Ann. Phys. (Leipzig) **14**, No. 1 – 3, 71 – 102 (2005) / **DOI** 10.1002/andp.200410127

Special Relativity and Lorentz Invariance

Claus Lämmerzahl*

ZARM, University of Bremen, Am Fallturm, 28359 Bremen, Germany

Received 18 November 2003
Published online 13 January 2005

Key words Constancy of speed of light, synchronization, time dilation.
PACS 03.30+p, 03.50-z

Lorentz Invariance, the main ingredient of Special Relativity, is one of the pillars of modern physics. Though Special Relativity has been replaced by General Relativity, Lorentz Invariance is still valid locally. All physical fields have to obey the laws of local Lorentz Invariance. This is also the reason why gravity within the theory of General Relativity has to be described by the metric tensor. Here we give a short introduction into the early experiments and show that they disproved the exact validity of the Galilean framework for the description of classical mechanics. After a short summary of Special Relativity, the procedure of synchronization is analyzed. It is emphasized that no experiment should depend on the synchronization. Otherwise it might be possible to simulate or compensate effects by choosing another synchronization. Accordingly, the requirement of synchronization independence is a guideline for the choice of appropriate measurable quantities which then reveal relativistic physics in an unambiguous manner. Examples are given. In a subsequent article the modern experiments implementing this kind of notions will be discussed. Also some remarks are made on the importance of Lorentz Invariance in daily life. Finally we comment on possible violations of Lorentz Invariance and their measurability.

Contents

* E-mail: laemmerzahl@zarm.uni-bremen.de

Abbreviations used in this article:
SR = Special Relativity, GR = General Relativity, LI = Lorentz Invariance, LT = Lorentz transformations.

1 Introduction

Hundered years ago, the theory of Special Relativity (SR) has been proposed by Einstein [1]. This theory revolutionized the view of the physical world – it led to a unified view of space and time, to the famous mass-energy relation $E = mc^2$, as well as to the formulation of General Relativity (GR) which replaced Special Relativity, see the article by G. Schäfer in this issue [2]. However, Lorentz Invariance (we restrict ourselves to the homogeneous part), the most important ingredient of Special Relativity, is still valid locally and leads to the result that the gravitational field in General Relativity has to be represented by a metrical tensor. SR is much more than a physical theory for the propagation of light and particles, it is necessary for the interpretation of many experimental data and, even more important, it is a theoretical framework for other physical theories, as for the Standard Model, for example, and is the basis for relativistic quantum field theory.

Today, Lorentz Invariance (LI) is indispensible for practical purposes: Together with GR it is necessary for the functioning of the Global Positioning System GPS as well as for spectroscopy and metrology. Atomic spectra, used in connection with the realization of the international atomic time (TAI), can be understood only by including LI, and the comparison and calibration of clocks in various countries, again necessary for the definition of TAI, requires to take into account the special (and general) relativistic effects due to the motion (and position) of these clocks.

Owing to this overwhelming importance of SR it is clear that the experimental basis for such a theory has to be as good as possible. Indeed, starting with Fizeau, Michelson, and others in the 19th century, there are a lot of experiments carried through which disproved the Galilean structure of space-time, and explored, instead, its relativistic structure and tested LI with ever increasing accuracy. It is remarkable that just before the 100th anniversary of SR a kind of a race started among many experimental groups in the world in order to obtain the best tests of LI showing that LI still is in the focus of modern phyiscs. The three famous experiments providing the basis of LI, namely the test of the isotropy of light propagation, of the constancy of the speed of light and of time dilation, were all improved in 2003 [3–5], and further improvements can be expected in the very near future. Until now no departure from LI has been observed. – The increasing accuracy in the tests of these basic principles are mainly based on the improvements of the precision and stability of clocks. Indeed, almost all tests of LI (and of GR) can be interpreted as clock-comparison tests (see below).

Beside the universal importance and applicability of LI, the situation becomes even more involved because almost all approaches to a quantum gravity theory like string theory, canonical loop quantum gravity, or non-commutative geometry predict small deviations from exact Lorentz symmetry. Therefore there is a strong pressure on the experimental side to improve their devices in order to get even more precise results than obtained up to now. In a first approach one expects that quantum gravity modifies any of today's valid standard theories at a scale of the Planck length, Planck time, or Planck energy. For standard laboratory experiments, the energy involved is of the order 1 eV so that deviations are expected to occur at the order 10^{-28} which seems to be far of any experimental access. It should be stressed, however, that all these predictions are in fact merely hypotheses, these predictions are not based on complete theories. In addition, there might perhaps also some mechanism at work which may lead to some enhancements of the expected effect as it is the case for deviations from Newton potential at small distances as predicted by higher dimensional theories. Therefore, there is still a possibility that deviations from standard physics may occur at lower levels. Consequently, *any improvement of the accuracy of experimental results is of great value*. Furthermore, even for laboratory experiments, as will be shown at the end of this review, there are instruments which possess at least in principle a capability to approach the 10^{-28} level.

This is the first of two review papers. Here we introduce the main notions of LI and SR, in the second paper we like to review the experimental status of LI. We start by showing that at the end of the 19th century inconsistencies in the theoretical description of mechnanis and electrodynamics had been recognized. More importantly, also the experimental situation became even worse since all the attempts clearly failed to verify the motion between the Earth and an ether, a prediction which follows form the theoretical understanding at that time. Then, we introduce the basic notions, namely the notion of an inertial system and of the relativity principle, which the derivation of the Lorentz and Poincaré trasformations were based on. One of the main results of the Lorentz and Poincaré transformations were the unification of space and time and, thus, the notion of simultaneity. While simultaneity is unimportant for the physical interpretation of SR and LI (only synchronization independent quantities can be taken for an unambigous description of experiments since otherwise it might be possible to simulate or compensate effects by choosing just another setting of clocks), it is important for the understanding of the physics behind LI and for the selection of meaningful observables reveal physical effects of LI in an unambigious way. Finally we summarize the meaning of SR and LI and give some outlook of "predictions" from quantum gravity theories suggesting a tiny violation of LI at very high energies.

2 Before Special Relativity

2.1 The theoretical frame

Prerelativistic physics is characterized by two theories: Newtonian or (non-relativistic) classical machanics and Mawxell's theory. While Newtonian mechanics is Galilean invariant, Maxwell's equations are not: The force in Newton's equation $\boldsymbol{F} = m\ddot{\boldsymbol{x}}$ transforms covariant under Galilei-Transformations

$$\boldsymbol{x}' = R\boldsymbol{x} + \boldsymbol{v}t + \boldsymbol{a}\,, \qquad t' = t + b\,, \tag{1}$$

where R is a rotation, $\boldsymbol{v}$ the relative velocity between two inertial systems, $\boldsymbol{a}$ a translation, and b a re-set of the clock which is not allowed to depend on the position, orientation or relative velocity. On the other hand, it has been shown by Bateman [6] that Maxwell's equations in vacuum are invariant under conformal transformations.

This will become inconsistent at last as one wishes to couple both theories, as in the Lorentz force equation $m\ddot{\boldsymbol{x}} = q\boldsymbol{E} + \frac{q}{c}\boldsymbol{v} \times \boldsymbol{B}$. If the left-hand-side is Galilei covariant while the right hand side is Poincaré covariant, then the relativity principle is violated: The same experiment will lead to different results when performed in different frames. There will be observer systems which are distinguished by a specific outcome of certain experiments.

The Galilean invariance of classical mechanics is a consequence of the invariance of the force equation $\boldsymbol{F} = m\ddot{\boldsymbol{x}}$ only if one assumes a certain $\boldsymbol{F}$ and an absolute time, $t' = t + b$. This assumption has been stated by Newton even though there is no need for doing that. It has been shown later that dropping the assumption of an absolute time leads to the Lorentz transformations.

One of the consequences of the different covariance groups defined by classical mechanics and Maxwell's equations is that the speed of light which is constant in Mawxell's theory may acquire any value within classical mechanics. This is due to the (Galilean) addition of velocities

$$\boldsymbol{u}' = \boldsymbol{u} + \boldsymbol{v}\,, \tag{2}$$

where $\boldsymbol{v}$ is the velocity of a second observer with respect to a first observer and $\boldsymbol{u}$ and $\boldsymbol{u}'$ are the velocities of a body with respect to the first and second inertial frame. One way to make both theories compatible is to assume that Maxwell's theory is valid in a preferred frame only in which the velocity of light is constant. This particular frame can be identified with an *ether* frame. By going to another frame, one should observe a velocity and orientation dependent velocity of light, as predicted by the classical law (2) of the addition of velocities

$$\begin{aligned} c'(\theta, v) &= \sqrt{c^2 + v^2 + 2cv\cos\theta} \\ &\approx c\left(1 + \frac{v}{c}\cos\theta + \frac{1}{2}\frac{v^2}{c^2}\left(1 + 3\cos\theta\right)\right)\,, \end{aligned} \tag{3}$$

where $\boldsymbol{v}$ is the velocity of the observer with respect to the ether and $\theta = \angle(\boldsymbol{c}, \boldsymbol{v})$. We also made an expansion for small velocities. This is the basis for searches for an orientation dependence or anisotropy of the velocity of light and for a velocity dependendence of the velocity of light.

The violation of Galilean invariance came in by comparing the dynamics of different realms of physics, namely mechanics and electrodynamics. The same one does today when one searches for a violation of LI: one compares the dynamics of the electromagnetic field and the dynamics of (various) quantum particles or fields.

2.2 First experiments

In principle, all experiments with light or with any other electromagnetic phenomenon that are peformed on Earth and, thus, in a frame moving with a variable orientation and velocity with respect to a frame defined by the Sun, by our galaxy, or by the cosmic background radiation, can be used for a search for the ether. In fact, already very early it became clear that at least a naive version of an ether cannot be compatible with the experimental results. These experiments played no decisive role in the course of establishing SR but at the end disproved classical mechanics and showed that some new elements had to be taken into consideration. Here we shortly describe some of these early experiments which are incompatible with standard classical mechanics and Galilean kinematics. The setups of some of these experiments are still used today.

2.2.1 Aberration and the experiment of Airy

Aberration is the effect that incoming light, when viewed from a moving system, seems to come from a different direction. The non-relativistic part of aberration has an analogue in everyday experience: falling rain, when observed from a moving car, seems to come from ahead. In the same way light from stars seem to come from a direction slightly shifted in the direction of motion. This has already been observed by

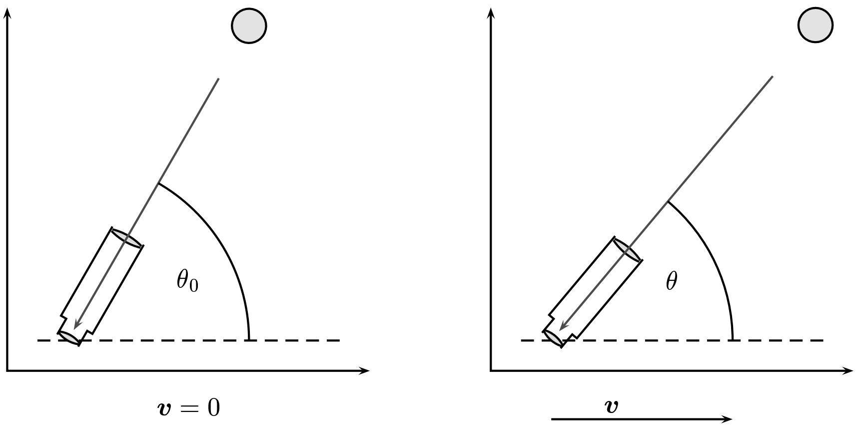


Fig. 1 (online colour at: www.ann-phys.org) Aberration: If for one observer the light from a distant star arrives from a direction θ_0 (left), then for a second observer moving with a velocity v with respect to the first observer (right) light arrives from another direction $\theta < \theta_0$.

Bradley in 1725 who recognized that the position of the stars is related to the state of motion of the Earth, see Fig. 1.

The non-relativistic aberration can be calculated very easily by referring to Fig. 2. If for an observer at rest in a given inertial frame light comes from a direction given by the angle θ_0, then an observer moving in that inertial frame has to tilt the ocular a bit in order to account for the motion of the ocular during the time the light propagates inside the ocular: light now seems to arrive from a direction θ

$$\tan\theta = \frac{ct\sin\theta_0}{ct\cos\theta_0 + vt} = \frac{\sin\theta_0}{\cos\theta_0 + \frac{v}{c}} . \tag{4}$$

The aberration angle $\delta\theta = \theta - \theta_0$ then is

$$\tan\delta\theta = \tan(\theta - \theta_0) = \frac{\tan\theta - \tan\theta_0}{1 + \tan\theta\tan\theta_0} = -\frac{v}{c}\frac{\sin\theta_0}{1 + \frac{v}{c}\cos\theta_0} \approx -\frac{v}{c}\sin\theta_0 , \tag{5}$$

where we approximated for small velocities v. If v is given by the motion of the Earth around the sun, then this results in a 1-year periodic change of the apparent direction of the incoming star light.

In an experiment designed by Airy in 1871, it should be possible to determine the motion of the telescope with respect to the ether. The trick is to use a second version of the same telescope filled with water. This

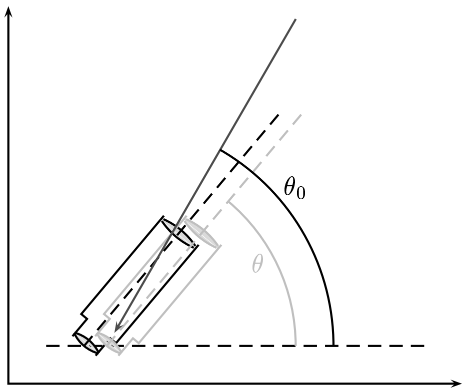


Fig. 2 (online colour at: www.ann-phys.org) After entering the telescope, light needs some time to reach the ocular. During this time the telescope and the ocular moves and, thus, has to be tilted a bit.

