Influence of a plasma on the light deflection by compact objects

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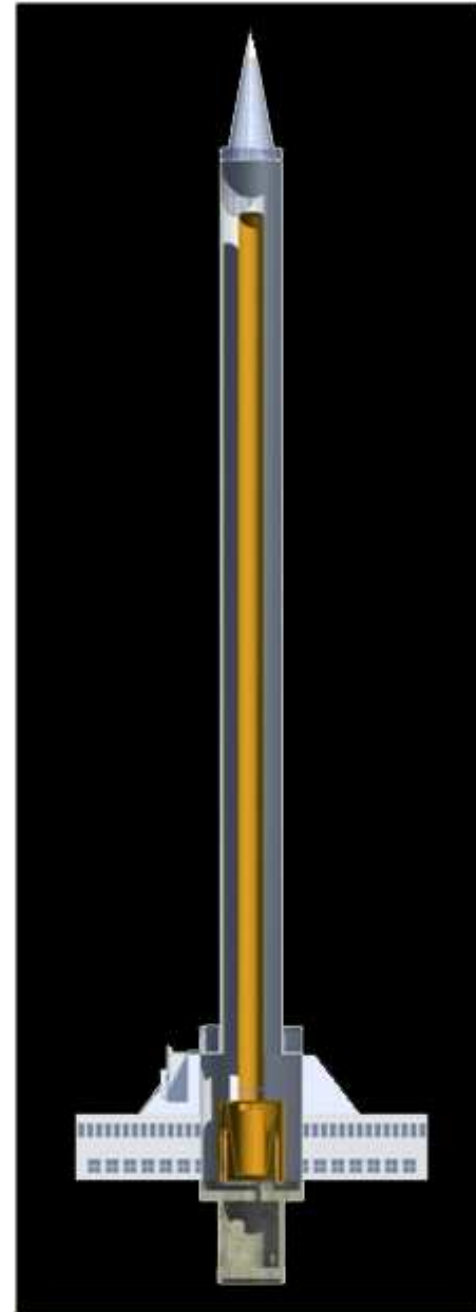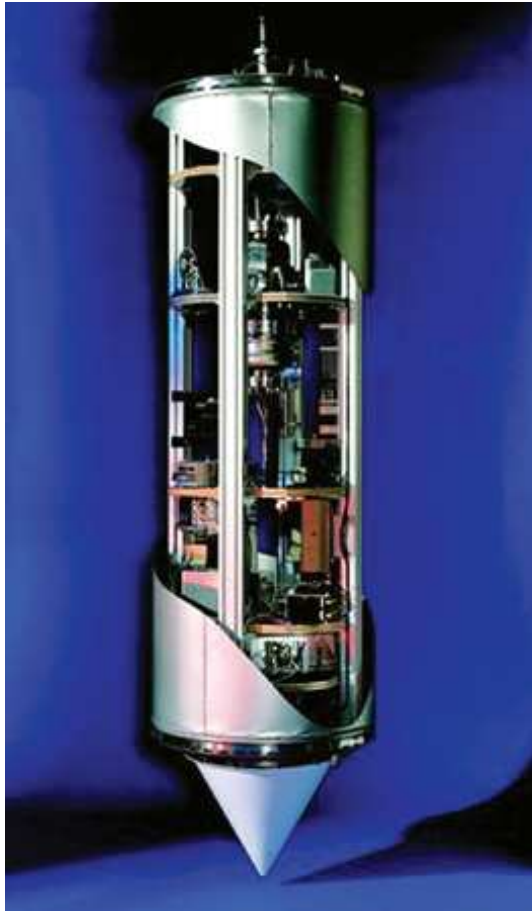**Mexico City, 2 December 2015**

Bremen Drop Tower

Height of tower: 146 m

Free fall distance: 110 m

Free fall time: 4.7 s

With catapult: 9.3 s

Goal: Analytic treatment of light propagation in a plasma on general-relativistic spacetimes

1. Light propagation in a non-magnetised, cold plasma [1]

2. Light deflection in a plasma on a spherically symmetric and static spacetime [2], in particular on Schwarzschild spacetime [1]

