Shadows of black holes

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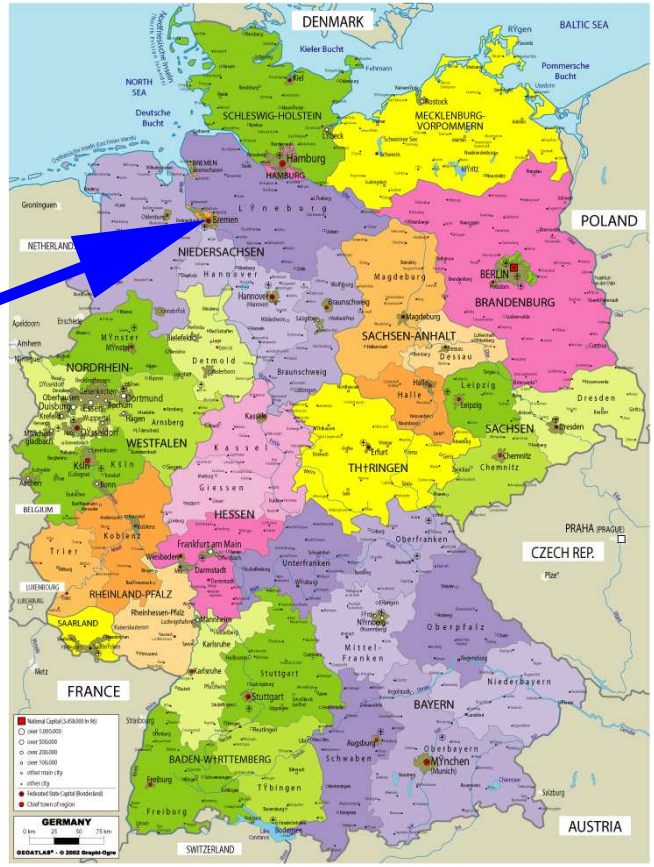
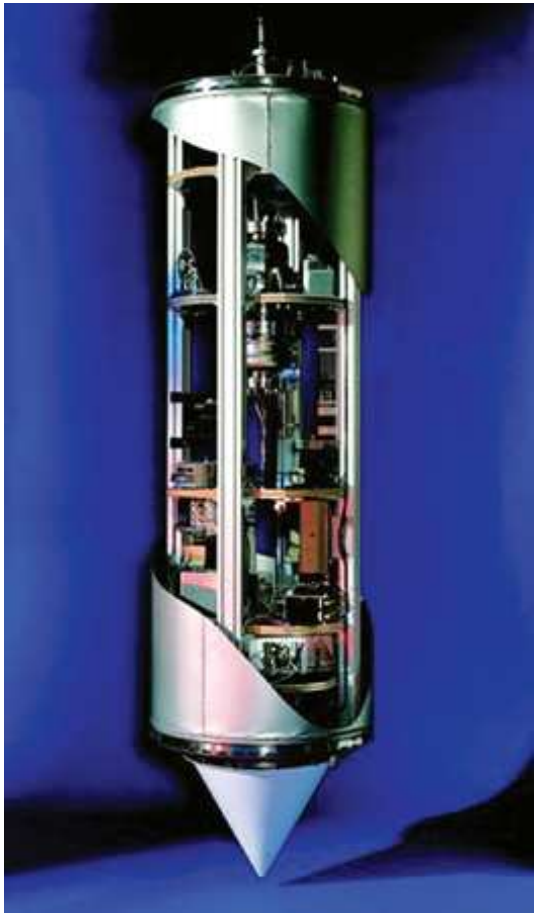
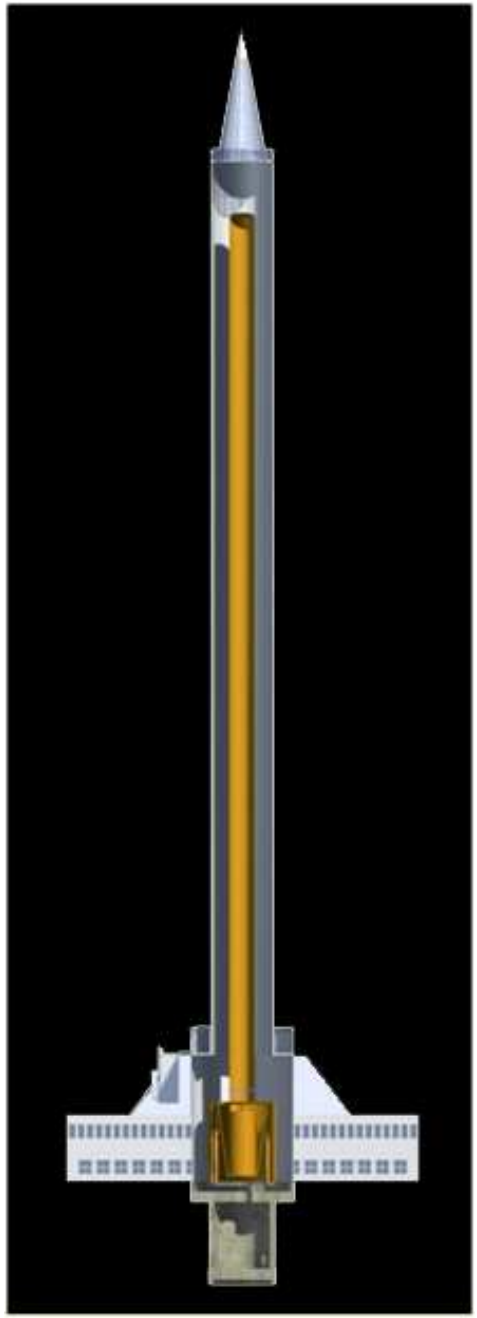


Tarusa, 10 September 2015

Bremen Drop Tower

Height of tower: 146 m
Free fall distance: 110 m
Free fall time: 4.7 s
With catapult: 9.3 s





Outline of talk

- **Shadows of Schwarzschild black holes**
- **Shadows of other spherically symmetric objects**
- **Shadows of rotating black holes**
- **Perspective of observations**

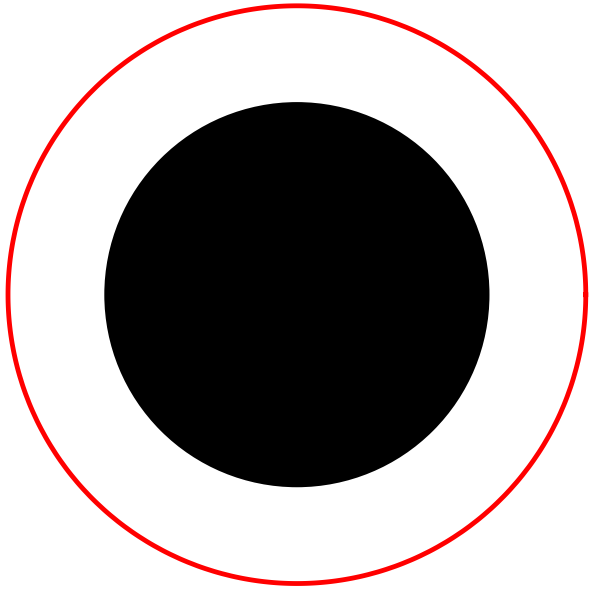
General reference for background material:

VP: "Gravitational Lensing from a Spacetime Perspective", http://www.livingreviews.org/lrr-2004-9

Schwarzschild black hole

$$g_{\mu\nu}dx^\mu dx^\nu = -\left(1-\frac{2m}{r}\right)c^2dt^2 + \frac{dr^2}{1-\frac{2m}{r}} + r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)$$

$$m = \frac{GM}{c^2}$$



Horizon:

$r = 2m$

Light sphere (photon sphere)

$r = 3m$

Schwarzschild black hole

$$g_{\mu\nu}dx^\mu dx^\nu = -\left(1-\frac{2m}{r}\right)c^2dt^2 + \frac{dr^2}{1-\frac{2m}{r}} + r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)$$

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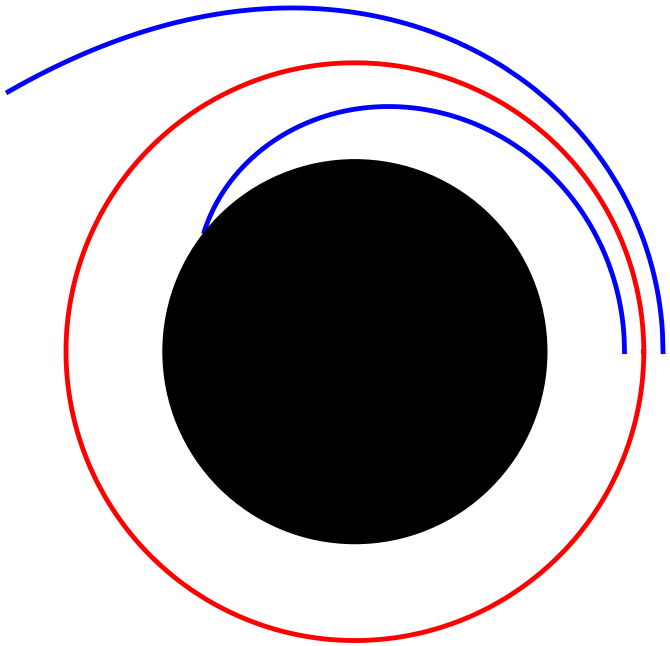

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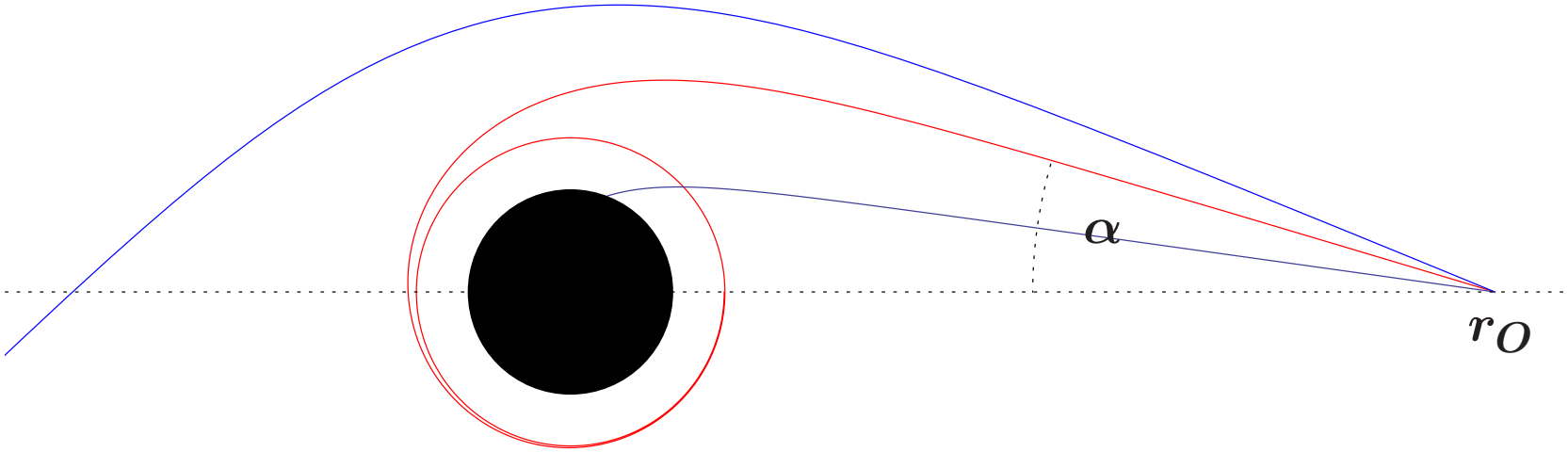
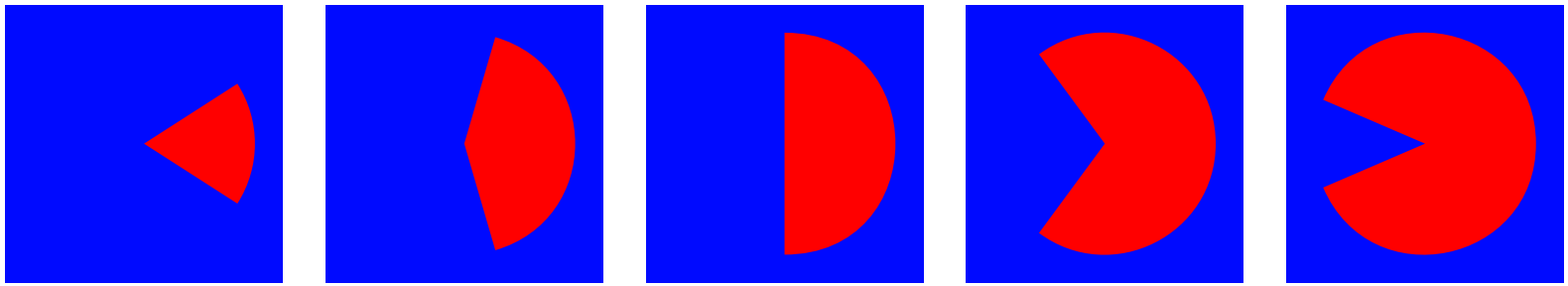


Angular radius α of the "shadow" of a Schwarzschild black hole:

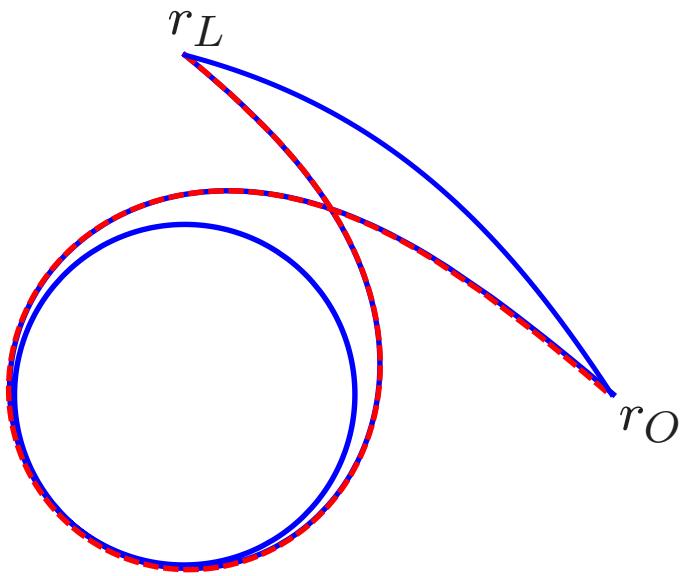
$$\sin^2\alpha = \frac{27\,m^2}{r_O^2}\left(1 - \frac{2m}{r_O}\right)$$