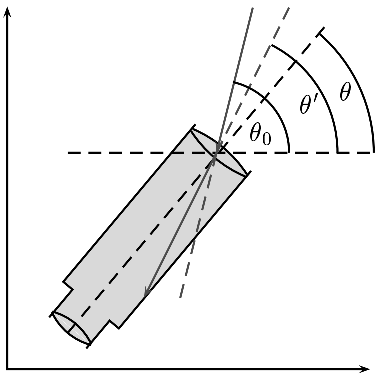


Fig. 3 (online colour at: www.ann-phys.org) The experiment of Airy. The direction of the incoming light is given by θ_0, in water the direction is θ'. The orientation of the telescope is described by the angle θ.

modifies the velocity of the light inside the telescope and, thus, the angle the telecope has to be tilted in order to account for the finite velocity of light inside the telescope. Then we have one more condition that allows the calculation of the velocity relative to the ether. The description is a bit more complicated than before since we have to take into account an additional refraction when entering the optically more dense medium,

$$\sin(\theta_0 - \theta_n) = n \sin(\theta' - \theta_n) \,. \tag{6}$$

Here n is the refractive index and θ_0 and θ' are the directions of the propagation of light outside the telescope and within the water and θ_n the new angle of the telescope. We find

$$\tan(\theta_n) = \frac{(c/n)t \sin(\theta')}{(c/n)t \cos(\theta') + vt} = \frac{\sin(\theta')}{\cos(\theta') + \frac{nv}{c}} \,, \tag{7}$$

where θ' still depends on θ. The aberration is given by (5) where we have to use $\tan\theta_n$ insetad of $\tan\theta$. Being a transcendental equation, it cannot be solved exactly. However, using the approximation $|\theta_n - \theta_0|, |nv/c| \ll 1$ we obtain the aberration $\delta\theta_n = \theta_n - \theta_0$, with

$$\delta\theta_n = -\frac{n^2 v}{c} \sin\theta_0 \,. \tag{8}$$

The difference between the aberrations of the air and water filled telescopes is

$$\delta\theta_n - \delta\theta = -\left(n^2 - 1\right) \frac{v}{c} \sin\theta_0 \,. \tag{9}$$

This change in the aberration has never been observed. Therefore, one of the above assumption cannot be true. The n^2 term comes in during use of the Galilean addition of velocities. If we use Einstein addition law, then this term is not present, in agreement with observation. The same phenomenon happens in the experiment by Fizeau.

2.2.2 The Fizeau “ether drag” experiment

In this interference experiment carried through by Fizau in 1851, see Fig. 4, light will be split coherently. Both parts of the light ray propagate through a moving medium where the motion is in opposite direction. A change in the velocity of the medium should result in moving interference fringes.

The velocity of light in the moving medium, for $v \ll c$, is

$$c_{\rm lab} \approx \frac{c}{n} \pm \sigma v \,, \tag{10}$$

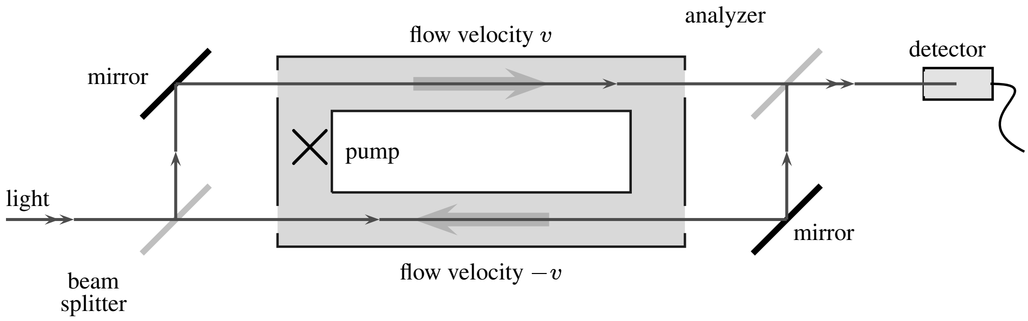


Fig. 4 (online colour at: www.ann-phys.org) The experiment of Fizeau: The split light beams propagates through differently moving media. The interference fringes are sensitive on the magnitude of the flow speed.

where $\sigma = 1 - \epsilon/n^2$ and $\epsilon = 0$ stands for the Galilean (2) and $\epsilon = 1$ for the Einstein addition of velocities. Since it does not interfere with any result from SR, we can describe light by plane waves

$$\varphi = \exp\left(i(kx - \omega t)\right) = \exp\left(i\omega\left(\frac{1}{c_{\mathrm{lab}}}x - t\right)\right), \tag{11}$$

what results in the observable intensity

$$I = \frac{1}{2}\left\{1 + \cos\left[\omega\left(\frac{1}{c/n + \sigma v} - \frac{1}{c/n - \sigma v}\right) l\right]\right\} \approx \frac{1}{2}\left[1 + \cos\left(2n^2\sigma\frac{\omega}{c}\frac{v}{c}l\right)\right], \tag{12}$$

where l is the distance light travels in the medium and where we used $v \ll c$. The phase shift depends on v so that a change in v allows to determine ϵ. The result is not compatible with $\epsilon = 0$. Therefore, either the Galilean addition of velocities is not correct or one tries to explain the result by assuming a dragging of the ether with the motion of the moving medium.

2.2.3 The experiment of Michelson and Morley

The first experiment searching for the ether was the Michelson-Morley experiment, see Fig. 5, which in its advanced version was capable to disprove the orientation dependence of the velocity of light (3) as originating from the Galiean addition of velocities (2). No orientation dependence has been observed in this experiment though it has the accuracy to detect the motion of the Earth through the ether assuming the ether to be attached to the Sun.

We describe this experiment by calculating the difference in the time-of-flight Δt of the light along the two interferometer arms. This depends on the orientation of the interferometer and leads to a phase shift $\Delta\phi = \omega\Delta t$ where ω is the frequency of the light. The difference Δt can be calculated either in the frame of the interferometer using the direction dependent velocity of light, or in the ether frame taking into account the motion of the interferometer. The calculation in the ether frame refers to Fig. 6 showing the propagation of the light rays along one interferometer arm from the beam splitter A to the mirror B where during the flight the mirror continues to move so that the light effectively propagates the distance from A to B'. This yields the time-of-flight along one interferometer arm

$$\Delta t(l, \vartheta) = \frac{2lc}{c^2 - v^2}\sqrt{1 - \frac{v^2}{c^2}\left(1 - \cos^2\vartheta\right)}, \tag{13}$$

where ϑ is the angle between the interferometer arm and the velocity $\boldsymbol{v}$ of the interferometer with respect to the ether. Doing this for both interferometer arms, which are assumed to be orthogonal, yields the difference

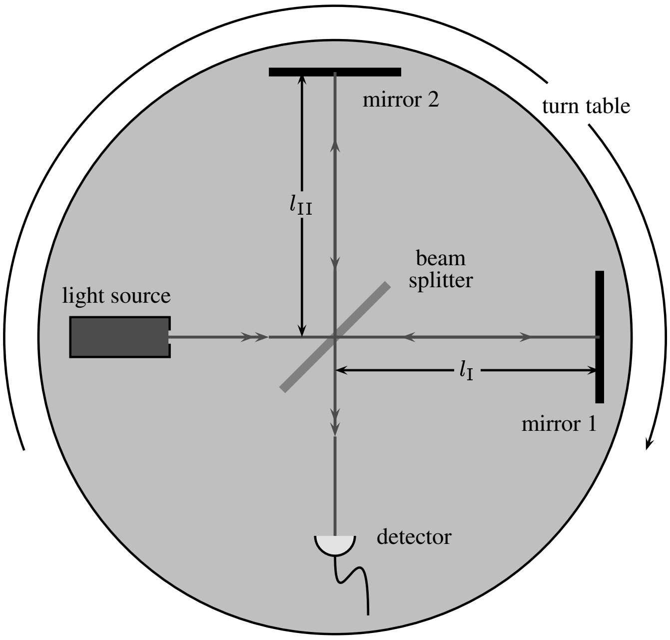


Fig. 5 (online colour at: www.ann-phys.org) The interference experiment of Michelson and Morley. Light is split coherently at the beam splitter and propagates in two orthogonal directions. After reflection at the mirrors it recombines again at the beam splitter. If the velocity of light is direction dependent then the interference fringes should vary during the rotation of the apparatus.

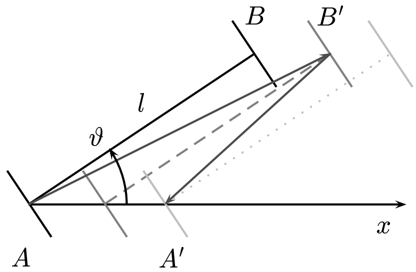


Fig. 6 (online colour at: www.ann-phys.org) The trajectories of light in an interferometer arm that moves through the ether along the x-axis.

in the time of flight $\Delta t = \Delta t(l, \vartheta) - \Delta t(l, \vartheta + \pi/2)$ and, thus, the phase shift

$$\begin{aligned} \Delta\phi &= \frac{2l\omega}{c} \frac{1}{1 - \frac{v^2}{c^2}} \left(\sqrt{1 - \frac{v^2}{c^2} \left(1 - \cos^2\vartheta\right)} - \sqrt{1 - \frac{v^2}{c^2} \left(1 - \sin^2\vartheta\right)} \right) \\ &\approx \frac{2l\omega}{c} \frac{v^2}{c^2} \cos(2\vartheta) \,, \end{aligned} \tag{14}$$

where we made an expansion for $v \ll c$. A calculation of the same process in the interferometer system using the orientation dependent velocity of light yields exactly the same result.

For an armlength of 11 m, light with a wavelength of 550 nm and a velocity of the Earth around the sun of about 30 km/s, this yields a phase shift of $\Delta\phi = 0.8\pi$. The sensitivity of the Michelson-Morley apparatus was $\Delta\phi \sim 0.01\pi$. However, no effect has been observed. This leads to a maximum velocity of the apparatus with respect to the ether system of $v \leq 8$ km/s. Using the velocity of the Earth with respect to the cosmic background, that is, taking the microwave background as ether frame, should result in a more than 10-fold larger effect and makes the failure even more drastic.

Therefore either the addition of velocities is not correct or some process has to be modified. A modification may be the dragging of the ether by massive bodies. Another explanation was the hypothesis of Lorentz and Fitzgerald proposing a physical contraction of the length of the interferometer by a factor $\sqrt{1 - v^2/c^2}$ in the direction of the motion. This was an important step towards establishing the Lorentz transformations. This physical contraction has to be universal, that is, independent of the material used for the interferometer arms. In fact, subsequently, Morley and Miller [7, 8] repeated the experiment with different materials, like pine wood, sand stone etc. Of course, they didn'd find any material dependent effect.

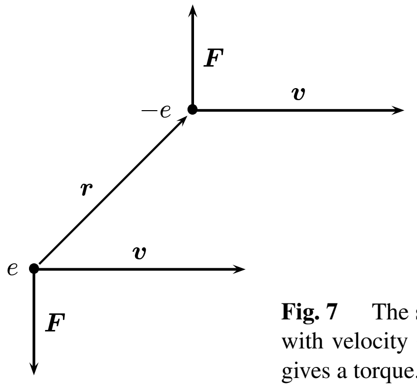


Fig. 7 The setup of the experiment of Trouton and Noble. Two opposite charges move with velocity $\boldsymbol{v}$ through the ether. The resulting magnetic field acts on the charges and gives a torque.

For arms of unequal lengths $l_1 \neq l_2$, one obtains as phase difference

$$\begin{aligned}\Delta\phi &= \frac{2\omega}{c}\frac{1}{1-\frac{v^2}{c^2}}\left(l_1\sqrt{1-\frac{v^2}{c^2}\left(1-\cos^2\vartheta\right)} - l_2\sqrt{1-\frac{v^2}{c^2}\left(1-\sin^2\vartheta\right)}\right)\\ &= \frac{2\omega}{c}\left(l_1 - l_2 + \frac{v^2}{c^2}\frac{1}{2}\left(l_1 - l_2 + l_1\cos^2\vartheta - l_2\sin^2\vartheta\right)\right) + \mathcal{O}(v^4/c^4)\,,\end{aligned} \tag{15}$$

so that a change in the velocity should lead to a change in the phase. This is the issue of the experiment of Kennedy and Thorndike performed as late as in 1932 [9].

2.2.4 The experiment of Trouton and Noble

In this experiment [10], Trouton and Noble checked whether electromagnetic phenomena single out an ether frame. If the Maxwell equations possess their well known form in the ether system and if an absolute motion of a charge with respect to the ether creates a magnetic field

$$\boldsymbol{B} = \mathrm{e}\frac{\boldsymbol{v}\times\boldsymbol{r}}{r^3}\,, \tag{16}$$

then both charges of a dipole moving with a velocity v with respect to the ether feel a Lorentz force due to the magnetic field of the other charge. This leads to the torque

$$\boldsymbol{M} = \boldsymbol{r}\times\boldsymbol{F} = \mathrm{e}^2\frac{\boldsymbol{r}\times(\boldsymbol{v}\times(\boldsymbol{v}\times\boldsymbol{r}))}{r^3}\,, \tag{17}$$

see Fig. 7. For a capacitor C, the torque is

$$T = \tfrac{1}{2}CV^2\frac{v^2}{c^2}\sin(2\psi)\sin^2\varphi\,, \tag{18}$$

where V is the voltage applied to the capacitor, ψ the angle between $\boldsymbol{v}$ and the condensor, and φ the angle between $\boldsymbol{v}$ and the suspension. Though the sensitivity of the experiment of Trouton and Noble was not sufficient, better experiments performed later have shown no effect.

3 Special Relativity

3.1 Basic notions

The basic notions in the derivation of the Lorentz transformations (LT) are the inertial systems, the relativity principle, and the notion of simultaneity (or synchronization) which is needed for an operational meaning of the velocity of bodies moving with respect to an observer. These notions are already known from classical mechanics and can be taken over except the notion of simultaneity. The definition of a frame of reference to be an inertial system relies on the uniform motion of force-free particles. The relativity principle states that by performing experiments inside a box in an inertial system with identical intial and boundary conditions and perfect shielding from the environment, it is not possible to determine the state of motion of this box. The latter requires the covariance of physical equations: all laws of physics have to acquire the same form in all boxes, that is, in all inertial systems.

