \( r = r_{max} \)
- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:
- Continuity of the surface;
- Continuity of the null tangent vector.

AH:
- Expansion is zero.
Outer boundary:  \( r = r_{max} \)

- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:
- Continuity of the surface;
- Continuity of the null tangent vector.

AH:
- Expansion is zero.
Outer boundary: \( r = r_{\text{max}} \)
- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:
- Continuity of the surface;
- Continuity of the null tangent vector.

AH:
- Expansion is zero.
Outer boundary: \( r = r_{\text{max}} \)

- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:
- Continuity of the surface;
- Continuity of the null tangent vector.

AH:
- Expansion is zero.
Outer boundary: \( r = r_{\text{max}} \)
- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:
- Continuity of the surface;
- Continuity of the null tangent vector.

AH:
- Expansion is zero.
Outer boundary: \( r = r_{max} \)

- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:
- Continuity of the surface;
- Continuity of the null tangent vector.

\[ u = h(r, \phi) \]

**AH:**
- Expansion is zero.
Outer boundary: \( r = r_{max} \)

- Continuity of the surface;
- Continuity of the null tangent vector.

Inner boundary:
- Continuity of the surface;
- Continuity of the null tangent vector.

AH:
\[
(\rho^{D-2} - h)^2 \left\{ h_{rr} + (D - 3) \frac{h_r}{r} \left[ 1 + \frac{(D - 2)h - (3/2)rh_r}{r^{D-2} + (D - 3)h} + \frac{(D - 2)h - (1/2)rh_r}{r^{D-2} - h} \right] \right\} \\
+ r^{-2} (\rho^{D-2} + (D - 3)h)^2 \left\{ h_{\phi\phi} + (D - 4) \cot \phi h_{\phi} + \frac{h_{\phi\phi}^2}{2} \left[ \frac{(D - 3)}{r^{D-2} + (D - 3)h} - \frac{(D - 7)}{r^{D-2} - h} \right] \right\} = 0.
\]

\( u = h(r, \phi) \)
Outer boundary: \( r = r_{max} \)

\( h = r^D_{max} \)

\( h,r = (D - 2)r^D_{max} \left( 1 + \frac{1 - B^2}{1 + B^2 + 2B \cos \phi} \right) \),

Inner boundary:

\( h(r, \phi) = 0 \)

\( (h^2_{,r} + r^{-2}h^2_{,\phi}) \left|_x (h^2_{,r} + r^{-2}h^2_{,\phi}) \right|_x^* = 16, \)

AH:

\( (r^{D-2} - h)^2 \left\{ h_{,rr} + (D - 3) \frac{h_{,r}}{r} \left[ 1 + \frac{(D - 2)h - (3/2)rh_{,r}}{r^{D-2} + (D - 3)h} + \frac{(D - 2)h - (1/2)rh_{,r}}{r^{D-2} - h} \right] \right\} 

+ r^{-2} (r^{D-2} + (D - 3)h)^2 \left\{ h_{,\phi\phi} + (D - 4) \cot \phi h_{,\phi} + \frac{h^2_{,\phi}}{2} \left[ \frac{(D - 3)}{r^{D-2} + (D - 3)h} - \frac{(D - 7)}{r^{D-2} - h} \right] \right\} = 0. \)
CONTENTS

- Introduction
- High-energy two-particle system
- Finding the apparent horizon
- **Numerical results**
- Summary and discussion
Results of D=5
Results of $D=5$
Results of $D=5$
Results of $D=5$
Results of $D=5$

$b=0.4$

$b=0.9$

$b=1.1$

$b=1.145$

$D=5$

$D=5$

$D=5$

$D=5$
<table>
<thead>
<tr>
<th>$D$</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b^{(\text{new})}_{\text{max}} / r_0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Main result

<table>
<thead>
<tr>
<th>$D$</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{\text{max}}^{(\text{new})}/r_0$</td>
<td>0.843</td>
<td>1.145</td>
<td>1.33</td>
<td>1.44</td>
<td>1.51</td>
<td>1.57</td>
<td>1.61</td>
<td>1.65</td>
</tr>
</tbody>
</table>
### Main result

<table>
<thead>
<tr>
<th>$D$</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{\text{max}}^{(\text{new})}/r_0$</td>
<td>0.843</td>
<td>1.145</td>
<td>1.33</td>
<td>1.44</td>
<td>1.51</td>
<td>1.57</td>
<td>1.61</td>
<td>1.65</td>
</tr>
</tbody>
</table>

**Diagram:**

- **$b_{\text{max}}/r_h(2\mu)$**
  - YR (Red circles)
  - YN (Blue crosses)

---

- **$D$ values:** 4, 5, 6, 7, 8, 9, 10, 11
- **$b_{\text{max}}^{(\text{new})}/r_0$ values:** 0.843, 1.145, 1.33, 1.44, 1.51, 1.57, 1.61, 1.65
## Main result