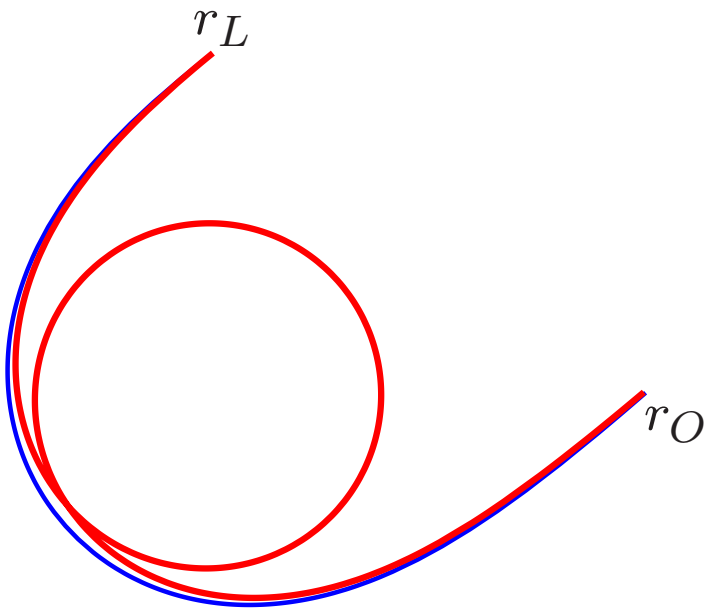
J. L. Synge, Mon. Not. R. Astr. Soc. 131, 463 (1966)



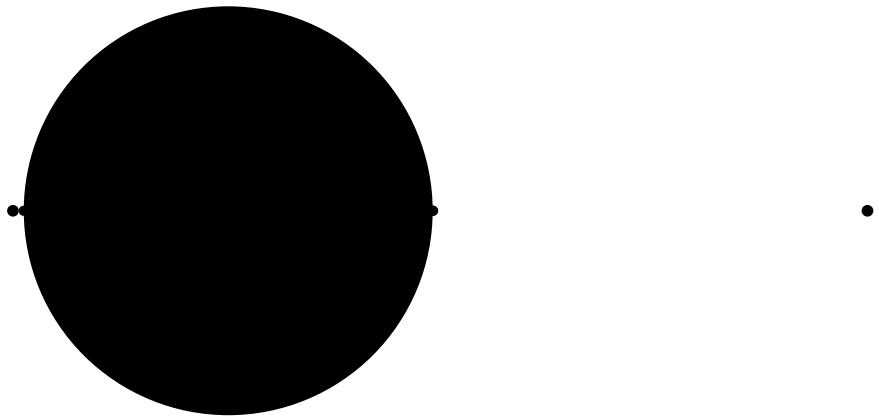
$r_O = 1.05\,r_S$ $\quad$ $r_O = 1.3\,r_S$ $\quad$ $r_O = 3\,r_S/2$ $\quad$ $r_O = 2.5\,r_S$ $\quad$ $r_O = 6\,r_S$

Schwarzschild black hole produces infinitely many images:





Visual appearance of a Schwarzschild black hole



Other spherically symmetric and static black holes:

- Reissner-Nordström
- Kottler (Schwarzschild-(anti)deSitter)
- Janis-Newman-Winicour
- Newman-Unti-Tamburino (NUT)
- Black holes from nonlinear electrodynamics
- Black holes from higher dimensions, braneworld scenarios, ...

All of them have an unstable photon sphere $\Longrightarrow$ Qualitative lensing features are similar to Schwarzschild

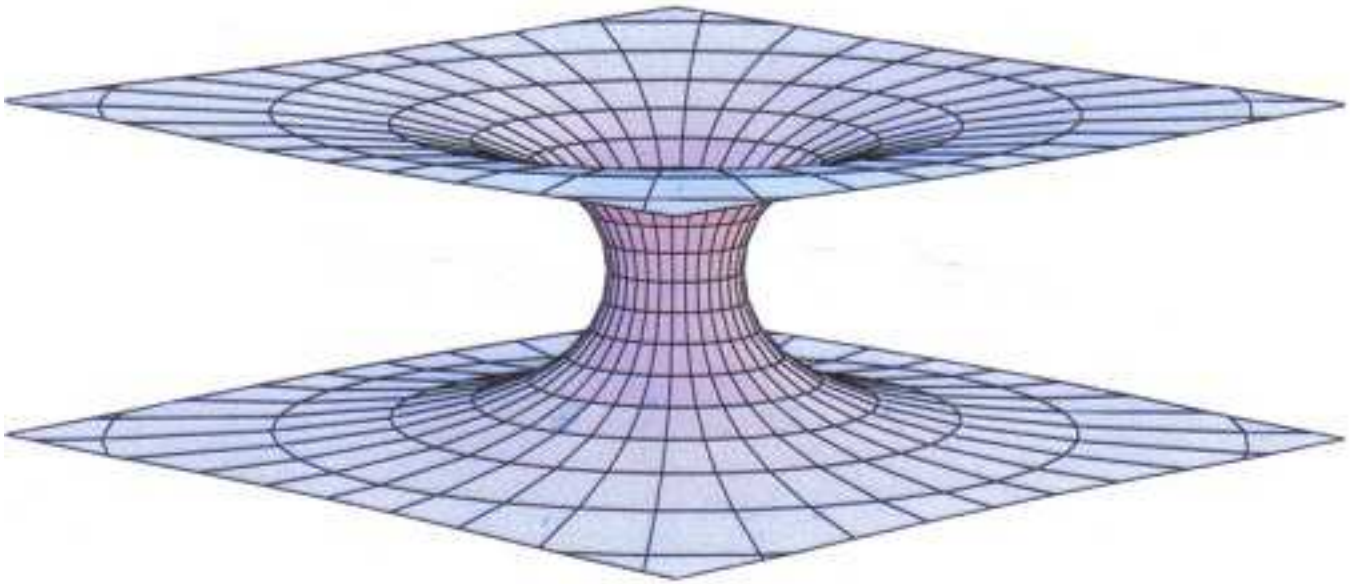
Quantitative features (ratio of angular separations of images, ratio of fluxes of images) are different, see V.Bozza: Phys. Rev. D 66, 103001 (2002)

The shadow is always circular. Its angular radius depends on r_O and the parameters of the black hole.