3. Influence of a plasma on the shadow of spherically symmetric compact objects [2]

4. Light deflection and shadow in a plasma on Kerr spacetime [3]

[1] VP: "Ray optics, Fermat's principle and applications to general relativity" Springer (2000)

[2] VP, O. Yu. Tsupko, G. S. Bisnovatyi-Kogan: "Influence of a plasma on the shadow of a spherically symmetric black hole" Phys. Rev. D 92, 104031 (2015)

[3] VP, O. Yu. Tsupko, G. S. Bisnovatyi-Kogan: in preparation

Light rays on a general-relativistic spacetime with metric $g_{ik}(x)$:

$$\dot{x}^i = \frac{\partial H(x,p)}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H(x,p)}{\partial x^i}, \quad H(x,p) = 0$$

In vacuo:

$$H(x,p) = \frac{1}{2} g^{ik}(x) p_i p_k$$

Light rays are lightlike geodesics of the spacetime metric g_{ik}

In a non-magnetised cold plasma:

$$H(x,p) = \frac{1}{2}\left(g^{ik}(x) p_i p_k + \omega_p(x)^2 \right),$$

plasma frequency: $\omega_p(x)^2 = \frac{4\pi e^2}{m} N(x)$

e: charge of the electron , $\qquad$ m: mass of the electron
$N(x)$: number density of the electrons

Light rays are timelike geodesics of the conformally rescaled metric $\omega_p^{-2} g_{ik}$

Rigourous derivation from Maxwell's equation, even for magnetised cold plasma:

R. Breuer, J. Ehlers: Proc. Roy. Soc. London, A 370, 389 (1980), A 374, 65 (1981)

for non–magnetised cold plasma:

VP: "Ray Optics, Fermat's Principle and Applications to General Relativity" Springer (2000)

A plasma is a dispersive medium; propagation of light rays depend on the frequency $\omega = -p_i U^i$

For a cold non-magnetised plasma, only the plasma frequency matters, not the 4-velocity of the electrons

Light rays are characterised by a Lorentz invariant index of refraction

$$n(x, \omega)^2 = 1 - \frac{\omega_p(x)^2}{\omega^2} \, .$$

J. Synge: "Relativity: The General Theory", North-Holland (1960)

Spherically symmetric and static case

$$g_{ik}(x)dx^i dx^k = -A(r)dt^2 + B(r)dr^2 + D(r)(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2)$$

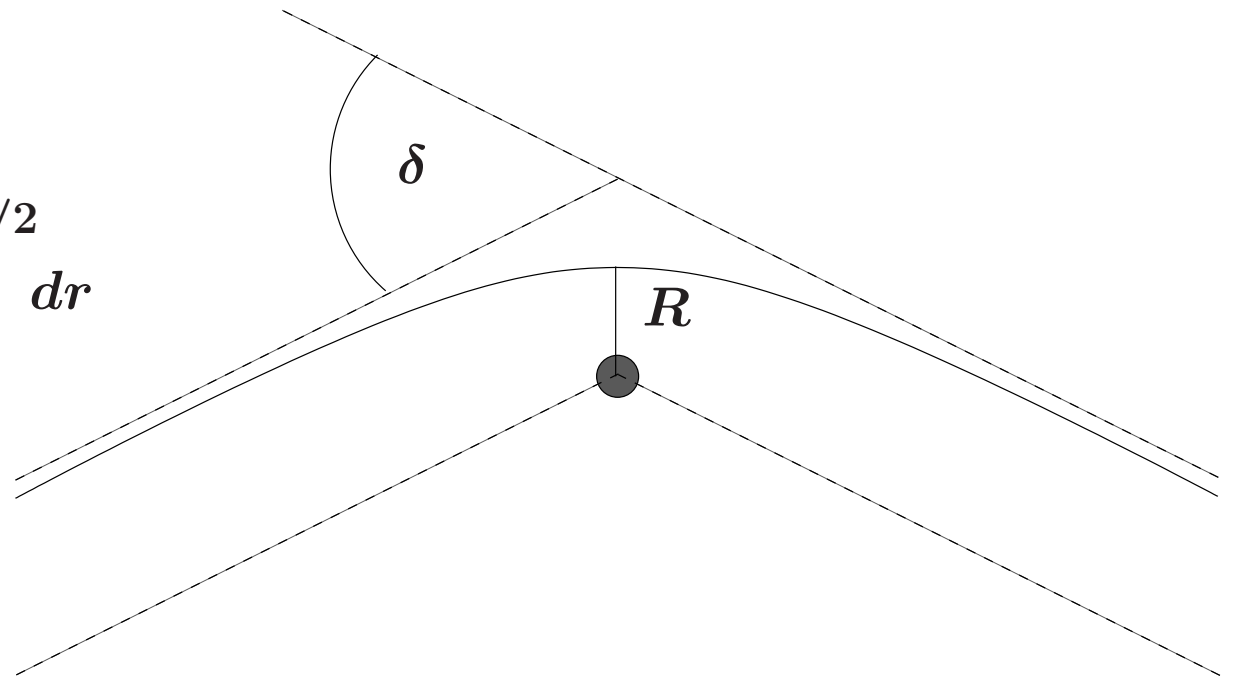$$H(x,p) = \frac{1}{2}\Big(g^{ik}(x)p_i p_k + \omega_p(r)^2\Big)$$