Since in classical mechanics time is absolute, all inertial systems carry the same time and are, thus, automatically synchronized. If one does not accept the absolute time and likes to replace the synchronization with a physical process with finite propagation speed, then ambiguities may occur. In principle one may take any physical process to synchronize clocks at different positions, but it is preferable to use a distinguished phenomenon. Since the propagation of light is a universal and unique phenomenon (the velocitiy does not depend on the velocity of the source and there is only one light signal connecting two events) light is a good choice for that. Since the measured velocity of a body moving with respect to the observer depends on the synchronization of clocks at different positions, it is clear that the statement that the velocity of light is constant depends on the synchronization procedure chosen. However, the two-way velocity of light does not depend on the synchronization. Furthermore, since in all experiments only the two-way velocity of light plays a role – in fact: *can* play a role – synchronization is, in principle, a completely unimportant issue in SR. The description of all the effects characteristic for SR should lead to the same result irrespective of the chosen synchronization. The particular effects which are characteristic for special relativistic physics, remain unaffected by the choice of a synchronization. However, the condition of being independent of any synchronization serves as a guide for the selection of unambiguous tests of special relativistic effects where it is not possible to manipulate the experimental result by choosing another synchronization procedure. We will address this issue below in Sect. 4.

3.2 Derivation of the Poincaré and Lorentz transformation

Since the derivations of the Poincaré transformations and LT have been exposed many times, we just mention that there are at least three ways to derive them, see Fig. 8. The first one was used by Einstein [1] where he assumed the constancy of the speed of light and the relativity principle in the sense that there is no preferred inertial system. This leads to the transformations of the coordinates $(t(p), \boldsymbol{x}(p))$ and $(t'(p), \boldsymbol{x}'(p))$ of one event E at point p:

$$t'(p) = \frac{1}{\sqrt{1 - v^2/c^2}} \left(t(p) - \frac{\boldsymbol{v}}{c^2} \cdot \boldsymbol{x}(p) \right) + t_0 \, , \tag{19}$$

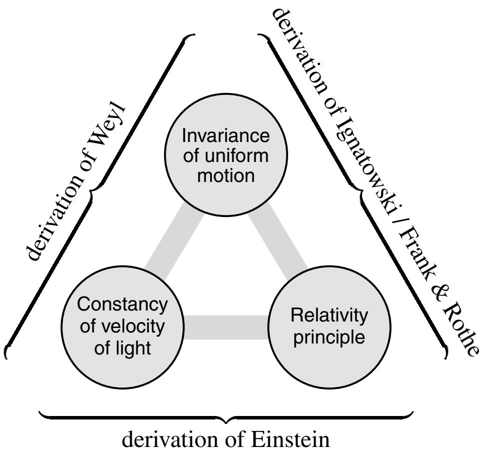


Fig. 8 (online colour at: www.ann-phys.org) The "magic triangle" of SR. From two of the three conditions one can derive the Poincaé transformation.

$$\boldsymbol{x}'(p) = \boldsymbol{x}_\perp(p) + \frac{1}{\sqrt{1 - v^2/c^2}} \left(\boldsymbol{x}_\parallel(p) - \boldsymbol{v}t(p)\right) + \boldsymbol{x}_0\,, \tag{20}$$

$(t, \boldsymbol{x})$ and $(t', \boldsymbol{x}')$ are the coordinates of two inertial systems S and S' where S' moves with a relative velocity $\boldsymbol{v}$ with respect to S. Here $\boldsymbol{x}_\parallel$ and $\boldsymbol{x}_\perp$ are the components of the position vector parallel and orthogonal with respect to the relative velocity $\boldsymbol{v}$. In (19), (20) also a translation in the space and time can appear. The relations (19), (20) are the Poincaré transformations. For $t_0 = 0$ and $\boldsymbol{x}_0 = 0$ eqs. (19), (20) reduce to the LT. The LT are homogeneous in position and time. The velocity of light is a limiting velocity. The relative velocity between two inertial systems never can exceed c. For $v \ll c$, the Poincaré transformations degenerate to the Galilean transformations.

Another derivation, intiated by v. Ignatowski [11, 12] and Frank and Rothe [13, 14], shows that the invariance of the uniform motion together with the relativity principle already is enough to derive Poincaré transformations with an undefined invariant velocity which has to be identified with the velocity of light.

A third derivation uses the uniform motion as well as the constancy of the speed of light. The coordinate transformations which leave a uniform motion invariant are related by projective transformations, while the conformal transformations leave the velocity of light invariant. Requiring both, breaks down the transformations to the Poincaré transformations, see Fig. 8.

3.3 The consequences

The consequences of the LT are well known and some are listed below. All these relativistic effects can be divided into two classes: (i) Effects that rely on the particular chosen Einstein synchronization. These effects are usually presented in the literature. The result of these effects are different from the nonrelativitsic results but depend on the synchronization and, thus, may be simulated or compensated by a choice of another synchronization. (ii) Effects which are independent of the synchronization, which we are going to discuss in the next section. The synchronization independent effects are more important since these effects are realized in the experiments which, as described above, should not depend on the synchronization.

Here we first describe the effects of the first category:

- If one event E at point p which is located at the spatial origin of an observer is viewed from another observer moving with velocity v with respect to the first one, then the time coordinates of that event determined by the two observers are related by

$$t'(p) = \frac{1}{\sqrt{1 - v^2/c^2}}\, t(p)\,. \tag{21}$$

  That means that two observers measure different times intervals. The time interval is larger the larger the relative velocity is. This is the *time dilation*. It is also easy to see that the time of a light clocks moving in an inertial system goes slower by the the factor $1/\sqrt{1 - v^2/c^2}$ compared to a clock at rest in that inertial system, see Fig. 9.
- The contraction of lengths,

$$L' = \sqrt{1 - v^2/c^2}\, L\,, \tag{22}$$

  which can also be understood in terms of the light clock, see Fig. 9
- the addition of velocities

$$\boldsymbol{u} = \frac{\boldsymbol{v} + \frac{1}{\gamma(v)}\boldsymbol{v}' + \frac{\gamma(v)-1}{\gamma(v)}\frac{\boldsymbol{v}'\cdot\boldsymbol{v}}{v^2}\boldsymbol{v}}{1 + \boldsymbol{v}\cdot\boldsymbol{v}'/c^2} \tag{23}$$

  which, when specialized to case of parallel velocities, reads

$$u = \frac{v + v'}{1 + vv'/c^2}\,. \tag{24}$$

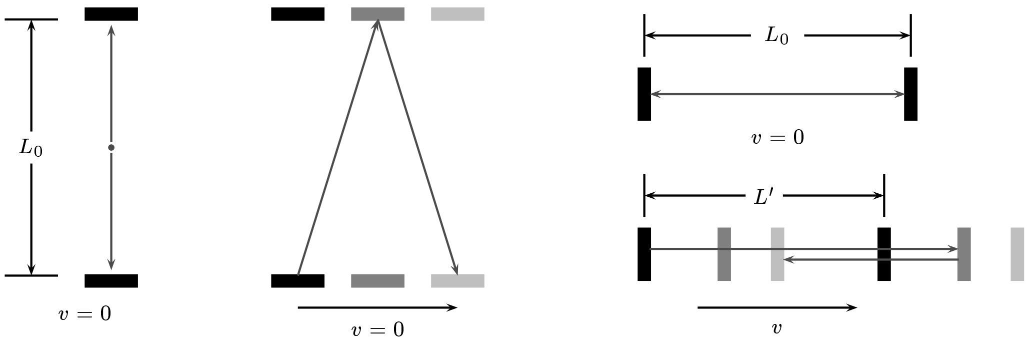


Fig. 9 (online colour at: www.ann-phys.org) A light clock consists of two mirrors at a distance L_0 and a photon propagating back and forth between the two mirrors. This defines a unit of time by $T_0 = 2L_0/c$ (left). In case the clock moves orthogonally with respect to distance between the two mirrors, then for the observer at rest the distance the photon has to travel is longer, namely $L = 2\sqrt{L_0^2 + (vT)^2}$, where T is the time-of-flight of the photon from one mirror to the other. Since photons have a constant velocity c, the unit of time is now given by $T = L/c$. If we use the longer L, the unit of time of the moving clock is longer by $1/\sqrt{1 - v^2/c^2}$. If the light clocks moves along its axes (right), then the constancy of the speed of light and the requirement of the ordinary time dilation, which is a consequence of the relativity principle, leads to the necessity that the length has to be shortened according to (22).

These equations show that the velocity of light has the same value independent of the velocity of the observer in an inertial system. Thus, we are back at the constancy of the speed of light.

- The Doppler effect

$$\nu' = \frac{\sqrt{1 - v^2/c^2}}{1 + (v/c)\cos\theta'}\nu \,, \tag{25}$$

relates the frequency ν measured by an observer at rest in an inertial system with the frequency ν' measured by an observer moving in that inertial system. θ' is the direction the moving observes sees the light coming from.

- Aberration

$$\cos\theta' = \frac{\cos\theta - v/c}{1 - (v/c)\cos\theta} \,, \tag{26}$$

where θ is the direction the observer at rest sees the light coming from.

- Finally, effects based on accelerated motion, namely the Sagnac effect and the Thomas precession.

The aberration (26) is a direct consequence of the addition of velocities (23) if one takes as one of the velocities the velocity of light, and the Doppler effect is a consequence of time dilation. While time dilation is a consequence of the constancy of the speed of light, the addition of velocities also rely on the relativity principle. Therefore, also from the consequences of the LT it is very clear that the constancy of the speed of light and the relativity principle are directly behind all these effects and are, thus, the postulates which have to be confronted with experiments. The length contraction is no proper effect of the LT since it always depends on the synchronization and, thus, can be simulated or transformed away, see below.

Instead of going into the details of all these well known effects we only cover two points which may be of particular interest. The first one is the twin paradox, and the other the Sagnac effect.

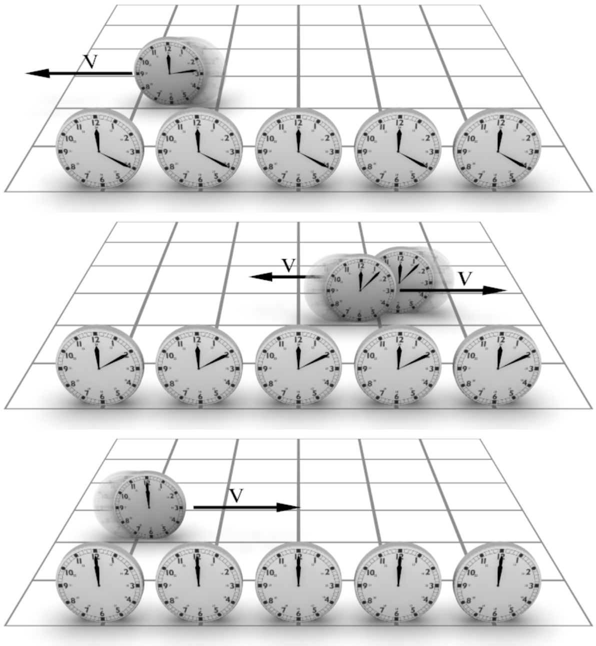


Fig. 10 (online colour at: www.ann-phys.org) The twin paradox as realized by three clocks (from bottom to top). The clocks in the first row indicate the time elapsing for the twin staying at rest. The clocks with an attached arrow, which indicates the motion of that clock with respect to the twin at rest, show the time of the moving twin as seen from the twin at rest.

3.3.1 The twin paradox

Here we emphasize that contrary to what is often stated, it is not necessary to have an acceleration of one of the twins or of the moving clock. This is important since sometimes the effect is said to be caused by the acceleration. Since accelerations are beyond the Poincaré transformations, this would mean that the effect is not due to the LT and, thus, no genuine SR effect. This is not true. By using a third clock, we have a purely kinematical version of the twin paradox. The procedure is as follows: The first clock is at rest in the inertial system. Then a moving clock is given the same time as the clock at rest. After some time, a third clock is used which moves in the opposite direction and overtakes the time of the moving clock when they meet, see Fig. 10. The twin paradox now is the time difference between the first and the third clock upon arrival at the original position. No acceleration is necessary to observe this time difference. Therefore, this is a purely kinematical effect fully describable within SR [15, 16]. In addition, we will see below that the twin paradox is the synchronization invariant version of the time dilation. However, it has been demonstrated experimentally that clocks based on atomic phenomena are inert against accelerations. Therefore, for tests of the twin paradox it is no problem to use accelerated clocks.

3.3.2 The Sagnac effect

The Sagnac effect is the shift of interference fringes in the case the interferometer starts to rotate. This holds for laser interferometers as well as for interferometers for neutrons or atoms. Though many derivations of the Sagnac effect for matter waves use a non-relativistic Hamiltonian only or even use non-relativistic particle motion, we will show that the Sagnac effect is a truly relativistic effect which can be understood only by using SR.

We model the geometry of the interferometer by a disk of radius R rotating with angular velocity Ω with respect to the laboratory attached to an inertial system (again, though in principle a rotating system is accelerating and, thus, not related to an inertial system by a Poincaré transformation and therefore beyond the range of application of SR, we safely can apply the LT locally acting on vectors defined at a point p). The source and analyzer are attached to the disc and, thus, are co-rotating. The phase shift is $\delta\phi = \omega\delta t$, where ω is the frequency of the matter or light wave in the rotating system at the position of the source and

δt the time difference in the arrival time of the two counter-propagating matter or light waves. The source emits particles that move with velocities $\pm v$ with respect to the source. Since the source is moving with velocity ΩR with respect to the non-rotating laboratory, the velocities of these particles with respect to the laboratory are

$$v'_\pm = \frac{\Omega R \pm v}{1 \pm \Omega R v/c^2} \,. \tag{27}$$

From this velocity we calculate the time $t'_\pm$ in the laboratory that the particles need to reach the recombiner which is placed on the disc opposite to the source. This time is determined by $v'_\pm t'_\pm = \pm\pi R + \Omega R t'_\pm$:

$$t'_\pm = \frac{\pm\pi R}{v'_\pm - \Omega R} \,. \tag{28}$$

Therefore, by substituting (27) the times of arrival of the two particles differ by

$$\delta t' = t'_+ - t'_- = \frac{1}{1 - \Omega^2 R^2/c^2} \frac{2\Omega\Sigma}{c^2} \,. \tag{29}$$

In the co-rotating system we find

$$\delta t = \frac{1}{(1 - \Omega^2 R^2/c^2)^{1/2}} \frac{2\Omega\Sigma}{c^2} = \frac{2\Omega\Sigma}{c^2} + \mathcal{O}\left[(\Omega R/c)^4\right] \,, \tag{30}$$

where $\Sigma = \pi R^2$ is the area of the disc. The observed phase shift is given by the phase difference between the two points of arrival, that is, by

$$\delta\phi = \omega\delta t = \frac{1}{(1 - \Omega^2 R^2/c^2)^{1/2}} \frac{2\omega\Omega\Sigma}{c^2} = \frac{2\omega\Omega\Sigma}{c^2} + \mathcal{O}(c^{-4}) \,. \tag{31}$$

Thus, the Sagnac effect is a purely relativistic effect. In a non-relativistic approach $\delta t = 0$ and no phase shift can occur.