Black hole impostor: Ellis wormhole

H. Ellis: J. Math. Phys. 14, 104 (1973)

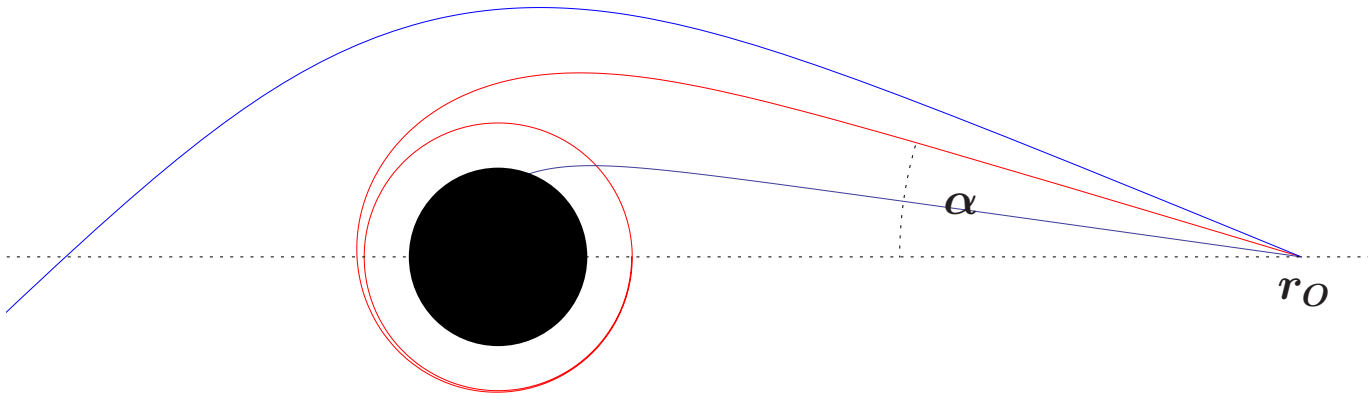
$$g = -c^2 dt^2 + dr^2 + (r^2 + a^2)\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)$$



Angular radius α of shadow: $\sin^2\alpha = \dfrac{a^2}{r_O^2 + a^2}$

Black hole impostor: Ultracompact star

Uncharged dark star with radius between $2m$ and $3m$



Lensing features indistinguishable from Schwarzschild black hole

It seems to be that ultracompact objects are necessarily unstable, see
V. Cardoso, L. Crispino, C. Macedo, H. Okawa, P. Pani: Phys. Rev. D 90, 044069 (2014)

Influence of a plasma on the shadow:

VP, O. Tsupko, G. Bisnovatyi-Kogan: arXiv:1507.04217

Schwarzschild spacetime, plasma frequency $\omega_p(r)^2 = \beta_0\,\omega_0^2\left(\frac{GM}{c^2 r}\right)^{3/2}$

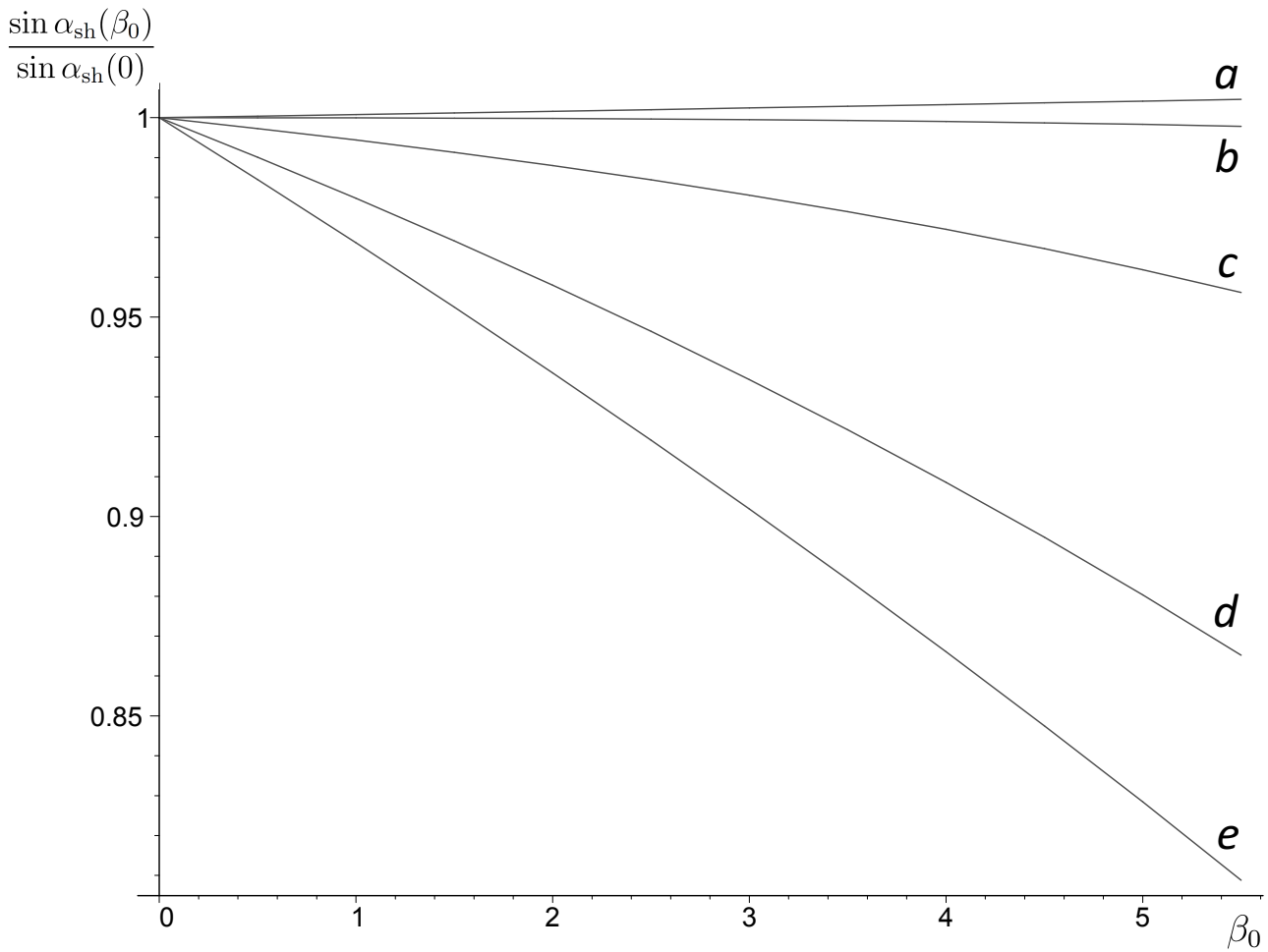
$a : r_{\rm O} = 3.3\,GM/c^2$

$b : r_{\rm O} = 3.8\,GM/c^2$

$c : r_{\rm O} = 5\,GM/c^2$

$d : r_{\rm O} = 10\,GM/c^2$

$e : r_{\rm O} = 50\,GM/c^2$



Rotating black holes:

Shadow no longer circular

Shape of shadow can be used for discriminating between different black holes

Shape of the shadow of a Kerr black hole for observer at infinity:

J. Bardeen in C. DeWitt and B. DeWitt (eds.): "Black holes" Gordon & Breach (1973)

cf. S. Chandrasekhar: "The mathematical theory of black holes" Oxford UP (1983)

Shape and size of the shadow for black holes of the Plebański-Demiański class for observer at coordinates (r_O, ϑ_O) (analytical formulas):