<table>
<thead>
<tr>
<th>$D$</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{max}^{(new)}/r_0$</td>
<td>0.843</td>
<td>1.145</td>
<td>1.33</td>
<td>1.44</td>
<td>1.51</td>
<td>1.57</td>
<td>1.61</td>
<td>1.65</td>
</tr>
<tr>
<td>$\sigma_{AH}/\pi \left[ r_h(2\mu) \right]^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Graph of $b_{max}/r_h(2\mu)$ vs $D$]

- **YR**
- **YN**
## Main result

<table>
<thead>
<tr>
<th>$D$</th>
<th>$4$</th>
<th>$5$</th>
<th>$6$</th>
<th>$7$</th>
<th>$8$</th>
<th>$9$</th>
<th>$10$</th>
<th>$11$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{max}^{(\text{new})}/r_0$</td>
<td>$0.843$</td>
<td>$1.145$</td>
<td>$1.33$</td>
<td>$1.44$</td>
<td>$1.51$</td>
<td>$1.57$</td>
<td>$1.61$</td>
<td>$1.65$</td>
</tr>
<tr>
<td>$\sigma_{AH}/\pi [r_h(2\mu)]^2$</td>
<td>$0.76$</td>
<td>$1.54$</td>
<td>$2.15$</td>
<td>$2.52$</td>
<td>$2.77$</td>
<td>$2.95$</td>
<td>$3.09$</td>
<td>$3.20$</td>
</tr>
</tbody>
</table>

- $b_{max}/r_h(2\mu)$
  - $D$
  - $4$  $5$  $6$  $7$  $8$  $9$  $10$  $11$
  - $b_{max}/r_h(2\mu)$
  - $0.25$  $0.5$  $0.75$  $1$  $1.25$  $1.5$  $1.75$  $\sigma_{AH}/\pi [r_h(2\mu)]^2$
  - $0.76$  $1.54$  $2.15$  $2.52$  $2.77$  $2.95$  $3.09$  $3.20$

- YR
- YN
Main result

<table>
<thead>
<tr>
<th>$D$</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{\text{max}}^{(\text{new})}/r_0$</td>
<td>0.843</td>
<td>1.145</td>
<td>1.33</td>
<td>1.44</td>
<td>1.51</td>
<td>1.57</td>
<td>1.61</td>
<td>1.65</td>
</tr>
<tr>
<td>$\sigma_{\text{AH}}/\pi \left[r_h(2\mu)\right]^2$</td>
<td>0.76</td>
<td>1.54</td>
<td>2.15</td>
<td>2.52</td>
<td>2.77</td>
<td>2.95</td>
<td>3.09</td>
<td>3.20</td>
</tr>
</tbody>
</table>

$\sigma_{\text{AH}}/\pi \left[r_h(2\mu)\right]^2 < 3.2$
### Main result

<table>
<thead>
<tr>
<th>$D$</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{max}^{(new)}/r_0$</td>
<td></td>
<td>0.843</td>
<td>1.145</td>
<td>1.33</td>
<td>1.44</td>
<td>1.51</td>
<td>1.57</td>
<td>1.61</td>
</tr>
<tr>
<td>$\sigma_{AH}/\pi [r_h(2\mu)]^2$</td>
<td></td>
<td>0.76</td>
<td>1.54</td>
<td>2.15</td>
<td>2.52</td>
<td>2.77</td>
<td>2.95</td>
<td>3.09</td>
</tr>
</tbody>
</table>

### Graph

- $b_{max}/r_h(2\mu)$

- 1.75
- 1.5
- 1.25
- 1
- 0.75
- 0.5
- 0.25

- YR
- YN

**BH production rate is fairly larger than 1BH/1s.**

(If the energy loss is small).
Final state restriction
Final state restriction

$A_{AH}$
Final state restriction

$A_{AH} < A_{EH}$
Final state restriction

\[ A_{AH} \ < \ A_{EH} \ < \ A_{BH} \]
Final state restriction

\[ A_{AH} \quad < \quad A_{EH} \quad < \quad A_{BH} \]

\[ M_{AH} \quad := \quad \frac{(D-2)\Omega_{D-2}}{16\pi G_D} \left( \frac{A_{AH}}{\Omega_{D-2}} \right)^{(D-3)/(D-2)} \]
Final state restriction

\[ A_{AH} < A_{EH} < A_{BH} \]

\[ M_{AH} := \frac{(D - 2)\Omega_{D-2}}{16\pi G_D} \left( \frac{A_{AH}}{\Omega_{D-2}} \right)^{(D-3)/(D-2)} \]

\[ < \frac{(D - 2)\Omega_{D-2}}{16\pi G_D} \left( \frac{A_{BH}}{\Omega_{D-2}} \right)^{(D-3)/(D-2)} \]
Final state restriction

\[ A_{AH} < A_{EH} < A_{BH} \]