With constant of motion $\omega_0 = -p_t$, define

$$h(r)^2 = \frac{D(r)}{A(r)}\left(1 - A(r)\frac{\omega_p(r)^2}{\omega_0^2}\right)$$

Deflection angle

$$\pi + \delta =$$

$$2\int_R^\infty \frac{\sqrt{B(r)}}{\sqrt{D(r)}}\left(\frac{h(r)^2}{h(R)^2} - 1\right)^{-1/2} dr$$

VP, O. Yu. Tsupko, G. S. Bis-
novatyi-Kogan: Phys. Rev. D
92, 104031 (2015)

Schwarzschild spacetime:

$$A(r) = B(r)^{-1} = 1 - \frac{2m}{r}, \quad D(r) = r^2, \quad m = \frac{GM}{c^2}$$

$$\pi + \delta = 2 \int_R^\infty \left(\frac{r^2 \left(\frac{r}{r-2m} - \frac{\omega_p(r)^2}{\omega_0^2} \right)}{R^2 \left(\frac{R}{R-2m} - \frac{\omega_p(R)^2}{\omega_0^2} \right)} - 1 \right)^{-1/2} \frac{dr}{\sqrt{r}\sqrt{r-2m}}$$

In the weak-field approximation:

D. O. Muhleman and I. D. Johnston: Phys. Rev. Lett. 17, 455 (1966)

Exact formula:

VP: "Ray optics, Fermat's principle and applications to general relativity" Springer (2000)
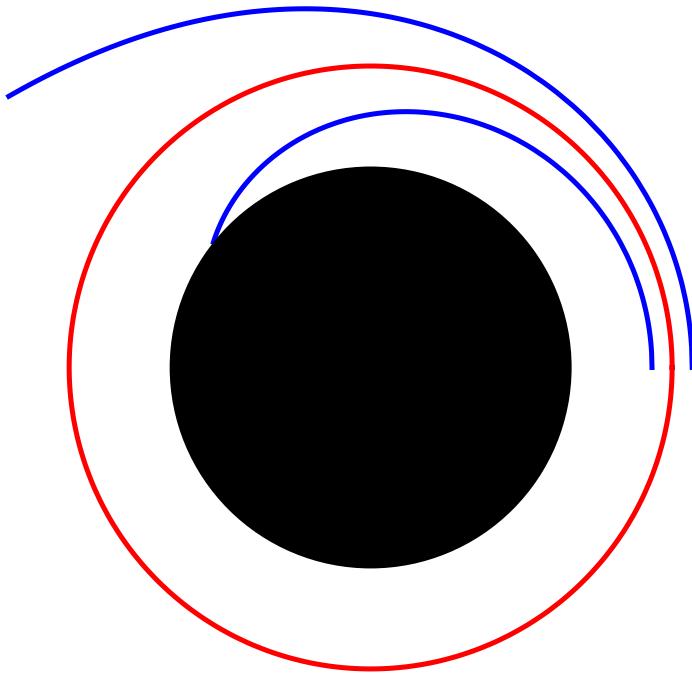
Astrophysical applications:

O. Yu. Tsupko and G. S. Bisnovatyi-Kogan: Phys. Rev. D 87, 124009 (2013)

X. Er and S. Mao: Mon. Not. Roy. Astron. Soc. 437, 2180 (2013)

A. Rogers: Mon. Not. Roy. Astron. Soc. 451 4536 (2015)

Effect of a plasma on the shadow
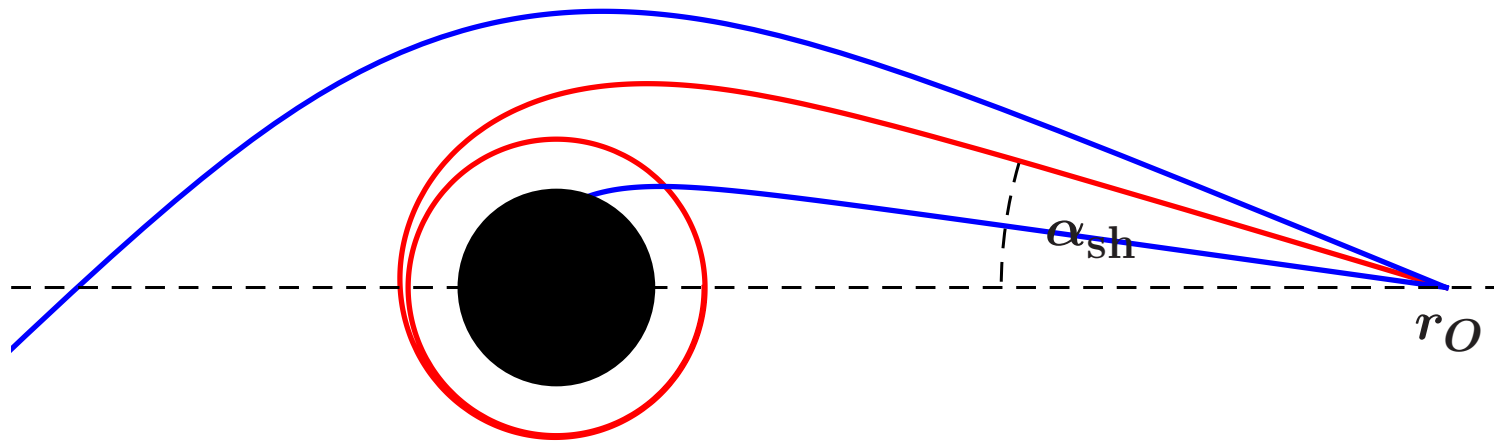
Recall: Shadow in vacuum of a Schwarzschild black hole
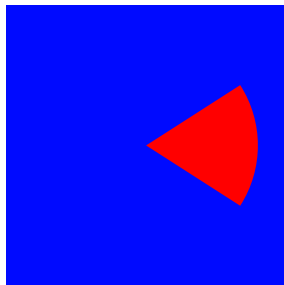

Horizon:

$$r_S = \frac{2GM}{c^2} = 2m$$

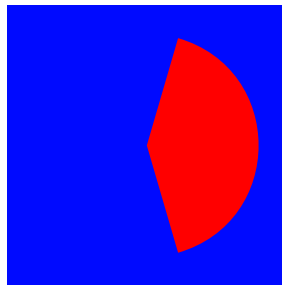**Light sphere (photon sphere)**

$$\frac{3}{2}\, r_S = \frac{3GM}{c^2} = 3m$$

Angular radius α_{sh} of shadow: $\sin^2\alpha_{\mathrm{sh}} = \dfrac{27m^2}{r_O^2}\left(1 - \dfrac{2m}{r_O}\right)$
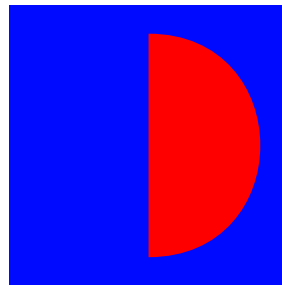
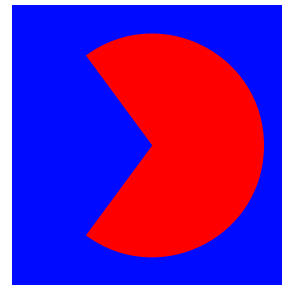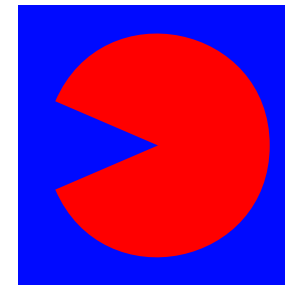J. L. Synge, Mon. Not. R. Astr. Soc. 131, 463 (1966)

$r_O = 1.05\,r_S$ $r_O = 1.3\,r_S$ $r_O = 3\,r_S/2$ $r_O = 2.5\,r_S$ $r_O = 6\,r_S$

Picture from VP: "Gravitational Lensing from a Spacetime Perspective", http://www.livingreviews.org/lrr-2004-9

Perspectives of observations

Object at the centre of our Galaxy:

Mass $= 4 \times 10^6 \, M_\odot$

Distance $= 8 \, \text{kpc}$

Synge's formula gives for the diameter of the shadow $\approx 54 \mu\text{as}$

(corresponds to a grapefruit on the moon)

Object at the centre of M87:

Mass $= 3 \times 10^9 \, M_\odot$
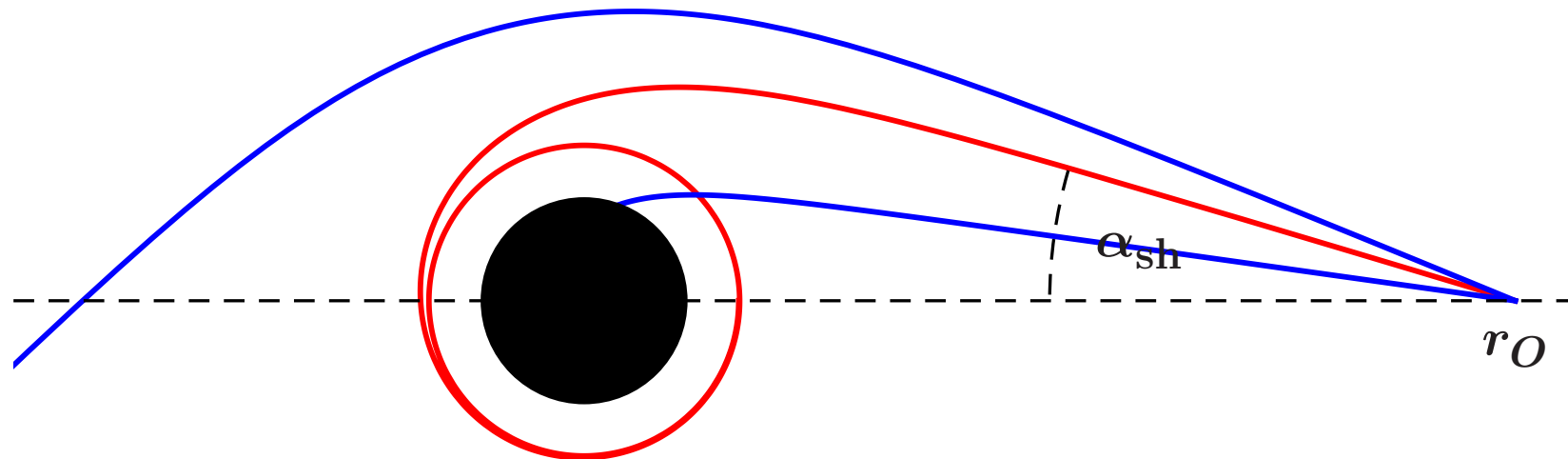
Distance $= 16 \, \text{Mpc}$

Synge's formula gives for the diameter of the shadow $\approx 20 \mu\text{as}$

Perhaps observable in the near future with submillimeter VLBI (Event Horizon Telescope, BlackHoleCam)