We can now assume that there is a dispersion relation of the form $\hbar\omega = E = \left(m^2c^4 + p^2c^2\right)^{1/2}$, which for slow particles yields approximately mc^2. Then

$$\delta\phi = \frac{E\delta t}{\hbar} = \frac{1}{\left(1 - \frac{\Omega^2 R^2}{c^2}\right)^{1/2}} \frac{2E\Omega\Sigma}{c^2} = \frac{2m\Omega\Sigma}{\hbar} + \mathcal{O}(c^{-2}) \,. \tag{32}$$

It is only due to the multiplication by c^2 from re-expressing the energy in terms of the mass (instead of the frequency) that the speed of light disappears in the dominating term. However, in the frame of wave mechanics it is more appropriate to describe properties of a wave in terms of its frequency rather than in terms of the (mechanical) energy E. Only in the form (31) all notions appearing in the Sagnac effect are defined in an operational way. This phase shift has the same form as the phase shift for light. This indeed has to be the case because both, light and matter waves, are wave phenomena. A wave is characterized by its frequency and its wave vector. The "mass" of a wave is only a derived concept since it assumes a particular dynamical equation for the wave, either in terms of a wave equation or of a dispersion relation. As a consequence, the result (31) is valid for *any* matter wave, irrespective of the dynamical equation it obeys. This is also apparent from the fact that, beside the frequency of the matter wave at the position of the beam splitter, the result (31) does not depend on any particle properties, and in particular not on the mass. Thus, the Sagnac effect (31) is a *universal* special relativistic wave phenomenon.

This result looks a bit contradictory to the fact that the first term in (32) can be derived from the non-relativistic Schrödinger Hamiltonian in a rotating frame, $H = \boldsymbol{p}^2/(2m) + \boldsymbol{\Omega}\cdot\boldsymbol{L}$, where $\boldsymbol{L}$ is the angular momentum. However, this Hamiltonian can be derived as a non-relativistic limit of the Dirac equation [17] which directly leads to the first term in (32). For the massless case, one has to consider the Maxwell equations in a rotating frame. In an eikonal approximation it leads to (31) with ω as the frequency of the light beam. In terms of fundamental equations one has to treat the massive and massles cases separately; in terms of wave propagation, both cases can be treated in a unified manner. In any case, the exact result as well as the approximation $2m\Omega\Sigma/\hbar$ are of purely relativistic origin.

We can push these ideas a bit further if we distinguish between the velocity of light which appears in the LT and, thus, in the formula for the addition of velocities, and the velocity of "light" which appears in relativistic field equations for the matter wave. If we denote by c_{D} the velocity of "light" appearing in the Compton wavelength of the Dirac particle, e. g., $\lambda_{\mathrm{C}} = \hbar/mc_{\mathrm{D}}$, then the above formula (32) for the Sagnac effect yields

$$\delta\phi = \frac{1}{\left(1 - \frac{\Omega^2 R^2}{c^2}\right)^{1/2}} \frac{2E\Omega\Sigma}{c^2} = 2\frac{c_{\mathrm{D}}^2}{c^2}\frac{2m\Omega\Sigma}{\hbar} + \mathcal{O}(c^{-2})\,. \tag{33}$$

As a consequence, one may, at least in principle, use the Sagnac effect as a test of the universality of the limiting velocity for various elementary particles. The ratio of the various masses can be determined by different and independent methods as collision processes, for example.

4 The issue of synchronization

One of the consequences of the derivations of the LT is a well-defined notion of synchronization between different events. However, experiments like the Michelson-Morley, Kennedy-Thorndike experiments or experiments searching for the correct time-dilation using absorption of photons by moving atoms cannot depend on the the way clocks at different positions are related. For none of these experiments the experimentator has to synchronize clocks. And for the atom and the photon it absorbs it is completely irrelevant how the clocks are set. Therefore, physics must be invariant under the procedure of how to synchronize clocks. As a consequence, we have to establish a formalism for introducing arbitrary synchronizations. Subsequently, we have to show that all the experiments testing LI do not depend on the parameter which characterizes the chosen synchronization. We can go a step further and claim that only those experiments, which can be described such that the synchronization parameter drops out, are real experiments testing LI. If the description of an experiment depends on the synchronization, then the effect can be *simulated* or *transformed away* by choosing some (probably curious but still viable) synchronization.

In this section we define how to set up different synchronizations, reformulate the LT for arbitrary synchronizations and describe various basic effects. We will see that time dilation effects like the ordinary time dilation or the Doppler effect which depend on the chosen synchronization, can be reformulated in a synchronization independent way while length contraction always depends on the synchronization.

4.1 The definition of synchronization

In order to introduce these notions we first restrict to one spatial dimension. We use an observer with a clock moving along a straight line which we choose as time axis, and light propagating between the observer to an event B and back to the observer. According to a choice of a parameter $\widehat{\epsilon}$ in

$$t(B) = t_1 + \widehat{\epsilon}(t_2 - t_1) \quad \text{where} \quad 0 < \widehat{\epsilon} < 1 \tag{34}$$

the event B will be assigned the same time as the event A on the observer's worldline, see Fig. 11. This new definition of a synchronization can be regarded as a coordinate transformation relating the (t, x) coordinates (with the dashed line in Fig. 11 as x–axis) to the new coordinates where the new x'–axes is the line through A and B:

$$t' = t - (2\widehat{\epsilon} - 1)x\,, \qquad x' = x\,. \tag{35}$$

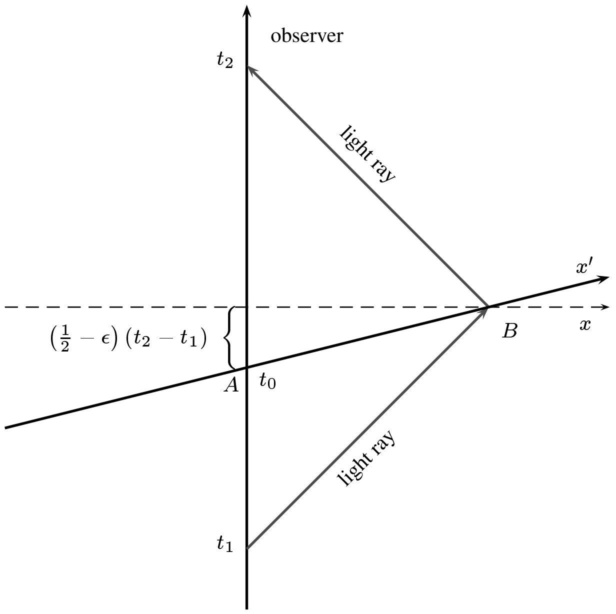


Fig. 11 (online colour at: www.ann-phys.org) Synchronization of two events using light rays. At time t_1 an observer, represented by its worldline, sends a light signal to the event B where it is reflected back to the observer and arrives at t_2. Assigning B the time of the event A on the observer's worldline, $t(B) = t(A) = t_1 + \widehat{\epsilon}(t_2 - t_1)$ for some $0 < \widehat{\epsilon} < 1$, then all events on the line through A and B are simultaneous. Einstein synchronization is given by $\epsilon = \frac{1}{2}$ and the simultaneous events are shown by the dashed line.

In three dimensions this can be generalized to

$$t' = t - \widehat{\boldsymbol{\epsilon}} \cdot \boldsymbol{x}\,, \qquad \boldsymbol{x}' = \boldsymbol{x}\,, \tag{36}$$

where $\widehat{\boldsymbol{\epsilon}}$ is a 3-vector characterizing the hyperplane of simultaneity. (In "real life" one can observe a synchronization in the *Volksgarten* in Düsseldorf, see Fig. 12.)

The round-trip velocity of light is twice the spatial distance between the events A and B devided by the round-trip time

$$c := \frac{2(x_B - x_A)}{t_3 - t_1}\,. \tag{37}$$

It does not depend on the synchronization. However, the synchronization will affect the measured one-way velocities. The velocity of light propagating in the $\pm x$–directions is given by

$$c_+ = \frac{x_B - x_A}{t_0 - t_1} = \frac{c}{2\widehat{\epsilon}}\,, \qquad c_- = \frac{x_B - x_A}{t_3 - t_0} = \frac{c}{2(1-\widehat{\epsilon})}\,. \tag{38}$$

The values of the measured one-way velocities of light is directly related to the synchronization parameter

$$\widehat{\epsilon} = \frac{c}{2c_+} \qquad \text{and} \qquad \widehat{\epsilon} = 1 - \frac{c}{2c_-}\,. \tag{39}$$

Note that

$$\frac{1}{c_+} + \frac{1}{c_-} = \frac{2}{c}\,, \tag{40}$$

that is, the two one-way velocities can be combined to give the synchronization independent two-way velocity of light.

The velocity of massive bodies in $\pm x$–direction

$$v_+^\epsilon = \pm\frac{x_B - x_A}{t_B - t_A} \tag{41}$$

Fig. 12 (online colour at: www.ann-phys.org) A two-dimensional field of synchronized clocks at the *Volksgarten* in Düsseldorf.

also depends on the synchronization. However, with

$$\frac{1}{v} - \frac{1}{c} = \frac{1}{v^{\epsilon}_{\pm}} - \frac{1}{c_{\pm}} \tag{42}$$

we are able to define a velocity v which is independent of the choice of $\widehat{\epsilon}$. Therefore, for massive bodies it is also possible to get synchronization invariant velocities from certain combinations of synchronization dependent ones.

4.2 The general formalism

Now we turn to the description of arbitrary synchronizations in three dimensions. We start from an Einstein synchronized Lorentz system Σ with coordinates $(T, \boldsymbol{X})$ and perform a transformation into a system S with coordinates $(t, \boldsymbol{x})$, which moves with velocity $\boldsymbol{w}$ with respect to the Σ. In S an arbitrary synchronization is assumed. Then, the combination of the ordinary LT and the synchronization (36) yields

$$T = \gamma(w)\,(t - \boldsymbol{\epsilon} \cdot \boldsymbol{x}) \tag{43}$$

$$\boldsymbol{X} = \boldsymbol{x} - \left(1 - \frac{1}{\gamma(w)}\right) \frac{(\boldsymbol{w} \cdot \boldsymbol{x})\boldsymbol{w}}{w^2} + \boldsymbol{w}T\,, \tag{44}$$

where we used a redefined synchronization parameter $\boldsymbol{\epsilon}$. For a given value of $\boldsymbol{\epsilon}$ the synchronization is transitive.

A very important relation is the modified addition of velocities according to the diagram

$$\begin{array}{ccc} & S(t, \boldsymbol{x}) & \\ {\scriptstyle w}\nearrow & & \searrow{\scriptstyle u^{\epsilon}} \\ \Sigma(T, \boldsymbol{X}) & \xrightarrow{\quad w' \quad} & S'(t', \boldsymbol{x}') \end{array} \tag{45}$$

For deriving the dependence of $\boldsymbol{w}'$ on $\boldsymbol{w}$ and $\boldsymbol{u}^\epsilon$, we use (43), (44) and

$$T = \gamma(w')\left(t - \boldsymbol{\epsilon}' \cdot \boldsymbol{x}'\right) \tag{46}$$

$$\boldsymbol{X} = \boldsymbol{x}' - \left(1 - \frac{1}{\gamma(w')}\right) \frac{(\boldsymbol{w}' \cdot \boldsymbol{x}')\boldsymbol{w}'}{w'^2} + \boldsymbol{w}'T\,. \tag{47}$$

With (43), (44) and (46), (47) and using that the origin of S', namely $\boldsymbol{x}' = 0$, moves with velocity $\boldsymbol{u}^\epsilon$ with respect to S, we obtain

$$\boldsymbol{w}' = \boldsymbol{w} + \frac{\boldsymbol{u}^\epsilon - (1 - \sqrt{1 - w^2})\widehat{\boldsymbol{w}}(\boldsymbol{u}^\epsilon \cdot \widehat{\boldsymbol{w}})}{\gamma(w)\,(1 - \boldsymbol{\epsilon} \cdot \boldsymbol{u}^\epsilon)}\,. \tag{48}$$

For $\boldsymbol{\epsilon} = -\boldsymbol{w}$ we obtain (23). We also get

$$\gamma(w') = \gamma(w)\gamma_\epsilon(u^\epsilon)(1 - \boldsymbol{\epsilon} \cdot \boldsymbol{u}^\epsilon) \tag{49}$$

with the generalized Lorentz factor

$$\gamma_\epsilon(u^\epsilon) := \frac{1}{\sqrt{(1 - (\boldsymbol{\epsilon} + \boldsymbol{w}) \cdot \boldsymbol{u}^\epsilon)^2 - (u^\epsilon)^2}}\,. \tag{50}$$

It is possible to display the transformation between the two arbitrarily synchronized systems S and S'. Since they are rather complicated and since we do not need these transformations we will not display them.