\[ M_{AH} := \frac{(D - 2)\Omega_{D-2}}{16\pi G_D} \left( \frac{A_{AH}}{\Omega_{D-2}} \right)^{(D-3)/(D-2)} \]

\[ < \frac{(D - 2)\Omega_{D-2}}{16\pi G_D} \left( \frac{A_{BH}}{\Omega_{D-2}} \right)^{(D-3)/(D-2)} \leq M_{BH} \]
Final state restriction

\[ A_{AH} < A_{EH} < A_{BH} \]

\[ M_{AH} := \frac{(D - 2) \Omega_{D-2}}{16 \pi G_D} \left( \frac{A_{AH}}{\Omega_{D-2}} \right)^{(D-3)/(D-2)} \]

\[ < \frac{(D - 2) \Omega_{D-2}}{16 \pi G_D} \left( \frac{A_{BH}}{\Omega_{D-2}} \right)^{(D-3)/(D-2)} \leq M_{BH} \]
Final state restriction

\[ A_{AH} < A_{EH} < A_{BH} \]

\[ M_{AH} := \frac{(D - 2) \Omega_{D-2}}{16\pi G_D} \left( \frac{A_{AH}}{\Omega_{D-2}} \right)^{(D-3)/(D-2)} \]

\[ < \frac{(D - 2) \Omega_{D-2}}{16\pi G_D} \left( \frac{A_{BH}}{\Omega_{D-2}} \right)^{(D-3)/(D-2)} \leq M_{BH} \]
Implication for the LHC
Implication for the LHC


\[ M_{\text{BH}} = 2\mu \]
\[ M_{\text{BH}} = M_{\text{AH}} \]
Implication for the LHC


\[ M_{\text{BH}} = M_{\text{AH}} \]
\[ M_{\text{BH}} = 2\mu \]

\( M_P \) (TeV)

\[ \frac{M_{\text{threshold}}}{M_P} \]

BH production rate highly depends on the amount of radiated energy.
CONTENTS

- Introduction
- High-energy two-particle system
- Finding the apparent horizon
- Numerical results
- Summary and discussion
Summary
Summary

We studied the AH formation in the collision of high-energy particles
Summary

- We studied the AH formation in the collision of high-energy particles
- The problem was reduced to solving the 2-dim elliptic equation with unusual boundary conditions
Summary

- We studied the AH formation in the collision of high-energy particles
- The problem was reduced to solving the 2-dim elliptic equation with unusual boundary conditions
- We developed a numerical code to solve this problem and found the maximal impact parameter
Summary

We studied the AH formation in the collision of high-energy particles.

The problem was reduced to solving the 2-dim elliptic equation with unusual boundary conditions.

We developed a numerical code to solve this problem and found the maximal impact parameter.

The value of $\sigma_{AH}/\pi [r_h(2\mu)]^2$ ranges from 1.5 to 3.2.
Summary

We studied the AH formation in the collision of high-energy particles.

The problem was reduced to solving the 2-dim elliptic equation with unusual boundary conditions.

We developed a numerical code to solve this problem and found the maximal impact parameter.

The value of $\sigma_{AH}/\pi [r_h(2\mu)]^2$ ranges from 1.5 to 3.2.

If the energy loss by gravitational radiation is small, the production rate is fairly larger than 1BH/1s.
Related studies
Related studies

Effect of charge

HY and R.B. Mann, PRD74 (06) 044003 [gr-qc/0605131].
Related studies

Effect of charge
HY and R.B. Mann, PRD74 (06) 044003 [gr-qc/0605131].

Effects of Spin and duration
HY, A. Zelnikov and V.P. Frolov, PRD75, 124005 (2007).
Related studies

Effect of charge
HY and R.B. Mann, PRD74 (06) 044003 [gr-qc/0605131].

Effects of Spin and duration
HY, A. Zelnikov and V.P. Frolov, PRD75, 124005 (2007).

Further discussion POSTER!
Appendix
Final state restriction

Quasi-local mass of the horizon

\[ M_{AH} := \frac{(D - 2) \Omega_{D-2}}{16\pi G_D} \left( \frac{A_{AH}}{\Omega_{D-2}} \right)^{(D-3)/(D-2)} \]

Area theorem

\[ A_{Kerr}(M, J) > A_{AH} \]
Angular momentum & AH formation

The Kerr BH is extremal if \( J = J_\star(M) \)

\[
J_\star(M) = \begin{cases} 
(1/2)Mr_\star(M) & (D = 4) \\
(2/3)Mr_\star(M) & (D = 5) 
\end{cases}
\]

The BH (or AH) is expected to form only if

\[
q \equiv \frac{J_{\text{system}}}{J_\star(M_{\text{system}})} \lesssim 1,
\]

In our system, \( q = \begin{cases} 
0.84 & (D = 4) \\
0.93 & (D = 5) 
\end{cases} \)

This criterion was well confirmed in the collapse of rapidly rotating stars in 4-dim. by many authors, e.g., Sekiguchi & Shibata