A. Grenzebach, VP, C. Lämmerzahl: Phys. Rev. D 89, 124004 (2014), Int. J. Mod. Phys. D 24, 1542024 (2015)

Kerr metric in Boyer–Lindquist coordinates $(r, \vartheta, \varphi, t)$:

$$g_{\mu\nu} dx^\mu dx^\nu = \varrho(r,\vartheta)^2 \left(\frac{dr^2}{\Delta(r)} + d\vartheta^2 \right) + \frac{\sin^2\vartheta}{\varrho(r,\vartheta)^2} \Big(a\, dt - (r^2 + a^2) d\varphi \Big)^2$$

$$- \frac{\Delta(r)}{\varrho(r,\vartheta)^2} \Big(dt - a \sin^2\vartheta\, d\varphi \Big)^2$$

$$\varrho(r,\vartheta)^2 = r^2 + a^2 \cos^2\vartheta, \qquad \Delta(r) = r^2 - 2mr + a^2 \ .$$

$m = \dfrac{GM}{c^2}$ where M =mass , $a = \dfrac{J}{Mc}$ where J =spin

Plebański-Demiański black holes: Additional parameters

q_e =el. charge , q_m =magn.charge , ℓ =NUT parameter ,

Λ =cosmol. constant , α =acceleration

Consider in the following only the Kerr metric

Lightlike geodesics:

$$\varrho(r,\vartheta)^2\dot{t} = a\left(L - Ea\,\sin^2\vartheta\right) + \frac{(r^2+a^2)\left((r^2+a^2)E - aL\right)}{\Delta(r)},$$

$$\varrho(r,\vartheta)^2\dot{\varphi} = \frac{L - Ea\,\sin^2\vartheta}{\sin^2\vartheta} + \frac{(r^2+a^2)aE - a^2L}{\Delta(r)},$$

$$\varrho(r,\vartheta)^4\dot{\vartheta}^2 = K - \frac{(L - Ea\,\sin^2\vartheta)^2}{\sin^2\vartheta} =: \Theta(\vartheta),$$

$$\varrho(r,\vartheta)^4\dot{r}^2 = -K\Delta(r) + \left((r^2+a^2)E - aL\right)^2 =: R(r).$$

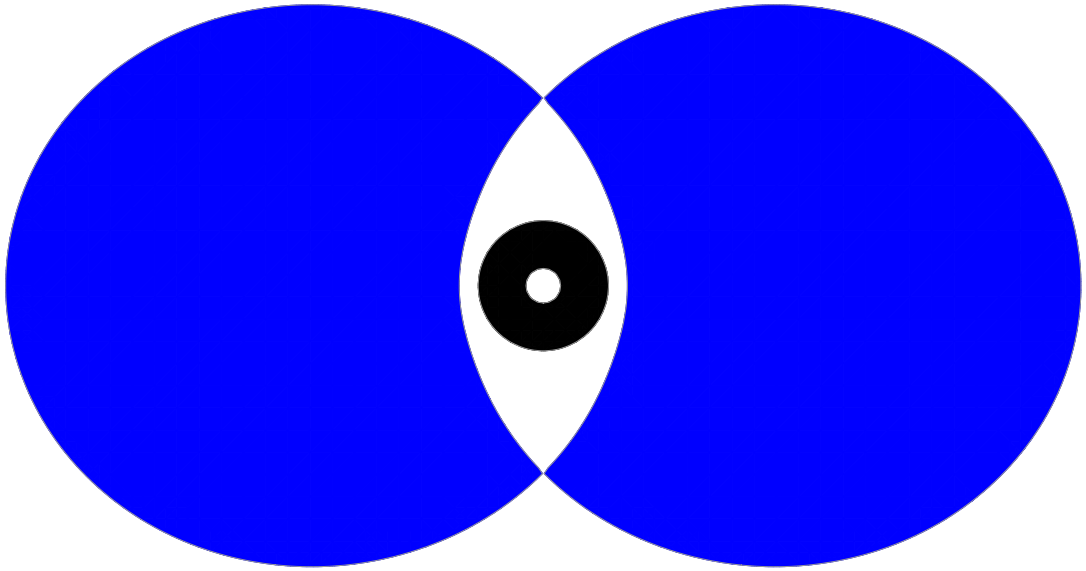
Spherical lightlike geodesics exist in the region where

$$R(r) = 0\,, \quad R'(r) = 0\,, \quad \Theta(\vartheta) \geq 0\,.$$

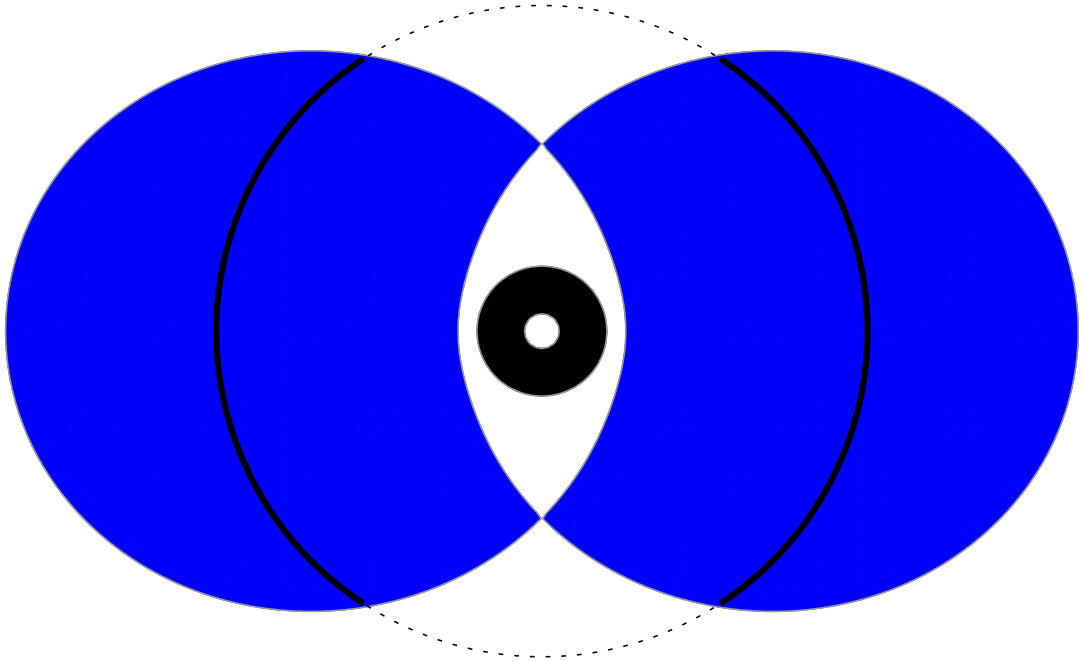
$$\left(2r\Delta(r) - (r-m)\,\varrho(r,\vartheta)^2\right)^2 \leq 4a^2r^2\Delta(r)\,\sin^2\vartheta$$

(unstable if $R''(r) \geq 0$)