4.3 A preliminary calculation of the effects

As preparation for the derivation of synchronization independent effects, we first treat the usual effects in a “naive”way. At first we calculate the one-way velocity of light measured in the system S. We start with the equation characterizing light rays in Σ, $(T_\mathrm{r} - T_\mathrm{s})^2 = |\boldsymbol{X}_\mathrm{r} - \boldsymbol{X}_\mathrm{s}|^2$, where the indices s and r refer to “sender” and “receiver”, respectively. With the transformations (43), (44) we get

$$T_\mathrm{r} - T_\mathrm{s} = \gamma(w)\left(x_\mathrm{rs} + \boldsymbol{w} \cdot \boldsymbol{x}_\mathrm{rs}\right)\,, \tag{51}$$

where $\boldsymbol{x}_\mathrm{rs} = \boldsymbol{x}_\mathrm{r} - \boldsymbol{x}_\mathrm{s}$. With (43) the time of sending and reception of the signal is $t_\mathrm{r} = t_\mathrm{s} + \frac{1}{\gamma}(T_\mathrm{r} - T_\mathrm{s}) + \boldsymbol{\epsilon} \cdot \boldsymbol{x}_\mathrm{rs}$. With (51) we obtain

$$t_\mathrm{r} = t_\mathrm{s} + x_\mathrm{rs} + (\boldsymbol{w} + \boldsymbol{\epsilon}) \cdot \boldsymbol{x}_\mathrm{rs}\,. \tag{52}$$

The velocity of light propagating in the direction $\widehat{\boldsymbol{x}}_\mathrm{rs} = \boldsymbol{x}_\mathrm{rs}/|\boldsymbol{x}_\mathrm{rs}|$ is then given by

$$c_{\mathrm{one-way}}(\widehat{\boldsymbol{x}}_\mathrm{rs}) = \frac{x_\mathrm{rs}}{t_\mathrm{r} - t_\mathrm{s}} = \frac{1}{1 + (\boldsymbol{w} + \boldsymbol{\epsilon}) \cdot \widehat{\boldsymbol{x}}_\mathrm{rs}}\,, \tag{53}$$

which clearly depends on the synchronization. In the case $\boldsymbol{\epsilon} \neq -\boldsymbol{w}$, the velocity not only depends on the direction but also on the state of motion of the system S. Only in the case $\boldsymbol{\epsilon} = -\boldsymbol{w}$ the velocity of light is the same for all directions and for all velocities of the system S. Accordingly, by postulating the constancy of the velocity of light, one implicitely makes an assumption about the synchronization of the reference system.

Now we are calculating in the usual manner the known effects of SR. We always refer to (45). For the time dilation of the moving system S' with respect to S we get

$$t = \gamma_\epsilon(u^\epsilon)t'\,. \tag{54}$$

The time dilation depends on the synchronization. Thus, it is not symmetric against interchange of the observers.

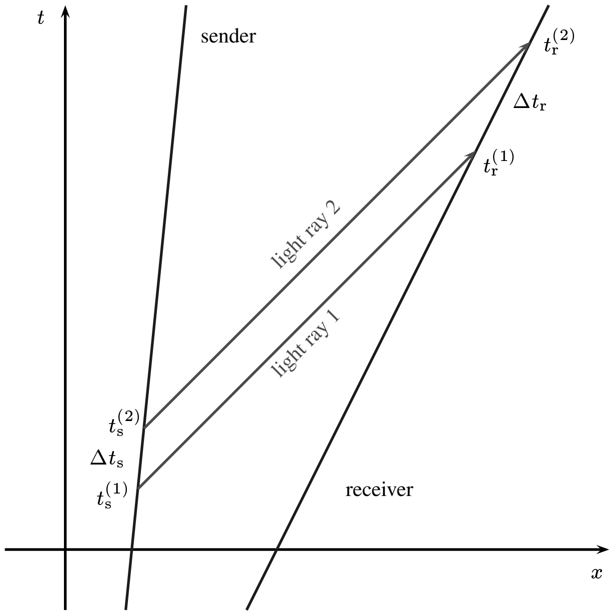
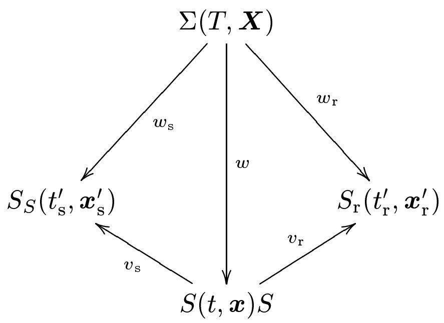


Fig. 13 (online colour at: www.ann-phys.org) For the calculation of the observed frequency $1/\Delta t_{\rm r}$ as function of the sent frequency $1/\Delta t_{\rm s}$.

Since the length depends on the spatial $t = \mathrm{const.}$ hypersurface which itself depends on the synchronization procedure, length contraction is no proper relativistic effect. Therefore we skip the discussion of length contraction here.

For the description of the Doppler effect, we refer to the reference systems outlined in the scheme



(55)

We describe two light rays that are sent at times $t_{\rm s}^{(1)}$ and $t_{\rm s}^{(2)}$ and reach the observer at times $t_{\rm r}^{(1)}$ and $t_{\rm r}^{(2)}$. The sender and reveiver are assumed to move with velocities $\boldsymbol{v}_{\rm s}$ and $\boldsymbol{v}_{\rm r}$ with respect to the system S. The latter moves with velocity $\boldsymbol{w}$ with respect to Σ. With $(t_s^{(1)}, \boldsymbol{x}_{\rm s}^{(1)})$ and $(t_{\rm s}^{(2)}, \boldsymbol{x}_{\rm s}^{(2)})$, as well as with $(t_{\rm r}^{(1)}, \boldsymbol{x}_r^{(1)})$ and $(t_{\rm r}^{(2)}, \boldsymbol{x}_{\rm r}^{(2)})$ as positions and times of sending and receiption of the signals we first get

$$\boldsymbol{x}_{\rm rs}^{(2)} - \boldsymbol{x}_{\rm rs}^{(1)} = \boldsymbol{x}_{\rm r}^{(2)} - \boldsymbol{x}_{\rm r}^{(1)} - \boldsymbol{x}_{\rm s}^{(2)} + \boldsymbol{x}_{\rm s}^{(1)} = \boldsymbol{v}_{\rm r}\Delta t_{\rm r} - \boldsymbol{v}_{\rm s}\Delta t_{\rm s}\,. \tag{56}$$

With (52) this yields

$$\Delta t_{\rm r} = t_{\rm r}^{(2)} - t_{\rm r}^{(1)} = \Delta t_{\rm s} + (\widehat{\boldsymbol{x}}_{\rm rs} + \boldsymbol{w} + \boldsymbol{\epsilon}) \cdot (\boldsymbol{v}_{\rm r}\Delta t_{\rm r} - \boldsymbol{v}_{\rm s}\Delta t_{\rm s})\,, \tag{57}$$

where we used $|\boldsymbol{x}_{\rm rs}^{(2)}| = |\boldsymbol{x}^{(1)}| + (\boldsymbol{x}_{\rm rs}^{(2)} - \boldsymbol{x}_{\rm rs}^{(1)})\boldsymbol{\nabla}|\boldsymbol{x}_{\rm rs}^{(1)}|$. This can be solved for $\Delta t_{\rm r}$

$$\Delta t_{\rm r} = \frac{1 - (\widehat{\boldsymbol{x}}_{\rm rs} + \boldsymbol{w} + \boldsymbol{\epsilon}) \cdot \boldsymbol{v}_{\rm s}}{1 - (\widehat{\boldsymbol{x}}_{\rm rs} + \boldsymbol{w} + \boldsymbol{\epsilon}) \cdot \boldsymbol{v}_{\rm r}}\Delta t_{\rm s}\,. \tag{58}$$

The time differences $\Delta t_{\rm s}$ and $\Delta t_{\rm r}$, still referring to the system S, have to be expressed by quantities measured in the moving systems $S_{\rm s}$ and $S_{\rm r}$ by

$$dt' = \frac{\gamma(v)}{\gamma(w)}(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{v})dt\,. \tag{59}$$

If we use this in (58) and introduce the observed and sent frequencies $\nu_{\rm r} = 1/\Delta t'_{\rm r}$ and $\nu_{\rm s} = 1/\Delta t'_{\rm s}$, then we obtain

$$\frac{\nu_{\rm s}}{\nu_{\rm r}} = \frac{\gamma(w_{\rm s})}{\gamma(w_{\rm r})}\,\frac{1 - \dfrac{(\widehat{\boldsymbol{x}}_{\rm rs} + \boldsymbol{w})\cdot\boldsymbol{v}_{\rm s}}{1 - \boldsymbol{\epsilon}\cdot\boldsymbol{v}_{\rm s}}}{1 - \dfrac{(\widehat{\boldsymbol{x}}_{\rm rs} + \boldsymbol{w})\cdot\boldsymbol{v}_{\rm r}}{1 - \boldsymbol{\epsilon}\cdot\boldsymbol{v}_{\rm r}}}\,. \tag{60}$$

The velocities $\boldsymbol{w}_{\rm s}$ and $\boldsymbol{w}_{\rm r}$ have to be calculated with the help of the law for the addition of velocities (48). In many cases one chooses $\boldsymbol{v}_{\rm s} = 0$ so that

$$\frac{\nu_{\rm r}}{\nu_{\rm s}} = \gamma_\epsilon(v_{\rm r})\left(1 - (\widehat{\boldsymbol{x}}_{\rm rs} + \boldsymbol{\epsilon} + \boldsymbol{w})\cdot\boldsymbol{v}_{\rm r}\right)\,. \tag{61}$$

In case of the Einstein synchronization, this reduces to the ordinary Doppler shift formula (25),

$$\frac{\nu_{\rm r}}{\nu_{\rm s}} = \gamma(v_{\rm r})\left(1 - \widehat{\boldsymbol{x}}_{\rm rs}\cdot\boldsymbol{v}_{\rm r}\right)\,. \tag{62}$$

As a result, both the time dilation as well as the Doppler shift depend on the chosen synchronization and are, thus, not applicable to a description of synchronization invariant experiments. We will see below that there are versions of these two effects which are indeed independent of any synchronization and which have been applied to real experiments. In the meantime we shortly discuss some synchronizations which can be realized by using some simple procedures.

4.4 Physically motivated synchronizations

4.4.1 Einstein-synchronization

In order to determine the coefficient $\boldsymbol{\epsilon}$ for the Einstein-synchronization we consider two clocks A and B which are at rest in a system S. This system S moves with a velocity $\boldsymbol{w}$ with respect to Σ. At $t = 0$, a signal is sent from A and arrives in B at $t = t_1$. This signal will be sent back immediately and reaches A at t_2, see Fig. 14. Einstein synchronization now requires (compare, e. g., [18])

$$t_2 = 2\,t_1\,. \tag{63}$$

According to the diagram (45) and the relations (43), (44) we represent the events E_1 and E_2 in the relations between S and Σ as well as in the relations between S' and Σ. Since the clock A is at rest in the moving system S, we have $\boldsymbol{x}_2 = 0$ and $\boldsymbol{X}_2 = \boldsymbol{w}T_2$. Therefore,

$$T_2 = \gamma(w)t_2\,. \tag{64}$$

From the equations for light propagation

$$|\boldsymbol{X}_1|^2 = T_1^2\,, \qquad |\boldsymbol{X}_2 - \boldsymbol{X}_1|^2 = (T_2 - T_1)^2\,, \tag{65}$$

we then obtain $(\boldsymbol{w} + \boldsymbol{\epsilon})\cdot\boldsymbol{x}_1 = 0$. Since this is true for all $\boldsymbol{x}_1$, we find

$$\boldsymbol{\epsilon} = -\boldsymbol{w}\,. \tag{66}$$

In this case the transformations (43), (44) reduce to the ordinary LT (19), (20).

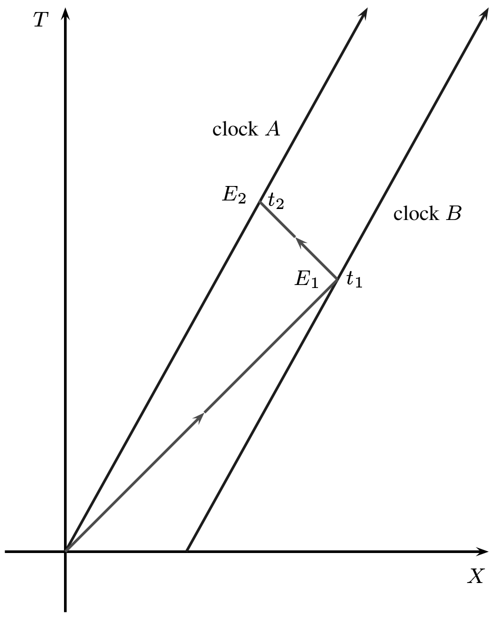


Fig. 14 (online colour at: www.ann-phys.org) The Einstein-synchronization: A and B are worldlines of two clocks at rest in a system which moves with respect to Σ. At $t = 0$, the observer A sends a light signal to B, where it arrives at time t_1. The signal sent back immediately arrives in A at time t_2. The Einstein-synchronization now requires $t_1 = \frac{1}{2} t_2$.

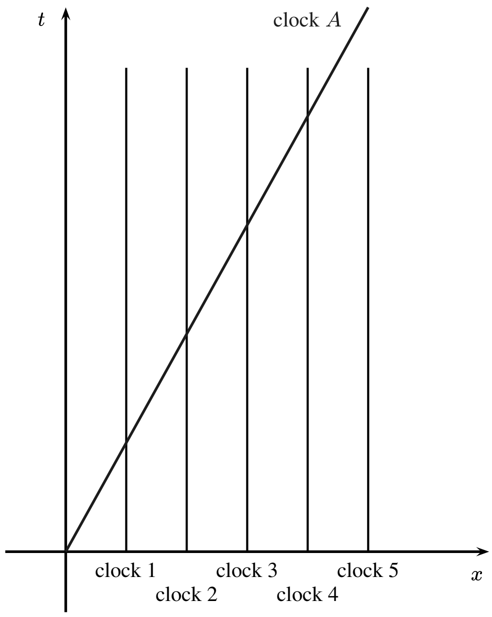


Fig. 15 (online colour at: www.ann-phys.org) Synchronization by using slow clocks: a clock A moves slowly in S and sets all clocks in S at its own time.