Plasma on spherical symmetric and static spacetime

Condition for photon sphere:

$$\frac{d}{dr}h(r)^2\Big|_{r=r_{\mathrm{ph}}} = 0$$
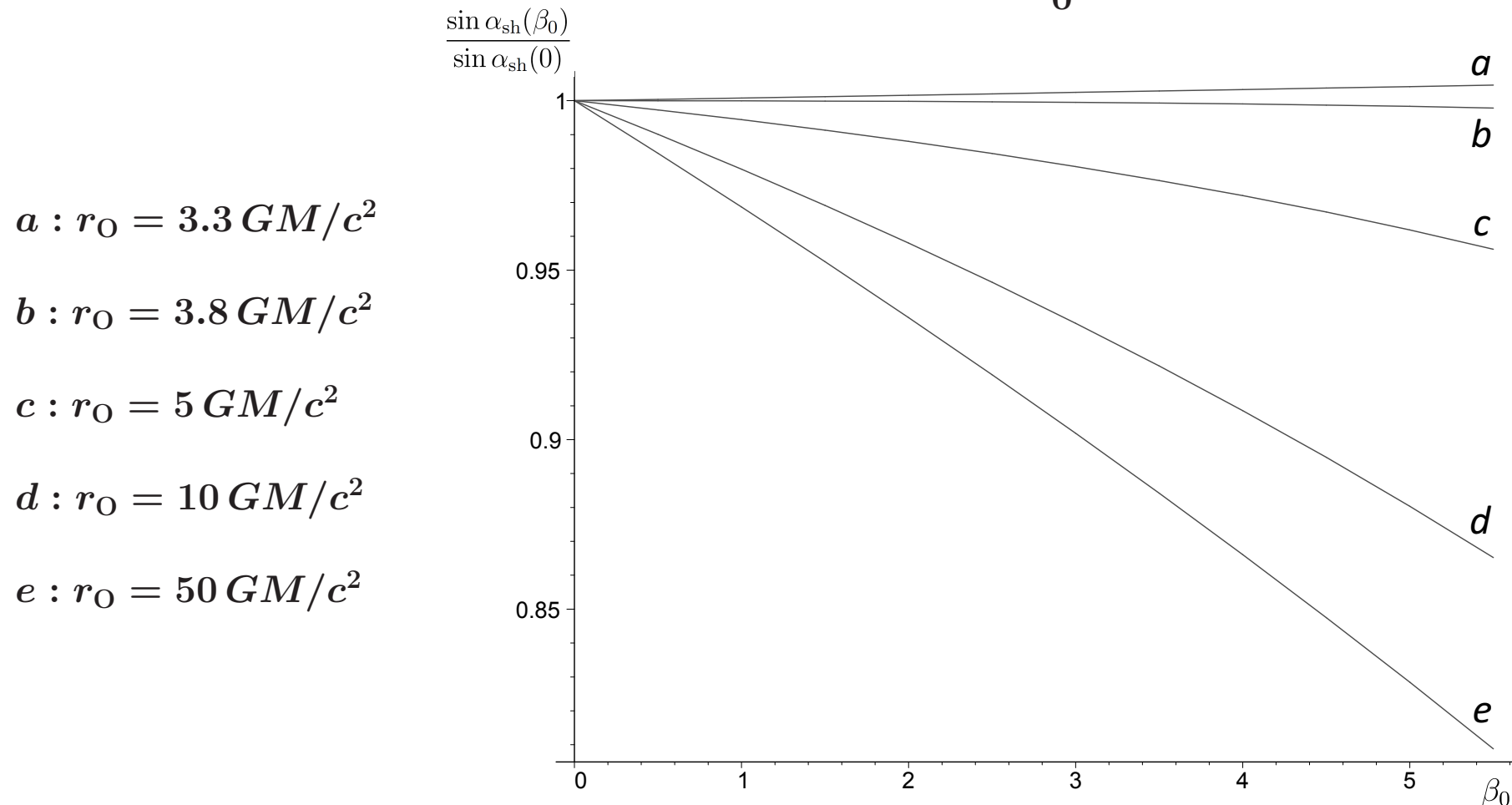

Angular radius α_{sh} of shadow:

$$\sin^2\alpha_{\mathrm{sh}} = \frac{h(r_{\mathrm{ph}})^2}{h(r_{\mathrm{O}})^2}$$

