ULTRARELATIVISTIC BOOSTS OF BLACK RINGS

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We summarize the main results of recent studies of Aichelburg-Sexl ultrarelativistic limits of five dimensional vacuum and charged black rings.

1. Introduction

In 1959 Pirani argued that the geometry associated with a fast moving mass resembles a “plane” gravitational wave.1 Later on, Aichelburg and Sexl (AS)2 considered a limiting (“ultrarelativistic”) boost of the Schwarzschild line element to determine the exact impulsive $pp$-wave generated by a lightlike particle. In higher dimensions $D \geq 4$, the AS limit of static black holes4 has been known for some time5

$$ds^2 = 2dudv + dz_i dz^i + H \delta(u)du^2 \quad (i = 2, \ldots, D - 1),$$

where

$$H = -8\sqrt{2}p_M \ln \rho - \frac{3\pi \sqrt{2} p_Q^2}{2 \rho} \quad (D = 4),$$

$$H = C_M \frac{p_M}{\rho^{D-4}} - C_Q \frac{p_Q^2}{\rho^{2(D-3)-1}} \quad (D > 4),$$

with

$$C_M = \frac{16\pi \sqrt{2}}{(D - 4) \Omega_{D-3}}, \quad C_Q = \frac{(2D - 9)!!}{(D - 3)!} \frac{2D - 5}{(D - 2)(D - 3)} 2^{D-4} \pi \sqrt{2},$$

$\rho^2 = z_i z^i$, $\delta(u)$ is the Dirac delta, $p_M/p_Q$ constants related to the mass/charge of the original spacetime4 and $\Omega_{D-3}$ the area of a unit $(D - 3)$-sphere. Such $pp$-waves have been employed in studies of classical formation of black holes in high energy collisions.6-11 In the case of zero charge they can be straightforwardly generalized to include an external magnetic field.12 The AS boost of rotating black holes13 has been studied in.14 Here we focus on $D = 5$ black rings (cf.15-17 for more details).

2. Boost of black rings

Ultrarelativistic boosts of spacetimes rely on first identifying a notion of Lorentz boost (e.g., with respect to asymptotic infinity). Then one applies such a transformation to the metric and takes the singular limit when the boost parameter tends to the speed of light. Simultaneously, the mass is appropriately rescaled to zero.2 The final metric depends on the boost direction. We shall use spatial “cartesian”
coordinates \((x_1, x_2, y_1, y_2)\) such that the 2-plane of the ring circle is \((y_1, y_2)\), and study boosts along the axes \(x_1\) and \(y_1\), “orthogonal” and “parallel” to it.

**Vacuum black ring** In the orthogonal case, the ultrarelativistic boost of the vacuum black ring of\(^{18}\) results\(^{15,16}\) in the following \(pp\)-wave propagating along \(x_1\)

\[
\text{ds}^2 = 2dudv + dx_1^2 + dx_2^2 + dy_1^2 + dy_2^2 + H_L(x_2, y_1, y_2)\delta(u)du^2, \tag{4}
\]

\[
H_L = \sqrt{2} \frac{3p_L L^2 + (2p_\nu - p_\lambda)\xi^2}{\sqrt{(\xi + L)^2 + x_2^2}} K(k) + \sqrt{2}(2p_\nu - p_\lambda) \times \left[ -\sqrt{(\xi + L)^2 + x_2^2} E(k) + \xi - L \frac{x_2^2}{\xi + L} \frac{x_2^2}{\sqrt{(\xi + L)^2 + x_2^2}} \Pi(\rho, k) + \pi|x_2|\Theta(L - \xi) \right], \tag{5}
\]

in which \(k = \sqrt{\frac{4\xi L}{(\xi + L)^2 + x_2^2}}, \quad \rho = \frac{4\xi L}{(\xi + L)^2}, \quad \xi = \sqrt{y_1^2 + y_2^2}; \tag{6}\)

and \(\Theta(L - \xi)\) denotes the step function (cf. the appendix of\(^{15}\) for definitions of the elliptic integrals \(K, E\) and \(\Pi\)). The null coordinates \(u\) and \(v\) are defined by

\[
t = \frac{-u + v}{\sqrt{2}}, \quad x_1 = \frac{u + v}{\sqrt{2}}, \tag{7}
\]

and \(p_\lambda, p_\nu\) and \(L\) are constants related to the mass, angular momentum and radius of the original ring. For black rings “in equilibrium” set \(p_\lambda = 2p_\nu\) in eq. (5).