4.4.2 Slow clock transport

A clock is moving with small velocity with respect to the system S. By passing the clocks at rest in S, these clocks will be given the time of the slowly moving clock, see Fig. 15. According to the law of addition of velocities (48), the clock possesses the velocity

$$\boldsymbol{w}' = \boldsymbol{w} + \frac{\boldsymbol{u}^\epsilon - (1 - \sqrt{1 - w^2})\widehat{\boldsymbol{w}}(\boldsymbol{u}^\epsilon \cdot \widehat{\boldsymbol{w}})}{1 - \boldsymbol{\epsilon} \cdot \boldsymbol{u}^\epsilon} \approx \boldsymbol{w} + \boldsymbol{u}^\epsilon - (1 - \sqrt{1 - w^2})\widehat{\boldsymbol{w}}(\boldsymbol{u}^\epsilon \cdot \widehat{\boldsymbol{w}}) \,, \tag{67}$$

with respect to Σ, where we used that $\boldsymbol{u}^\epsilon$ is small. From (43) we get $T = \gamma(w')t'$ and $T = \gamma(w)\,(t + \boldsymbol{\epsilon} \cdot \boldsymbol{x})$. The above described synchronization procedure states $t' = t$, that is,

$$\frac{1}{\gamma(w')} T = \frac{1}{\gamma(w)} T - \boldsymbol{\epsilon} \cdot \boldsymbol{x} \,. \tag{68}$$

The velocity of the clock in the system Σ is $\boldsymbol{X} = \boldsymbol{w}' T$. We use this in (44) what, together with (67), yields

$$\boldsymbol{\epsilon} \cdot \boldsymbol{x} = -\left(\frac{1}{\gamma(w')} - \frac{1}{\gamma(w)} \right) \frac{\boldsymbol{w} \cdot \boldsymbol{x}}{\boldsymbol{u} \cdot \boldsymbol{w}} = -\left((\boldsymbol{w}' - \boldsymbol{w}) \cdot \boldsymbol{\nabla}_{\boldsymbol{w}} \frac{1}{\gamma(w)} \right) \frac{\boldsymbol{w} \cdot \boldsymbol{x}}{\boldsymbol{u} \cdot \boldsymbol{w}} = -\boldsymbol{w} \cdot \boldsymbol{x} \,. \tag{69}$$

As a consequence, this kind of synchronization again requires (66).

4.4.3 Fast clock transport

According to an idea of Salmon [19], it is possible to define a synchronization by using clocks which are moving with arbitrary velocity. We call this “synchronization by fast clock transport”.

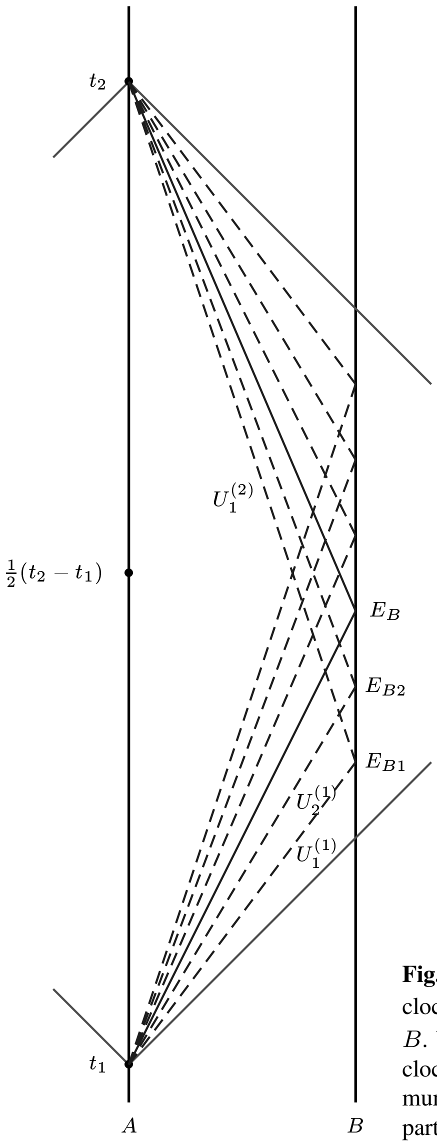


Fig. 16 (online colour at: www.ann-phys.org) Synchronization by fast clock transport. Numerous clocks $U_i^{(1)}$ start in A at time t_1 and move to B. Upon arrival, other clocks $U_i^{(2)}$ at B overtake the times of the arriving clocks and move back to A where these clock arrive at time t_2. The minimum time dilation characterizes a particular pair of clocks (solid lines). This particular synchronization assigns the time $\frac{1}{2}(t_2 - t_1)$ to the event E_B at B.

Again we have two worldlines A and B at a constant spatial distance. Both worldlines are equipped with clocks. At the moment t_1 a clock $U_1^{(1)}$ moves with a certain velocity from A to B where it arrives at t_{B1}. At that moment another clock $U_1^{(2)}$ with the time given by the arriving clock starts in B and moves with another velocity to A where it arrives at t_2. This procedure will be performed by numerous clocks with different velocities in such a way that at A all clocks start at t_1 and all clocks starting at B arrive A at t_2, see Fig. 16. The arriving clocks $U_i^{(2)}$ at A show a time t_i that is dilated with respect to $t_2 - t_1$. The rate of dilation depends on the velocity of $U_i^{(1)}$. We will show that there is a velocity which leads to a minimum dilation. The clock moving with that velocity defines a unique event E_B on B to which the time $\frac{1}{2}(t_2 - t_1)$ can be attributet. This again corresponds to $\boldsymbol{\epsilon} = -\boldsymbol{w}$.

From the time dilation we can relate the times of the moving clocks, $\Delta t'_{AB}$ and $\Delta t'_{BA}$ with the times in the system at rest,

$$\Delta t' = \Delta t'_{AB} + \Delta t'_{BA}$$

$$= \frac{1}{\gamma_\epsilon(v_1)}\Delta t_{AB} + \frac{1}{\gamma_\epsilon(v_2)}\Delta t_{BA}$$
$$= \left(\sqrt{\left(\frac{1}{v_1} - (\boldsymbol{\epsilon}+\boldsymbol{w})\cdot\boldsymbol{n}\right)^2 - 1} + \sqrt{\left(\frac{1}{v_2} + (\boldsymbol{\epsilon}+\boldsymbol{w})\cdot\boldsymbol{n}\right)^2 - 1}\right)\Delta x\,. \quad (70)$$

Here $\boldsymbol{n}$ is the spatial direction between A and B. The velocities $v_1 = \Delta x/\Delta t_{AB}$ and $v_2 = \Delta x/\Delta t_{BA}$, with $\Delta x = x_B - x_A$, yield

$$\frac{1}{v_1} + \frac{1}{v_2} = \frac{2}{v}\,. \quad (71)$$

The velocity v is independent of the synchronization. Thus,

$$\Delta t' = \left(\sqrt{\left(\frac{1}{v_1} - (\boldsymbol{\epsilon}+\boldsymbol{w})\cdot\boldsymbol{n}\right)^2 - 1} + \sqrt{\left(\frac{2}{v} - \frac{1}{v_1} + (\boldsymbol{\epsilon}+\boldsymbol{w})\cdot\boldsymbol{n}\right)^2 - 1}\right)\Delta x\,. \quad (72)$$

The velocity v_1 that minimizes this time dilation is given by

$$\frac{1}{v_1} = \frac{1}{v} + (\boldsymbol{\epsilon}+\boldsymbol{w})\cdot\boldsymbol{n}\,. \quad (73)$$

This uniquely defines an event E_B on the second worldline. Inserting this result into the time dilation (72) yields

$$\Delta t' = \left(\sqrt{\frac{1}{v^2} - 1} + \sqrt{\frac{1}{v^2} - 1}\right)\Delta x\,. \quad (74)$$

Thus, the particular clock which minimizes Δ moves both from A to B and back from B to A with the velocity v. The synchronization now assigns the time $\frac{1}{2}(t_2 - t_1)$ to the event E_B. Then $v_1 = v$ and $v_2 = v$. Hence, $\boldsymbol{\epsilon} = -\boldsymbol{w}$.

4.4.4 External synchronization

If we require all $t = const$ hypersurfaces to coincide with the $T = const$ hypersurface of the original Lorentz system, then $t \sim T$. This is possible only if

$$\boldsymbol{\epsilon} = 0\,. \quad (75)$$

This is called *external* synchronization [18]. This particular synchronization distinguishes an ether, a preferred frame. There is no unification of space and time. However, this theory is in agreement with all experimental tests of SR that, as stated above, should not depend on the synchronization.

4.5 Synchronization independent effects

4.5.1 Two-way velocity of light

It is easy to show that with the one-way velocity of light, defined in (53), the two-way velocity of light defined by

$$\frac{2}{c} = \frac{1}{c_{\text{one-way}}(\widehat{\boldsymbol{x}}_{ES})} + \frac{1}{c_{\text{one-way}}(-\widehat{\boldsymbol{x}}_{ES})} \quad (76)$$

is independent from the synchronization.

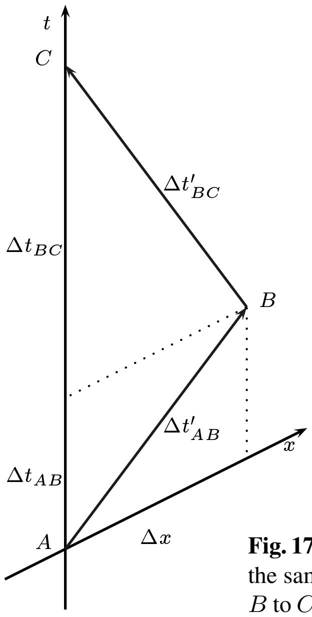


Fig. 17 (online colour at: www.ann-phys.org) The twin paradox: A clock which in A shows the same time as the clock at rest, moves from A to B and sets the clock which moves from B to C. The time shown by the second moving clock is behind the clock which stayed at rest.

4.5.2 Synchronization invariant velocities

Now we present the synchronization independent counterpart of the time dilation and Doppler effect derived above. For that purpose we again define synchronization independent velocities. As in the one-dimensional case discussed above, the difference of the inverse of velocities turns out to be synchronization independent. For the velocity of light $c_{\text{one-way}}(\boldsymbol{n})$ and the velocity of a particle $v^\epsilon(\boldsymbol{n})$, both moving in the same direction $\boldsymbol{n}$,

$$\frac{1}{v^\epsilon(\boldsymbol{n})} - \frac{1}{c_{\text{one-way}}(\boldsymbol{n})} \tag{77}$$

is independent from the synchronization. Using (53) this can be identified with a synchronization independent velocity V by

$$\frac{1}{v^\epsilon_\pm(\boldsymbol{n})} - (1 \pm (\boldsymbol{w} + \boldsymbol{\epsilon}) \cdot \boldsymbol{n}) = \frac{1}{V} - 1\,. \tag{78}$$

For $\boldsymbol{\epsilon} = -\boldsymbol{w}$, we have $V = v^\epsilon(\boldsymbol{n})$. Also the synchronization-dependent Lorentz factor can be reformulated:

$$\gamma_\epsilon(u) = \frac{1}{\sqrt{(1 - (\boldsymbol{\epsilon} + \boldsymbol{w}) \cdot \boldsymbol{v}^\epsilon)^2 - (v^\epsilon)^2}} = \frac{V}{v^\epsilon} \frac{1}{\sqrt{1 - V^2}}\,. \tag{79}$$

4.5.3 The twin paradox

Though the time dilation (54) is synchronization dependent, the twin paradox is not. A clock is moving from A to B and sets a clock which is moving from B to C, see Fig. 17.

We take two clocks with velocities $\boldsymbol{v}_1 = v_1 \boldsymbol{n}$ and $\boldsymbol{v}_2 = -v_2 \boldsymbol{n}$, with $v_1, v_2 \geq 0$. In S the duration between A and B is t_{AB} and the time given by the moving clocks after arrival is then

$$\Delta t'_{AB} + \Delta t'_{BC} = \frac{1}{\gamma_\epsilon(v_1)} \Delta t_{AB} + \frac{1}{\gamma_\epsilon(v_2)} \Delta t_{BC} = \sqrt{1 - V_1^2} \frac{\Delta x}{V_1} + \sqrt{1 - V_2^2} \frac{\Delta x}{V_2}\,, \tag{80}$$

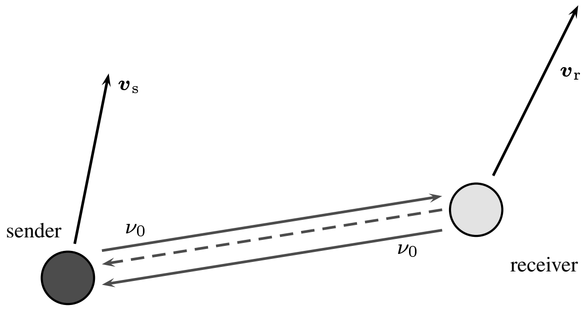


Fig. 18 (online colour at: www.ann-phys.org) The setup for the synchronization independent Doppler effect. The sender as well as the receiver are equipped with identical clocks, both emitting signals of frequency ν_0. In addition, the receiver sends back the signal it receives from the sender.

where we used (78). If the restrict to $V_1 = V_2 = V$, this yields

$$\Delta t'_{AC} = \Delta t'_{AB} + \Delta'_{BC} = 2\sqrt{1 - V^2}\frac{\Delta x}{V} = \sqrt{1 - V^2}\Delta t_{AC} \,, \tag{81}$$

with $\Delta t_{AC} = 2\Delta x/V$. This is a synchronization independent expression. Therefore, the twin paradox is a "true" manifestation of relativistic physics.

4.5.4 Synchronization invariant Doppler effect

According to the experiments performed there are at least two synchronization independent realizations of the Doppler effect. In both cases one adds a second measured quantity that at the end can be used to eliminate the synchronization parameter which is present in the "naive" effects described above. One may also consider the additional measurements as additional procedures to establish a certain synchronization in a clever way. However, in any case the synchronization is not introduced explicitly according to the way described above.

The synchronization independent Doppler ranging In order to obtain a synchronization independent Doppler effect, one uses three frequencies. For that purpose the sender as well as the receiver is equipped with two identical species of clocks, both establishing the same frequency ν_0 in their rest frame[1]. Now, the sender emits a signal with frequency ν_0 which the receiver observes as frequency $\nu_{\rm r}$. This is sent back with the same frequency $\nu_{\rm r}$ which the sender observes as frequency ν''. In addition, the receiver sends another signal with frequency ν_0 related to the clock he carries with him. This signal will be observed by the sender as possessing the frequency ν', see Fig. 18. This procedure, which is used for Doppler tracking, allows to eliminate the synchronization parameter.