VP, O. Yu. Tsupko, G. S. Bisnovatyi-Kogan: Phys. Rev. D 92, 104031 (2015)

Example: Accretion of dust onto Schwarzschild

$$A(r) = B(r)^{-1} = 1 - \frac{2GM}{c^2 r}\,, \ \ D(r) = r^2\,, \ \ \frac{\omega_p(r)^2}{\omega_0^2} = \beta_0 \left(\frac{GM}{c^2 r}\right)^{3/2}$$

$a : r_O = 3.3\,GM/c^2$

$b : r_O = 3.8\,GM/c^2$

$c : r_O = 5\,GM/c^2$

$d : r_O = 10\,GM/c^2$

$e : r_O = 50\,GM/c^2$

Light propagation in a plasma on Kerr spacetime

$$g_{ik}dx^i dx^k = \varrho(r,\vartheta)^2 \left(\frac{dr^2}{\Delta(r)} + d\vartheta^2 \right) + \frac{\sin^2\vartheta}{\varrho(r,\vartheta)^2}\left(a\,dt - (r^2+a^2)d\varphi \right)^2$$

$$- \frac{\Delta(r)}{\varrho(r,\vartheta)^2}\left(dt - a\sin^2\vartheta\,d\varphi \right)^2$$

$$\varrho(r,\vartheta)^2 = r^2 + a^2\cos^2\vartheta, \qquad \Delta(r) = r^2 - 2mr + a^2 \ .$$

$$m = \frac{GM}{c^2} \text{ where } M = \text{mass} , \qquad a = \frac{J}{Mc} \text{ where } J = \text{spin}$$

Bending angle for light rays in the equatorial plane for an r dependent plasma frequency:

VP: "Ray optics, Fermat's principle and applications to general relativity" Springer (2000)

Light deflection of a slowly rotating Kerr black hole with constant plasma density:

V. Morozova, B. Ahmedov, and A. Tursunov, Astrophys. Space Sci. 346, 513 (2013)

Still to be determined:

Bending of light in the general case

The shadow of a Kerr black hole in a plasma

Recall: Shadow of a Kerr black hole in vacuum

Shape of the shadow of a Kerr black hole for observer at infinity:

J. Bardeen in C. DeWitt and B. DeWitt (eds.): "Black holes" Gordon & Breach (1973)

Shape and size of the shadow of Kerr black holes (and other black holes) for observer at coordinates (r_O, ϑ_O) (analytical formulas):