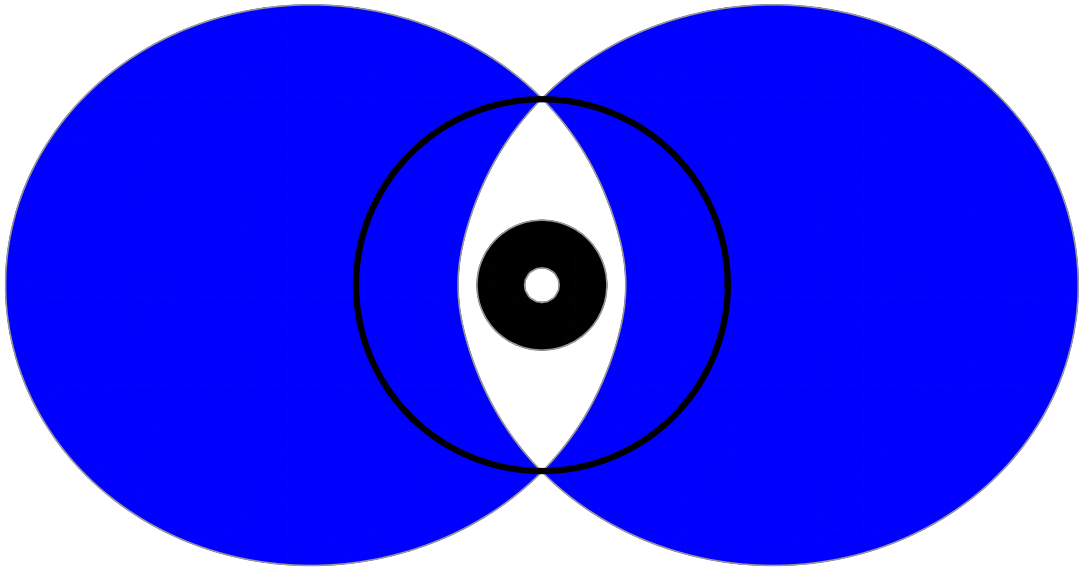
Photon region for Kerr black hole with $a = 0.75\,m$



Photon region for Kerr black hole with $a = 0.75\,m$



Photon region for Kerr black hole with $a = 0.75\,m$



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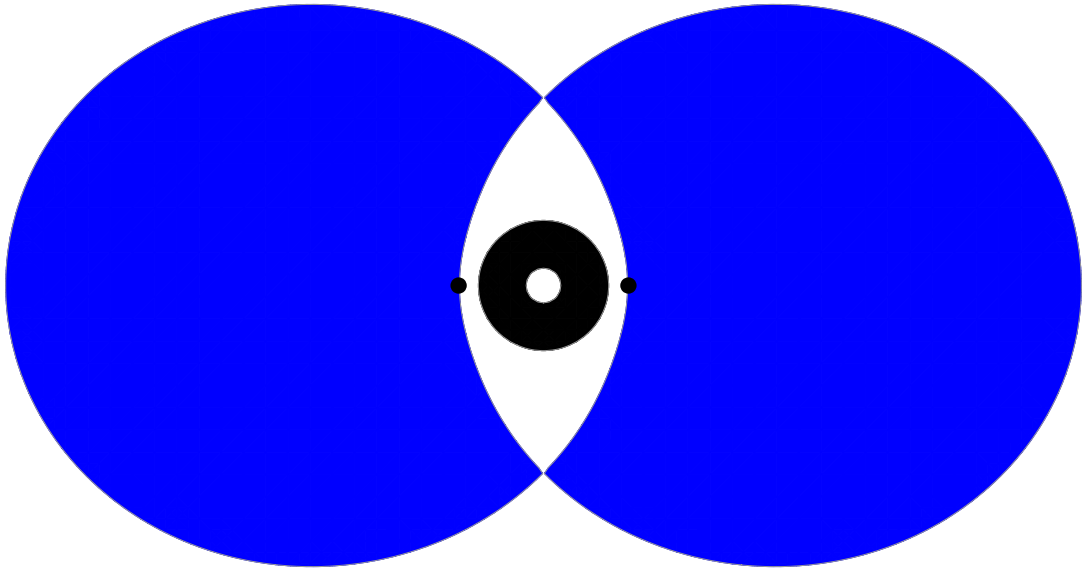
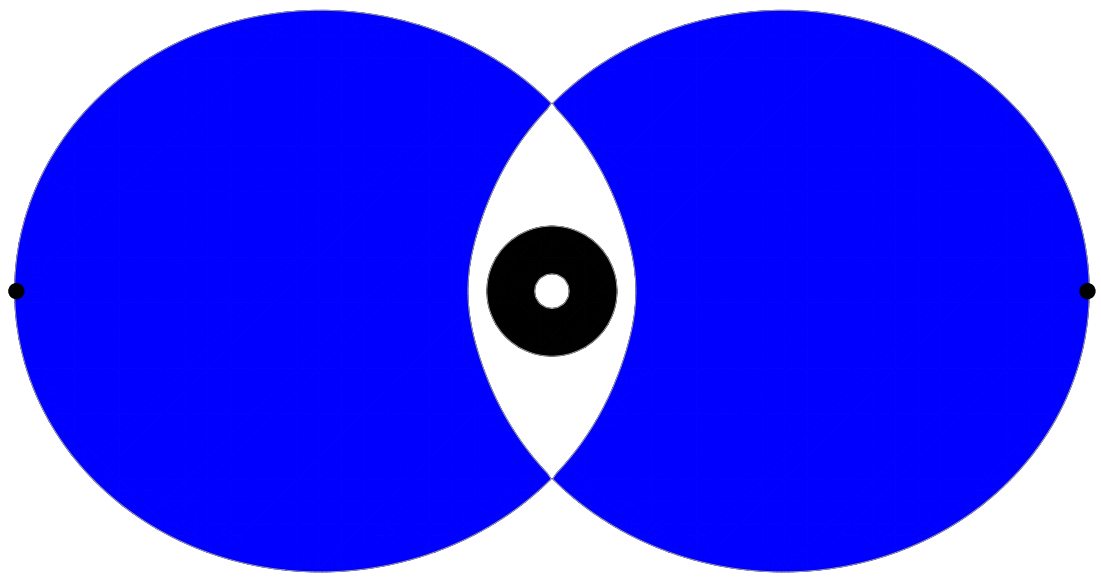


Photon region for Kerr black hole with $a = 0.75\,m$



The shadow is determined by light rays that approach an unstable spherical lightlike geodesic.