With the frequencies ν' and ν'', which are measured at the position of the sender only, we define the two-way Doppler-signal, see e. g. [20]

$$D := \frac{\nu'' - \nu_0}{\nu_0} \tag{82}$$

and the redshift signal

$$R := \frac{\nu' - \nu_0}{\nu_0} - \frac{1}{2}D \,. \tag{83}$$

[1] Here we have to make an additional assumption about the transport of clocks: We assume that clocks do not intrinsically change their frequency during transport into another frame. However, this does not interfere with the issue of synchronization. Our assumption is about the frequency, synchronization makes statements about the setting of clocks. Two points have to be checked in experiments: (i) The frequency of clocks is not allowed to depend on the history of the clock. This can be tested by first separating two clocks and then bringing them together again and compare the ticking rates. Nothing of this kind has been observed. (ii) the frequency does not depend on the frame (this would violate the relativity principle). (iii) The transport of clocks to other inertial frames has to be accomplished using accelerations. It has been experimentally verified with high accuracy that clocks, in particular atomic clocks, are very insensitive against accelerations.

Both quantities, which we have to calculate, depend only on frequencies which are measured at the sender and, thus, do not depend on any synchronization.

Using (60) twice we get

$$\nu'' = \frac{1 - (\widehat{\boldsymbol{x}}_{\rm rs} + \boldsymbol{\epsilon} + \boldsymbol{w}) \cdot \boldsymbol{v}_{\rm r}}{1 - (-\widehat{\boldsymbol{x}}_{\rm rs} + \boldsymbol{\epsilon} + \boldsymbol{w}) \cdot \boldsymbol{v}_{\rm r}} \, \nu_0 \, . \tag{84}$$

where at the end we set $\boldsymbol{v}_{\rm s} = 0$. Then

$$D = -2 \, \frac{\widehat{\boldsymbol{x}}_{\rm rs} \cdot \boldsymbol{v}_{\rm r}}{1 - (-\widehat{\boldsymbol{x}}_{\rm rs} + \boldsymbol{\epsilon} + \boldsymbol{w}) \cdot \boldsymbol{v}_{\rm r}} \, . \tag{85}$$

For the redshift signal we use again (60) and get for $\boldsymbol{v}_S = 0$

$$R = \frac{\sqrt{(1 - (\boldsymbol{\epsilon} + \boldsymbol{w}) \cdot \boldsymbol{v}_{\rm r})^2 - v_{\rm r}^2} - 1 + (\boldsymbol{w} + \boldsymbol{\epsilon}) \cdot \boldsymbol{v}_{\rm r}}{1 - (-\widehat{\boldsymbol{x}}_{\rm rs} + \boldsymbol{\epsilon} + \boldsymbol{w}) \cdot \boldsymbol{v}_{\rm r}} \, . \tag{86}$$

In this form D as well as R seems to depend on the synchronization $\boldsymbol{\epsilon}$. However, this dependence is only fictitious. We use D from (85) in order to determine $\boldsymbol{v}_{\rm r}$. Since we have a three-dimensional setup, we need three independent directions $\boldsymbol{n}_i$, $i = 1, 2, 3$, and get

$$D_i = -2 \, \frac{\boldsymbol{n}_i \cdot \boldsymbol{v}_{\rm r}}{1 - (-\boldsymbol{n}_i + \boldsymbol{\epsilon} + \boldsymbol{w}) \cdot \boldsymbol{v}_{\rm r}} \tag{87}$$

In principle we can solve this for $\boldsymbol{v}_{\rm r}$ and insert this into R. However, we use this synchronization-independent D_i in order to introduce a synchronization independent velocity $\boldsymbol{V}$ via

$$D_i =: -2 \, \frac{\boldsymbol{n}_i \cdot \boldsymbol{V}_{\rm r}}{1 + \boldsymbol{n}_i \cdot \boldsymbol{V}_{\rm r}} \, . \tag{88}$$

Eqs. (88) and (87) can now be solved for $\boldsymbol{v}_{\rm r}$

$$\boldsymbol{v}_{\rm r} = \frac{\boldsymbol{V}_{\rm r}}{1 + (\boldsymbol{w} + \boldsymbol{\epsilon}) \cdot \boldsymbol{V}_{\rm r}} \, , \tag{89}$$

which will be used in R,

$$R = \frac{\sqrt{1 - \boldsymbol{V}_{\rm r}^2} - 1}{1 + \widehat{\boldsymbol{x}}_{\rm rs} \cdot \boldsymbol{V}_{\rm r}} \, . \tag{90}$$

This result which is of the usual form does not depend on $\boldsymbol{\epsilon}$. Note that $\boldsymbol{V}$ is not the directly measured velocity, it is the velocity indirectly inferred from the two-way Doppler signal. In order to get a relation between directly measured quantities only, we can replace $\boldsymbol{V}_{\rm r}$ by D_i and find

$$R = \left(\sqrt{1 - \sum_i \frac{D_i^2}{(2 + D_i)^2}} - 1 \right) \Bigg/ \left(1 - \sum_i \frac{D_1}{2 + D_1} \right) \, . \tag{91}$$

This is the synchronization-independent Doppler-effect. This and *only* this particular dependence of R from D is specific for SR. Since this is a relation between measured quantities, it is an invariant manifestation of special relativistic physics. For non-relativistic physics, the relation between D and R is different.

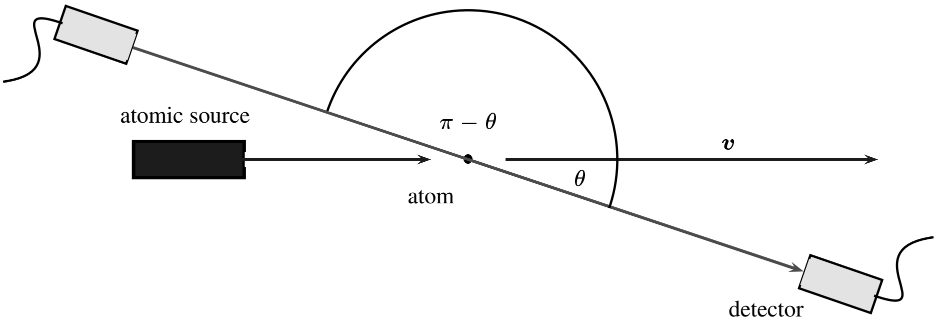


Fig. 19 (online colour at: www.ann-phys.org) The scheme of the experiment with 2-level systems as designed by Ives and Stilwell. Atoms in the excited state move with a velocity $\boldsymbol{v}$ and decay into the ground state by emitting electromagnetic radiation. In order to avoid any synchronization dependence, the frequency of the emitted radiation has been measured for two opposite directions, namely for $\theta = 0$ and $\theta = \pi$, that is, in the direction parallel and antiparallel to the velocity $\boldsymbol{v}$.

Two photon Doppler effect In this setup the ticks of a clock (or the frequency of the radiation of a atom) moving with respect to an inertial observer is measured simultaneously in opposite directions. In most cases the direction is given by the motion of the atom, see Fig. 19.

If the radiation emitted by the moving atom is detected in the direction of flight and in the opposite direction, then the measured frequencies ν_+ and ν_- are given by (61)

$$\nu_- = \gamma_\epsilon(v)\,(1 - (\widehat{\boldsymbol{x}}_{\rm rs} + \boldsymbol{\epsilon} + \boldsymbol{w}) \cdot \boldsymbol{v}_{\rm r})\,\nu_0\,, \tag{92}$$

$$\nu_+ = \gamma_\epsilon(v)\,(1 - (-\widehat{\boldsymbol{x}}_{\rm rs} + \boldsymbol{\epsilon} + \boldsymbol{w}) \cdot \boldsymbol{v}_{\rm r})\,\nu_0\,, \tag{93}$$

which clearly depends on the synchronization. With $\gamma_\epsilon(v)$ from (50) we get

$$\nu_-\nu_+ = \gamma_\epsilon^2(v)\,(1 - (\widehat{\boldsymbol{x}}_{\rm rs} + \boldsymbol{\epsilon} + \boldsymbol{w}) \cdot \boldsymbol{v}_{\rm r})\,(1 - (-\widehat{\boldsymbol{x}}_{\rm rs} + \boldsymbol{\epsilon} + \boldsymbol{w}) \cdot \boldsymbol{v}_{\rm r})\,\nu_0^2 = \nu_0^2\,, \tag{94}$$

which is a synchronization independent relation. Again, this and *only* this relation for the two frequencies is characteristic for SR. This relation should hold for all velocities. Though the notion of a velocity depends on the synchronization, we can use the measured frequencies to define an invariant velocity by

$$V := \frac{\nu_- - \nu_+}{\nu_- + \nu_+}\,. \tag{95}$$

This leads to

$$V = \frac{\widehat{\boldsymbol{x}}_{\rm rs} \cdot \boldsymbol{v}_{\rm r}}{1 - (\boldsymbol{\epsilon} + \boldsymbol{w}) \cdot \boldsymbol{v}_{\rm r}}\,. \tag{96}$$

The synchronization invariant statement now is that (94) should hold for all velocities defined in (95). Similar results can be achieved for the description of tests using the technique of two-photon spectroscopy.

5 Lorentz Invariance and Special Relativity

Einstein derived the Poincaré transformations from two postulates: the constancy of the speed of light and the relativity principle. The LT constitute the homogenous part of the Poincaré transformations. Though physics, due to GR, is no longer invariant against the translation part in the Poincaré transformation, it still is locally invariant with respect to the homogenous part, namely the LT. We have to restrict to local LI since, again due to GR, the characteristic length scale of the experimental setup has to be smaller than

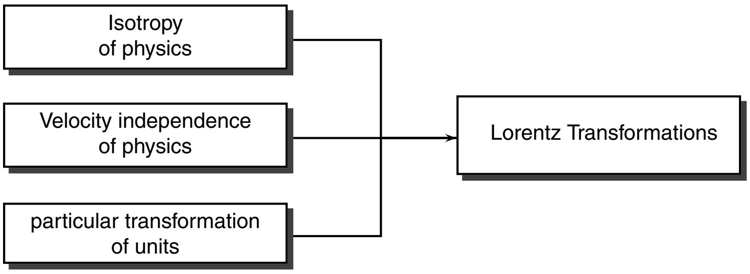


Fig. 20 Special Relativity is a consequence of the orientation and velocity independence of all experiments together with a statement about the relation of units in moving laboratories.

the length scale at which the curvature of space-time, or equivalently, the gravity gradient which cannot be transformed away, may influnce the experimental result. For example, if resonators, which are used for tests of the isotropy of the speed of light, are too large, then, even in free fall, the gravity gradient induces a distortion of the cavity which leads to an effect which looks like an anisotropy of the speed of light. This can be overcome only by making the resonator smaller (or by numerical compensation).

The constancy of the speed of light in the postulates means its independence from the orientation and from the velocity of the source and of the laboratory, and furthermore from its frequency and polarization (in vacuum). The relativity principle is a universality principle: it states that the results of experiments carried through in isolated laboratories in inertial frames of reference are independent from the orientation and the velocity of the laboratory. This should be true for *all* experiments, not only for those considering light rays. This means that for *all* physical phenomena the principle of LI has to be confirmed experimentally. This has the same logical status as tests of the weak equivalence principle where also for all types of test matter (including interactions) the universality of free fall has to be examined. Since the velocity independence of the outcome of experiments leads (except in the case of exotic synchronizations [18]) to a mixing of time and space with the consequence that the spatial axes of moving systems are realizations of quite different sets of events, there is a need to fix the units in the moving frame. This is done with the time dilation formula. Consequently, we have the scheme in Fig. 20 for the exploration of the physical content of the LT which also holds for kinematic and dynamics test theories.

We also address the question what happens if an experiment shows a violation of LI. This does not necessarily mean that one indeed has found a violation of LI. This effect may also be a result of a new interaction. This means that one first has to search whether this effect can be shielded or whether one can find a cause of this effect in the sense of a source which creates a field causing this effect, or whether this effect is universal or depends on the probe. In the first two cases the effect can be considered to be caused by a new interaction. In the latter case this new effect can be regarded to be of geometrical origin due to its universality. However, in this case the new gravitational interaction may violate LI, too, as it is the case in Finsler geometry, for example. Only if all these questions are answered appropriately one can speak about a violation of LI.

We also should mention that there is a difference between LI and Lorentz covariance. LI means that the result of an experiment, when prepared and performed under indentical conditions, is independent of the constant orientation and constant velocity of the laboratory with respect to an inertial system (the laboratory itself then constitutes an inertial system). Since the outcome is the same, the physical laws must have the same form in all inertial systems. This is equivalent to that the laws have to be transformed in a covariant way between the various inertial systems.

6 The importance of Lorentz Invariance

Though the LT can be derived mainly be considering the behaviour of light, the relativity principle makes these transformations universally applicable. That means, LI is a theoretical frame for all other theories of physics. In taking GR into account, LI has to be restricted to be applicable only locally. Beside that, LI is needed in order to successfully interpret the experimental data from molecular, atomic and nuclear spectroscopy. High energy physics is not possible without the use of the results of LI. In astrophysics, VLBI which gave the most precise observations of astrophysical phenomena works only by taking all effects of LI into account, besides GR effects, of course.

Not only in physics but also in daily life, the validity and use of LI (and of GR) is indispensible. The most well known application is positioning. Today's Global Positioning System, or later the European Galileo system, will give huge errors of more than 2 km per day when LI is not included in the processing of the signals (another 10 km error comes in if GR effects are not taken ionto account). The main effect in the relation between the time τ of a moving clock and the geocentric coordinate time t is described by

$$\Delta\tau = \left(1 - \frac{U(x_1) - U(x_0)}{c^2} - \frac{1}{2}\frac{v^2}{t^2} + \boldsymbol{\omega}\cdot\frac{d\boldsymbol{A}}{dt}\right) dt\,, \tag{97}$$

where $U(x)$ is the gravitational potential at position x including the mass quadrupole field of the earth and the centrifugal potential, v is the velocity of the clock with respect to the Earth, $\boldsymbol{\omega}$ the angular velocity of the Earth and $\boldsymbol{A}$ is the area a vector sweeps out from the center-of-mass of the Earth to the clocks [21,22]. This formula is built into the programs used in the GPS receivers, see [23] for a review. The importance is becoming clear by noting the numbers: While clocks today have a relative precision of the order 10^{-15}, the above contributions to GPS or Galileo satellites yield the following relative effects: gravitational potential $\sim 10^{-10}$, gravitational quadrupole $\sim 10^{-13}$, centrifugal force $\sim 10^{-12}$, velocity $\sim 10^{-10}$. For an orbit of a clock around the Earth, the Sagnac efect gives a time difference of $\sim 10^{-7}$. All these effects are much bigger that the present accuracy of clocks and, thus, have to be taken into account.

