Forced stationary gas-structure interaction vibrations in a rectangular parallelepiped tank

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A closed rigid rectangular parallelepiped tank is filled with gas and a part of one of its walls is a thin linearly elastic rectangular plate. The problem about the determination of the forced stationary vibrations of the obtained gas-structure interaction system under the action of a source is considered. The method of the crossed strips of Warburton and the Bubnov-Galerkin method are used to create a method of investigation of the dynamic behavior of this gas-structure interaction system.

1 Introduction

The rectangular thin elastic plates are often used as structural components closing or covering parallelepiped cavities filled with gas and subjected to dynamic loads. Such mechanical systems are applicable as covers of different tanks in chemical industry, as outside skin plates of supersonic air crafts, in the glass-skin technology of tall buildings, etc. In [1] the method of Bubnov-Galerkin together with the method of the crossed strips of G. Warburton is used and it is elaborated in the form of an easy scheme for application to the dynamic problem about the stationary vibrations of a special fluid-structure interaction system. It consists of a thin elastic plate, inserted into a rectangular orifice of an arbitrary wall of a parallelepiped tank, filled with an acoustic fluid.

2 The problem

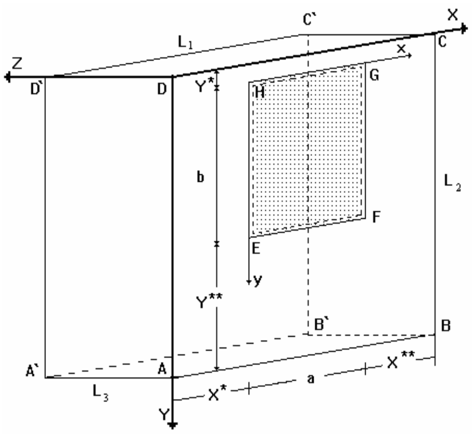

Fig. 1 The geometry of the gas-structure interaction system under consideration.

Thin linearly elastic rectangular plate EFGH with sizes a and b $\left(0 < a \le L_1, 0 < b \le L_2\right)$ and surface S is inserted into an orifice of the absolutely rigid wall ABCD of a rectangular parallelepiped tank $ABCDA^{'}B^{'}C^{'}D^{'}$, all its other walls are absolutely rigid (Figure 1). The tank volume is filled with gas with sound velocity c_0 and mass density ρ_0. Let ρ be the mass density per unit area of the plate, $\overline{D}$ - the flexural rigidity and h - the thickness of the elastic plate material. Two rectangular co-ordinate systems $DXYZ$ and $Hxyz$ are used. Let the velocity potential function of the gas motion be $\varphi(X, Y, Z; t)$ and the function, describing the middle surface vibrations of the plate, be $w(x, y; t)$. The problem about the stationary forced vibrations of the gas and the elastic plate under the action of a source, being situated in the gas tank, is under consideration. Let the source have a range of sizes which are small in comparison with the lengths of the excited waves - then it is possible to be accepted as a point source. It is supposed that the productivity Q and the frequency ω of the source are given and are not influenced by the earlier excited waves. The problem is considered in a linear approximation without giving an account of the dissipating forces. Then the velocity potential function of the gas motion satisfies the following wave equation with the corresponding boundary conditions:

$$\frac{\partial^2\varphi}{\partial X^2}+\frac{\partial^2\varphi}{\partial Y^2}+\frac{\partial^2\varphi}{\partial Z^2}-\frac{1}{c_0^2}\frac{\partial^2\varphi}{\partial t^2}=Qe^{i\omega t},\quad \frac{\partial\varphi}{\partial X}\Big|_{X=0,L_1}=0,\quad \frac{\partial\varphi}{\partial Y}\Big|_{Y=0,L_2}=0,\quad \frac{\partial\varphi}{\partial Z}\Big|_{Z=L_3}=0.$$

The function $w(x,y;t)$ satisfies the next partial differential equation together with the boundary conditions, which describe the way of supporting of the elastic plate - e.g. in a case of a simply-supported plate:

$$\overline{D}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 w+\rho h\frac{\partial^2 w}{\partial t^2}=-\rho_0\frac{\partial\varphi}{\partial t}\Big|_S,\quad w\Big|_{x=0,a}=\frac{\partial^2 w}{\partial x^2}\Big|_{x=0,a}=0,\quad w\Big|_{y=0,b}=\frac{\partial^2 w}{\partial y^2}\Big|_{y=0,b}=0.$$

Both functions φ and w have to satisfy the compatibility condition:

$$\frac{\partial\varphi}{\partial Z}\Big|_{Z=0}=\begin{cases}\dfrac{\partial w}{\partial t} & (x,y)\in S,\\ 0 & (x,y)\notin S.\end{cases}$$

3 Analytical solution

Let X_0, Y_0, Z_0 be the coordinates of the source. Then $Q=Q_0\delta(X-X_0, Y-Y_0, Z-Z_0)$, as the point source is presented by the Dirac delta function $\delta(X-X_0, Y-Y_0, Z-Z_0)$. The functions φ and w are separated into space-dependent modes and time-dependent terms:

$$\varphi(X,Y,Z;t)=\sum_{q=0}^{\infty}\sum_{r=0}^{\infty}C_{qr}(Z)\cos\left(\frac{q\pi}{L_1}X\right)\cos\left(\frac{r\pi}{L_2}Y\right)e^{i\omega t},\quad w(x,y;t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}W_m(x)W_n(y)e^{i\omega t},$$

where the trial functions $W_m(x)$ and $W_n(y)$ as well as the wave numbers or the dispersion equations are chosen correspondingly to the supporting conditions along the four plate edges [1]. After satisfying the compatibility condition and using the Bubnov-Galerkin method, some infinite system of nonhomogeneous algebraic equations about the unknown coefficients W_{mn} is obtained. Substituting $Q_0=0$ in it and taking the determinant of the homogeneous system equal to zero, the equation about the determination of the natural frequencies of the considered gas-structure interaction system is obtained.

4 Numerical calculations

The theoretical solution is very complicated - that is why an approximate solution is made at based on ignoring diffraction by the elastic plate waves. The approximate solution can be used when the frequencies of the source are not close to the resonance frequencies of the gas-structure interaction system and when the cavity is filled with air. Some numerical examples are made and they are represented graphically. If the frequency of the source $f=\frac{\omega}{2\pi}\to 0$, very strong increase of the amplitudes appears except at the resonance points. The approximate formula cannot be used if there is a heavy liquid in the rectangular tank.

5 Conclusions

In this paper a closed rigid rectangular parallelepiped tank is filled with gas as a part of one of its walls is a thin linearly elastic rectangular plate. The problem about the stationary forced vibrations of the gas and the elastic plate under the action of a source, being situated in the gas tank, is under consideration. A combination of the use of the method of the crossed strips of G. Warburton and the method of Bubnov-Galerkin is made to investigate the dynamic behavior of this gas-structure interaction system in the cases of arbitrary supporting conditions of the plate. Some numerical examples are made which demonstrate the necessity of taking into account which part of the spectrum of the natural frequencies of the elastic plate the forced frequency is located in.

References

[1] E. G. Gavrilova, Hydroelasticity of Thin-walled Prismatic Structures with Elastic Inclusions, PhD Thesis (BAS, Sofia, 1994).