Choose observer at r_O and ϑ_O

Choose tetrad

$$e_0 = \left.\frac{(r^2+a^2)\partial_t + a\partial_\varphi}{\varrho(r,\vartheta)\sqrt{\Delta(r)}}\right|_{(r_O,\vartheta_O)}$$

$$e_1 = \left.\frac{1}{\varrho(r,\vartheta)}\,\partial_\vartheta\right|_{(r_O,\vartheta_O)}$$

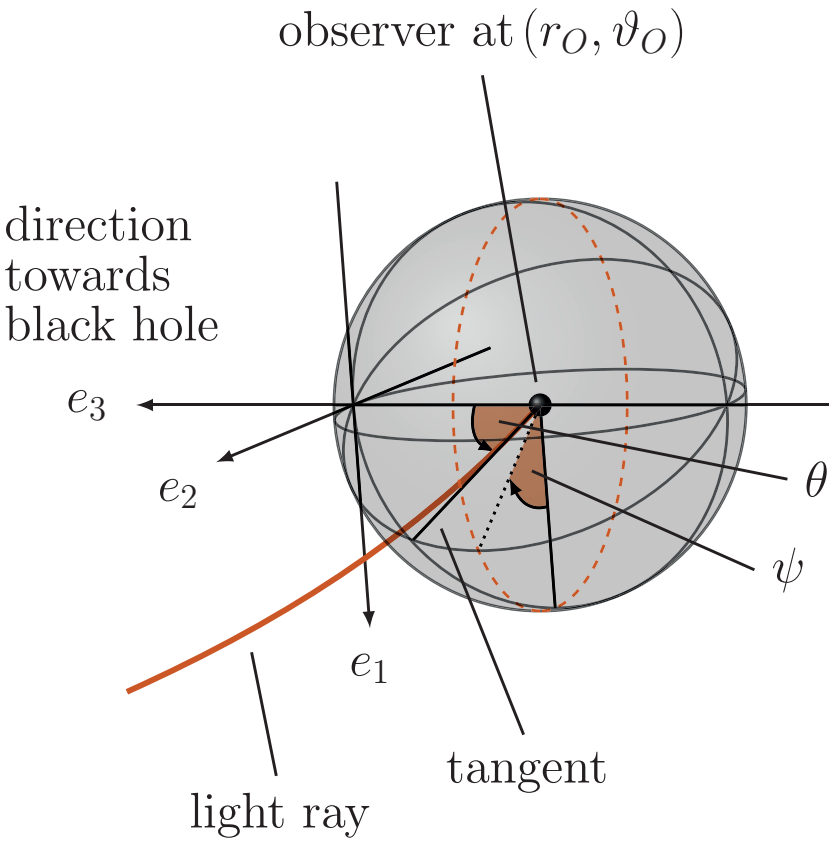
$$e_2 = -\left.\frac{(\partial_\varphi + a\sin^2\vartheta\partial_t)}{\sqrt{\varrho(r,\vartheta)^2}\sin\vartheta}\right|_{(r_O,\vartheta_O)}$$

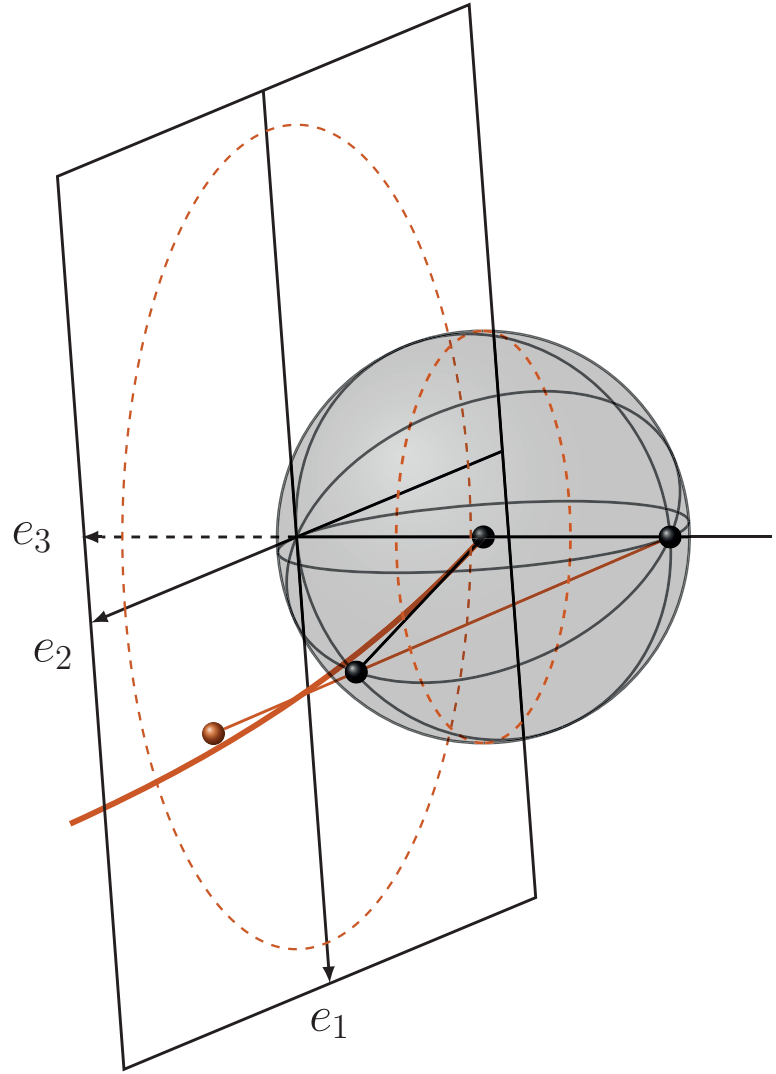
$$e_3 = -\left.\frac{\sqrt{\Delta(r)}}{\varrho(r,\vartheta)}\,\partial_r\right|_{(r_O,\vartheta_O)}$$

Observer with other 4-velocity: Aberration

A. Grenzebach, in D. Puetzfeld, C. Laemmerzahl, B.Schutz (eds.): “Equations of motion in relativistic gravity” Springer (2015)

celestial coordinates at observer (θ, ψ)





constants of motion $\left(K_E = \dfrac{K}{E^2}, L_E = \dfrac{L}{E} - a\right)$

$$\sin\theta = \left.\frac{\sqrt{\Delta(r)\ K_E}}{r^2 - aL_E}\right|_{r=r_O}, \qquad \sin\psi = \left.\frac{L_E + a\cos^2\vartheta + 2\ell\cos\vartheta}{\sqrt{K_E}\ \sin\vartheta}\right|_{\vartheta=\vartheta_O}$$

$$K_E = \left.\frac{16r^2\Delta(r)}{(\Delta'(r))^2}\right|_{r=r_p}, \qquad aL_E = \left.\left(r^2 - \frac{4r\Delta(r)}{\Delta'(r)}\right)\right|_{r=r_p}$$

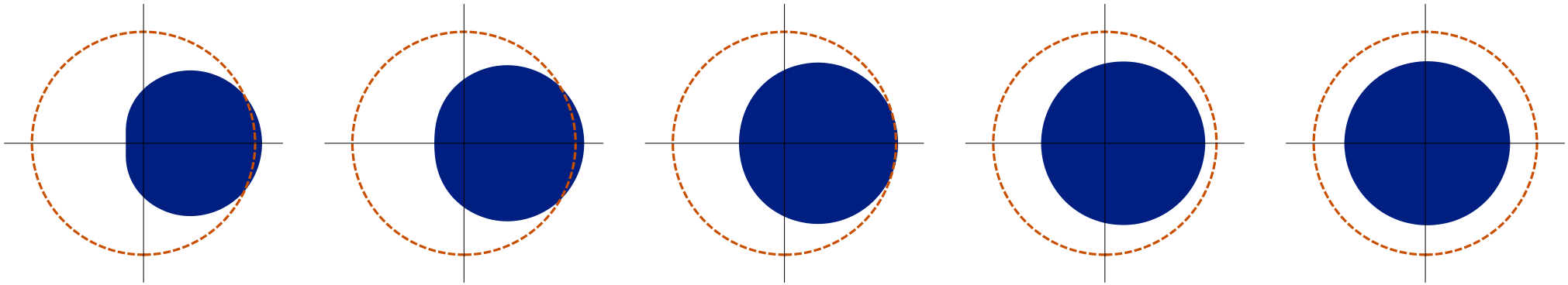
Analytic formula for shadow allows to extract parameters of the spacetime from the shape of the shadow