Not so well known but equally important is the consideraion of LI in the definition of the International Atomic Time TAI. This time is obtained by comparison, averaging an weighting the various clocks on Earth, mainly the clocks at the national bureaus of standards like the PTB (Germany), BIPM (France), NIST (USA), etc. All these clocks are on a different height above the geoid (surface of constant gravitational potential of the Earth) and move with different velocities due to the different geographical latitude. The comparison of clocks on the rotating Earth also needs to take into account the Sagnac effect. Therefore, Eq. (97) also applies to this comparison. Clocks are such precise so that, only by including LI, the observation of the rotation rate of the Earth reveals effects due to climatic changes.

Another application, which is of major importance for practical life, is today's definition of physical units, like the meter, the ohm and the volt. They directly rely on LI. The meter, for example, is the length light travels within the 299 792 458th part of a second. If the speed of light proves to be not isotropic, then the unit of length will depend on the direction. The ohm and the volt are operationally defined by means of the Quantum Hall and the Josephsen effect, which are described and interpreted by using standard Maxell and Schrödinger theory. This has to be modified if LI turns out to be violated.

7 Violations of Lorentz Invariance?

The search for a theory of quantum gravity is one of the important issues of today's theoretical as well as experimental physics [24,25]. There is a need for a new theory combining GR and quantum theory because both have been proven to be incompatible. This new theory may modify either GR or quantum theory or both. If GR is going to be modified, then also LI may have to undergo a modification since local LI is at the very basis of GR. Indeed, most of the approaches towards a quantum gravity theory predict not only tiny violations of, e. g., the equivalence principle, or deviations of PPN parameters from their Einsteinian values, but also a tiny violation of LI.

7.1 Main quantum gravity schemes

There are three main directions in the search for a theory of quantum gravity: (i) canonical, in particular loop quantum gravity, (ii) string theory, and (iii) non-commutative geometry. Each of these have characteristic features in their "predictions" of violations of standard physics.

- Canonical quantum gravity and in particular loop quantum gravity, being a scheme of quantizing the ordinary Einstein field equation and, thus, still emphasizing the geometrical nature of gravity, does not predict additional interactions but instead deviations from the ordinary Maxwell and Dirac equations in terms of higher derivatives and in terms of violations of LI which may arise from a spontaneous breaking of the underlying Lorentz symmetry. Many considerations predict such features [26–29] but there are also claims that loop gravity is still Lorentz invariant [30]: the hitherto found violations of LI are claimed to be due to particular choices of the quasiclassical states or result from a choice of the boundary conditions.

  The first order modifications of basic dynamical equations like the Maxwell or Dirac equations have the form

  $$(\eta^{\mu\rho}\eta^{\nu\sigma} + \chi^{\mu\nu\rho\sigma})\partial_\nu F_{\rho\sigma} + \chi^{\mu\rho\sigma} F_{\rho\sigma} = 4\pi j^\mu \, , \tag{98}$$

  where $\chi^{\mu\nu\rho\sigma}$ and $\chi^{\mu\rho\sigma}$ are tensors introducing frame dependent terms. In certain models these tensors are the result of spontaneous symmetry breaking. A first order modification of the Dirac equation is given by

  $$0 = i\gamma^\mu \partial_\mu \psi + M\psi \, , \tag{99}$$

  where the matrices γ^μ are not assumed to fulfill a Clifford algebra, rather $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = \frac{1}{2}\mathrm{tr}(\gamma^\mu\gamma^\nu) + X^{\mu\nu}$, and M is an arbitrary matrix. Here the $X^{\mu\nu}$ and the tracefree parts of M induce frame dependent effects. The $\chi^{\mu\nu\rho\sigma}$ as well as the $X^{\mu\nu}$ lead to birefringence and anisotropies in the characteristic cones [31–33]. Higher order deviations from ordinary equations consists in higher order derivatives and non-linearities. The parts that can be derived within a Lagrangian framework constitute the Standard Model Extension (SME) worked out by Kostelecký and co-workers [34, 35, 37].

- String theory in higher dimensions always predict many other fields which couple in different ways to the various matter sectors. Therefore, many types of additional interactions appear that lead to a violation of LI, of the Universality of Free Fall and the Universality of the Gravitational Redshift. In a string theory motivated dilaton scenario [36] deviations were predicted from the Universality of Free Fall at the order 10^{-13} and deviations of the PPN parameters γ and β at the order 10^{-5} and 10^{-9}, respectively. As far as the violation of LI is concerned, the main effects are collected in the universal phenomenological framework of the SME [34, 35, 37]. However, no specific predictions have been made in this case.

- In non-commutative geometry, translation and LI obviously is broken leading to some non-local features. This means that – in terms of partial differential equations – higher order derivatives may occur. This, in turn, leads to anomalous dispersion relations with higher order powers in the momentum and energy. The most simple example for such an anomalous dispersion relation is (see, e. g. [38, 39])

  $$\boldsymbol{p}^2 = E^2 \left(1 + \eta \, \frac{E}{E_{\mathrm{QG}}}\right) \, , \tag{100}$$

  where E_{QG} represents the quantum gravity energy scale which is of the order 10^{28} eV and η is a factor of the order 1 describing the strength of the quantum gravity modification.

  Furthermore, it has been shown in [40] that field theory in non-comuutative space-time leads to tiny violations of LI which can be considered as being part of the Standard Model Extension.

7.2 The magnitude of quantum gravity effects

Though many modern high precision devices are available for searching for new effects, the new effects that are expected to be induced by quantum gravity are assumed to be extremely small: Since the typical laboratory energies are of the order of 1 eV and the quantum gravity energy scale is assumed to be of the order of the Planck energy, which is about 10^{28} eV, the quantum gravity effects in laboratory experiments are likely to be of the order of 10^{-28}, which looks very unlikely to be accessible in laboratory experiments. Nevertheless, there are several reasons for pursuing this way of thinking:

- Since there is no theory of quantum gravity available that is worked out explicitely, all statements regarding the quantum gravity energy scale of 10^{28} eV have the status of a *speculation* only. Nobody knows about the "true" energy scale of the final quantum gravity theory.
- There could be sometimes mechanisms at work which magnify the quantum gravity induced effects by means of some multipliers. In theories leading to deviations from Newton's law at small distances, for example, the assumption of higher dimensions introduces additional constants which enhances the effect, see, e. g., [41]. Another example is the effect of quantum gravity induced fluctuations in interferometers. In the framework of models for such fluctuations, the magnitude of these fluctuations increase for small frequencies, that is, for long measurement times ($1/f$–noise) [42]. Therefore, searching for noise in high precision long-term stable devices (like optical resonators) may give new access to this domain of quantum gravity effects [43]. Further examples of this kind are the already mentioned predictions for a violation of the Universality of Free Fall and the deviation of PPN parameters from their Einsteinian values.
- There are ideas that the unification of the electroweak and strong interactions with the gravitational interaction will not occur at the Planck scale but at considerably lower energies, namely 10^{16} GeV which is three orders smaller than the Planck energy, see e. g. [44]. Therefore it is not the Planck energy we have to compare with the laboratory energies but rather the GUT energy scale of 10^{16} GeV.
- Using very high precision devices, it might be possible even in laboratory experiments to achieve a sensitivity that approaches, at least in principle, the 10^{-28} range. Such devices are gravitational wave interferometers, for example. Today's gravitational wave interferometers have a strain sensitivity of about $\Delta L/L \approx 10^{-22}$. Advanced LIGO is planned to reach the 10^{-24} level. For a periodic signal and a measurement time one can be better than 10^{-27}. Since this accuracy is related to phase sensitivity in an interferometric setup, and phase shifts can also be achieved by (100) using two beams of different frequencies, the modification in (100) might be accessible at the 10^{-27} level [39]. However, technical problems, as the frequency dependent properties of optical elements, might cause major problems. Nevertheless, it is interesting to note that, at least in principle, there is a "laboratory approach" to Planck scale effects.

Putting all these facts together it seems mandatory to search for quantum gravity induced effects. *All kinds of experimental tests should be considered and tried to be improved.*

Acknowledgements For ongoing collaboration and enlightening discussions I would like to thank M. Carrier, H. Dittus, F. Hehl, A. Kostelecký, A. Macías, B. Mashhoon, H. Müller, A. Peters, G. Schäfer and S. Schiller. Financial support of the German Space Agency DLR is acknowledged.

References

[1] A. Einstein, Ann. Phys. (Leipzig) **17**, 891 (1905).
[2] G. Schäfer, Ann. Phys. (Leipzig) **14**, this issue (2005).
[3] H. Müller, S. Herrmann, C. Braxmaier, S. Schiller, and A. Peters, Phys. Rev. Lett. **91**, 020401 (2003).
[4] P. Wolf, S. Bize, A. Clairon, A.L. Luiten, G. Santarelli, and M.E. Tobar, Phys. Rev. Lett. **90**, 060402 (2003).

[5] S. Saathoff, S. Karpuk, U. Eisenbarth, G. Huber, S. Krohn, R. Muñoz-Horta, S. Reinhardt, D. Schwalm, A. Wolf, and G. Gwinner, Phys. Rev. Lett. **91**, 190403 (2003).
[6] H. Bateman, Proc. Lond. Math. Soc. **8**, 223 (1910).
[7] E.W. Morley and D.C. Miller, Philos. Mag. **8**, 753 (1904).
[8] E.W. Morley and D.C. Miller, FitzGerald–Lorentz-effect, Philos. Mag. **9**, 680 (1905).
[9] R.J. Kennedy and E.M. Thorndike, Phys. Rev. **42**, 400 (1932).
[10] F.T. Trouton and H.R. Noble, Philos. Trans. R. Soc. Lond. A **202**, 165 (1904).
[11] W. v. Ignatowski, Phys. Z. **11**, 972 (1910).
[12] W. v. Ignatowski, Phys. Z. **12**, 779 (1911).
[13] P. Frank and H. Rothe, Ann. Phys. (Leipzig) **34**, 825 (1911).
[14] P. Frank and H. Rothe, Phys. Z. **13**, 750 (1912).
[15] J.A. Winnie, Philos. Sci. (USA) **37**, 81 (1968).
[16] J.A. Winnie, Philos. Sci. (USA) **37**, 223 (1968).
[17] F.W. Hehl and W.-T. Ni, Phys. Rev. D **42**, 2045 (1990).
[18] R. Mansouri and R.U. Sexl, Gen. Relativ. Gravit. (USA) **8**, 497 (1977).
[19] W.C. Salmon, Philos. Sci. (USA) **36**, 44 (1969).
[20] C.M. Will, Phys. Rev. D **45**, 403 (1992).
[21] G. Schäfer, Vernetzte Zeit in der Beobachtung von Doppelsternpulsaren, in: Mitteilungen des Bundesamtes für Kartographie und Geodäsie, vol. 5 (3. DFG-Rundgespräch zum Thema Bezugssysteme) (1999), p. 84.
[22] C. Audoin and B. Guinot, The Measurement of Time: Time, Frequency, and the Atomic Clock (Cambridge University Press, Cambridge, 2001).
[23] N. Ashby, Living Rev. Relativ. **6** (1), 2003; online article, available at http://www.livingreviews.org/lrr-2003-1.
[24] C. Kiefer, D. Giulini, and C. Lämmerzahl, editors, Quantum Gravity – From Theory to Experimental Search, Lecture Notes in Physics, vol. 631 (Springer-Verlag, Berlin, 2003).
[25] C. Kiefer, Quantum Gravity (Oxford University Press, Oxford, 2004).
[26] R. Gambini and J. Pullin, Phys. Rev. D **59**, 124021 (1999).
[27] J. Alfaro, H.A. Morales-Tecotl, and L.F. Urrutia, Phys. Rev. Lett. **84**, 2318 (2000).
[28] J. Alfaro, H.A. Morales-Tecotl, and L.F. Urrutia, Phys. Rev. D **65**, 103509 (2002).
[29] J. Alfaro, H.A. Morales-Tecotl, and L.F. Urrutia, Phys. Rev. D **66**, 124006 (2002).
[30] C.N. Kozameh and M.F. Parisi, Class. Quantum Gravity **21**, 4063 (2004).
[31] C. Lämmerzahl, Class. Quantum Gravity **14**, 13 (1998).
[32] F.W. Hehl and Y. Obukhov, Foundations of Electrodynamics (Birkhäuser, Boston, 2003).
[33] C. Lämmerzahl and F.W. Hehl, Phys. Rev. D **70**, 105022 (2004).
[34] D. Colladay and V.A. Kostelecký, Phys. Rev. D **58**, 116002 (1998).
[35] A. Kostelecký and M. Mewes, Phys. Rev. D **66**, 056005 (2002).
[36] T. Damour, F. Piazza, and G. Veneziano, Phys. Rev. Lett. **89**, 081601 (2002).
[37] D. Colladay and V.A. Kostelecký, Phys. Rev. D **55**, 6760 (1997).
[38] G. Amelino-Camelia, J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, and S. Sarkar, Nature **393**, 763 (1998).
[39] G. Amelino-Camelia and C. Lämmerzahl, Class. Quantum Gravity **21**, 899 (2004).
[40] S.M. Carroll, J.A. Harvey, V.A. Kostelecký, C.D. Lane, and T. Okamoto, Phys. Rev. Lett. **87**, 141601 (2001).
[41] I. Antoniadis, Physics with large extra dimensions and non-newtonian gravity at sub-mm distances, in: Quantum Gravity – From Thory to Experimental Search, edited by D. Giulini, C. Kiefer, and D. Lämmerzahl (Springer-Verlag, Berlin, 2003), p. 337.
[42] G. Amelino-Camelia, Phys. Rev. D **62**, 0240151 (2000).
[43] S. Schiller, C. Lämmerzahl, H. Müller, C. Braxmaier, S. Herrmann, and A. Peters, Phys. Rev. D **69**, 027504 (2004).
[44] M.J. Duff, A layman's guide to M-theory, hep-th/9805177.