In the case of a parallel boost the final \(pp\)-wave, now propagating along \(y_1\), is

\[
\text{ds}^2 = 2dudv + dx_1^2 + dx_2^2 + dy_1^2 + dy_2^2 + H_L(x_1, x_2, y_2)\delta(u)du^2, \tag{8}
\]

\[
H_L = \left[ 2(2p_\lambda - p_\nu)L^2 + (2p_\nu - p_\lambda)a^2 \left(1 + \frac{L^2 + \eta^2}{a^2 - y_2^2}\right) \right] + 2\sqrt{p_\lambda(p_\lambda - p_\nu)}Ly_2 \left(1 - \frac{L^2 + \eta^2}{a^2 - y_2^2}\right) \frac{\sqrt{2}}{a} K(k') - 2\sqrt{2}(2p_\nu - p_\lambda)aE(k') + \frac{\sqrt{2}}{2} \left[ (2p_\nu - p_\lambda)y_2 - 2\sqrt{p_\lambda(p_\lambda - p_\nu)L} \left[ -\frac{\eta^2 + L^2}{a\eta_2} \frac{a^2 + y_2^2}{a^2 - y_2^2} \Pi(\rho', k') + \pi \text{sgn}(y_2) \right] \right], \tag{9}
\]

where \(k' = \frac{(a^2 - \eta^2 - y_2^2 + L^2)^{1/2}}{\sqrt{2}a}, \quad \rho' = -\frac{(a^2 - y_2^2)^2}{4a^2y_2^2}, \quad a = \left[ (\eta^2 + y_2^2 - L^2)^2 + 4\eta^2 L^2 \right]^{1/4}, \quad \eta = \sqrt{x_1^2 + x_2^2}; \tag{10}\)

and the null coordinates are now defined by

\[
t = \frac{-u + v}{\sqrt{2}}, \quad y_1 = \frac{u + v}{\sqrt{2}}. \tag{11}\]
Static charged black ring  Static charged black rings were found\textsuperscript{19} (up to a misprint in $F_{\mu \nu}$) in the Einstein-Maxwell theory, see also.\textsuperscript{20–22} After an orthogonal boost of such solutions, one obtains a $pp$-wave (4) with

\begin{equation}
H_\perp^c = \sqrt{2} p_\lambda \left[ \left( 3L^2 \frac{1 + e^2}{1 - e^2} + \xi^2 + x_2^2 \frac{\xi + L}{\xi - L} \right) \frac{K(k)}{\sqrt{(\xi + L)^2 + x_2^2}} - \sqrt{(\xi + L)^2 + x_2^2} E(k) - \frac{\xi + L}{\xi - L} \frac{\xi - L}{\sqrt{(\xi + L)^2 + x_2^2}} \Pi(\rho_0, k) + \frac{\pi}{2} \left| x_2 \right| \right],
\end{equation}

$k$ and $\xi$ as in eq. (6) and $\rho_0 = -(\xi - L)^2/x_2^2$. The parameter $e$ is related\textsuperscript{17} to the electric charge of the original static spacetime.

For a parallel boost, one finds a metric (8) with

\begin{equation}
H_\parallel^c = \sqrt{2} p_\lambda \left[ \left( 2L^2 \frac{1 + 2e^2}{1 - e^2} + a^2 + a^2 \frac{L^2 + \eta^2}{a^2 - y_2^2} \right) \frac{1}{a} K(k') - 2a E(k') - \frac{\eta^2 + L^2 a^2 + y_2^2}{2a} \frac{\eta^2}{a^2 - y_2^2} \Pi(\rho', k') + \frac{\pi}{2} |y_2| \right],
\end{equation}

with $k'$, $\rho'$, $a$ and $\eta$ as in eq. (10).

See\textsuperscript{16} for the AS limit of the supersymmetric black ring of.\textsuperscript{23}

References