A. Grenzebach, VP, C. Lämmerzahl: Phys. Rev. D 89, 124004 (2014), Int. J. Mod. Phys. D 24, 1542024 (2015)

Shadow of black hole with $a = m$ for observer at $r_O = 5m$

$\vartheta_O = \frac{\pi}{2}$ $\vartheta_O = \frac{3\pi}{8}$ $\vartheta_O = \frac{\pi}{4}$ $\vartheta_O = \frac{\pi}{8}$ $\vartheta_O = 0$
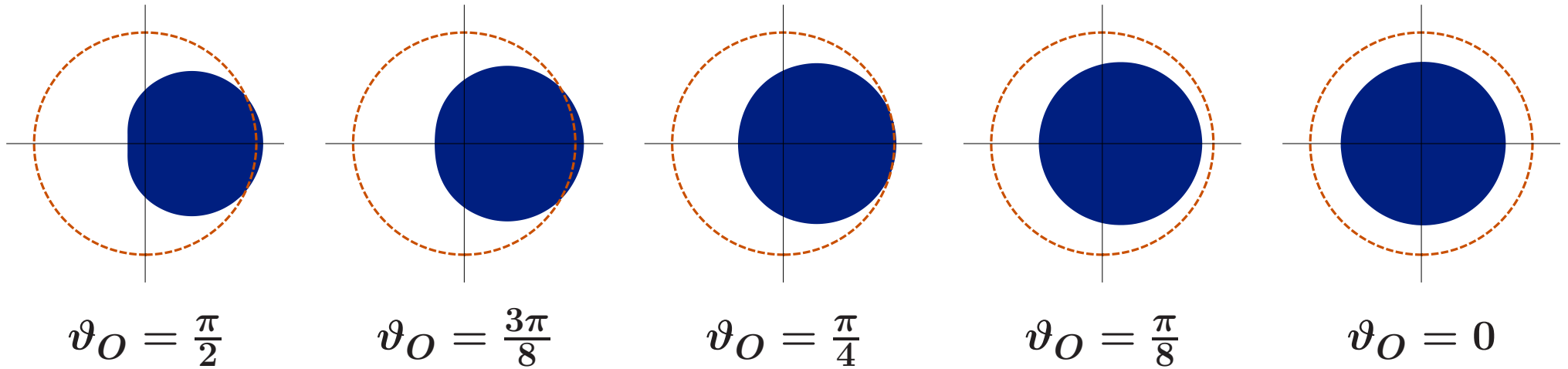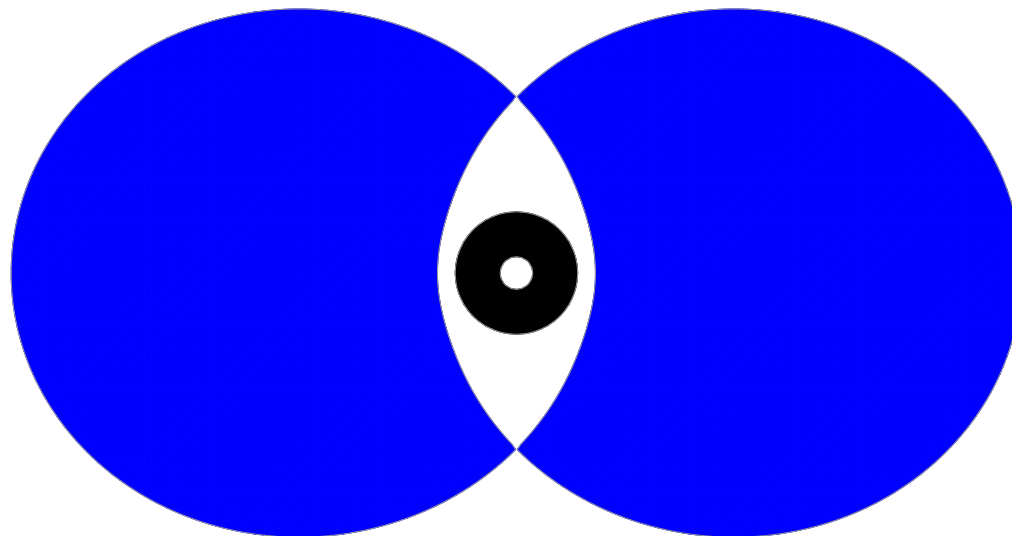
Analytical formula for the shadow is based on the following facts:

The equation for lightlike geodesics is completely integrable (because of the Carter constant)

There is a "photon region" filled with spherical lightlike geodesics (which generalises the photon sphere in Schwarzschild at $r = 3m$)

Photon region for $a = 0.75\,m$

Boundary curve of shadow corresponds to light rays that approach a spherical lightlike geodesic

Analytical formula for the shadow follows from identifying constants of motion of a light ray with the constants of motion of its limit curve

This gives, in particular, a formula for the vertical angular radius α_v of the shadow:

$$\sin^2 \alpha_v = \frac{27m^2 r_O^2 (a^2 + r_O(r_O - 2m))}{r_O^6 + 6a^2 r_O^4 + 3a^2(4a^2 - 9m^2)r_O^2 + 8a^6}$$

$$= \frac{27m^2}{r_O^2}\left(1 + O(m/r_O)\right), \qquad \vartheta_O = \frac{\pi}{2}$$

Up to terms of order $O(m/r_O)$, Synge's formula is still correct for the vertical diameter of the shadow

Generalisation to the plasma case requires:

Find out for which form of the plasma frequency $\omega_p(r, \vartheta)$ a generalised Carter constant exists

Determine, for these cases, the photon region

Derive, thereupon, an analytical formula for the shadow

VP, O. Yu. Tsupko, G. S. Bisnovatyi-Kogan: work in progress