Vertical angular radius α_v of the shadow:

$$\sin^2\alpha_v = \frac{27m^2r_O^2\big(a^2 + r_O(r_O - 2m)\big)}{r_O^6 + 6a^2r_O^4 + 3a^2(4a^2 - 9m^2)r_O^2 + 8a^6} = \frac{27m^2}{r_O^2}\Big(1 + O(m/r_O)\Big)$$

Up to terms of order $O(m/r_O)$, Synge's formula is still correct for the vertical diameter of the shadow

Shadow of black hole with $a = m$ for observer at $r_O = 5m$



$\vartheta_O = \frac{\pi}{2}$ $\quad$ $\vartheta_O = \frac{3\pi}{8}$ $\quad$ $\vartheta_O = \frac{\pi}{4}$ $\quad$ $\vartheta_O = \frac{\pi}{8}$ $\quad$ $\vartheta_O = 0$

Perspectives of observations

Object at the centre of our galaxy:

Mass $= 4 \times 10^6\, M_\odot$

Distance $= 8\,\mathrm{kpc}$

Synge's formula gives for the diameter of the shadow $\approx 54\mu$as

(corresponds to a grapefruit on the moon)

Object at the centre of M87:

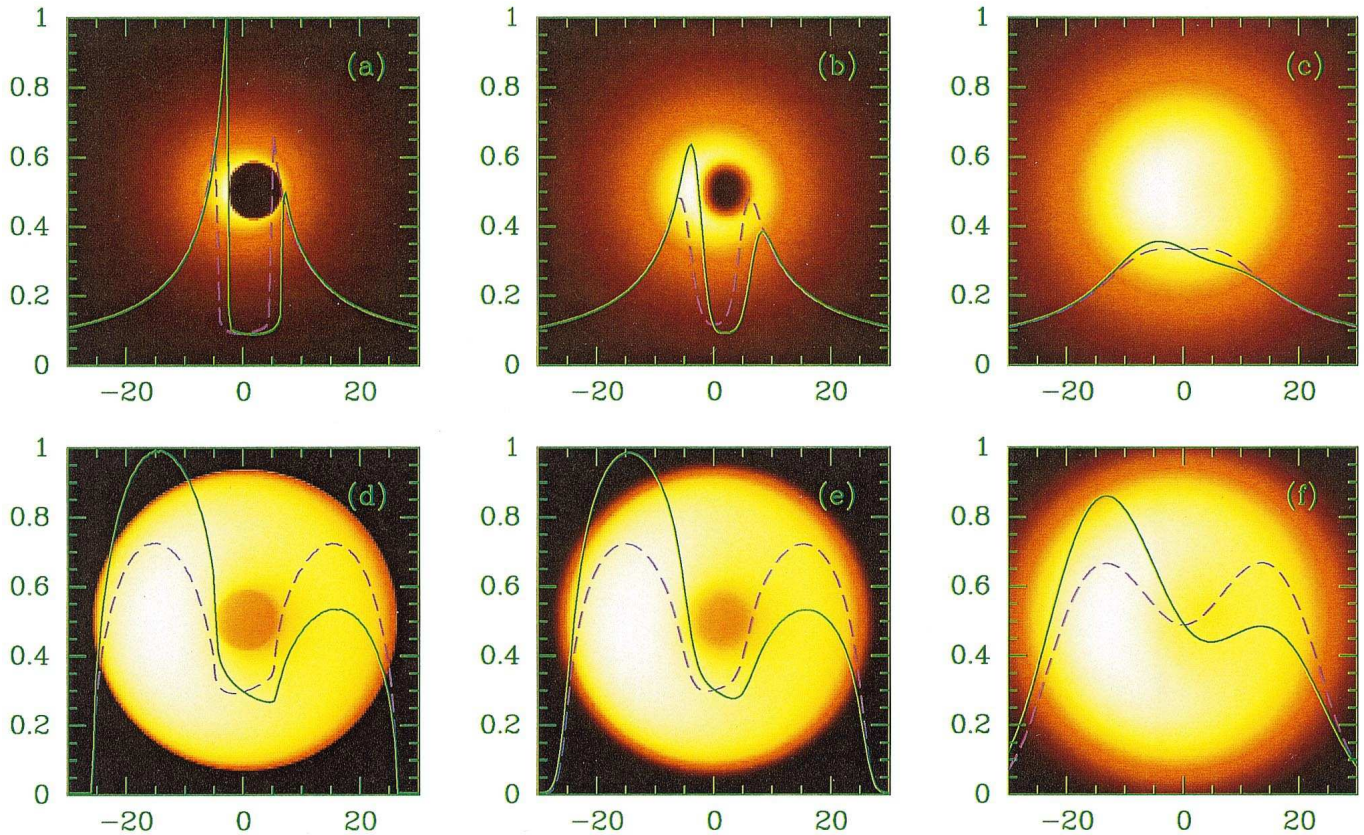
Mass $= 3 \times 10^9\, M_\odot$

Distance $= 16\,\mathrm{Mpc}$

Synge's formula gives for the diameter of the shadow $\approx 9\mu$as

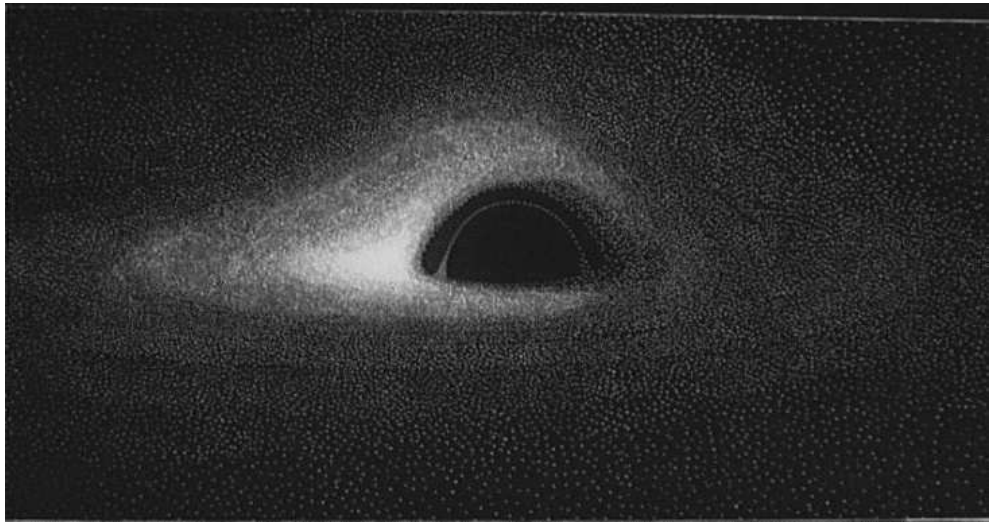
Kerr shadow with emission region and scattering taken into account:

no scattering | $\lambda = 0.6$ mm | $\lambda = 1.3$ mm

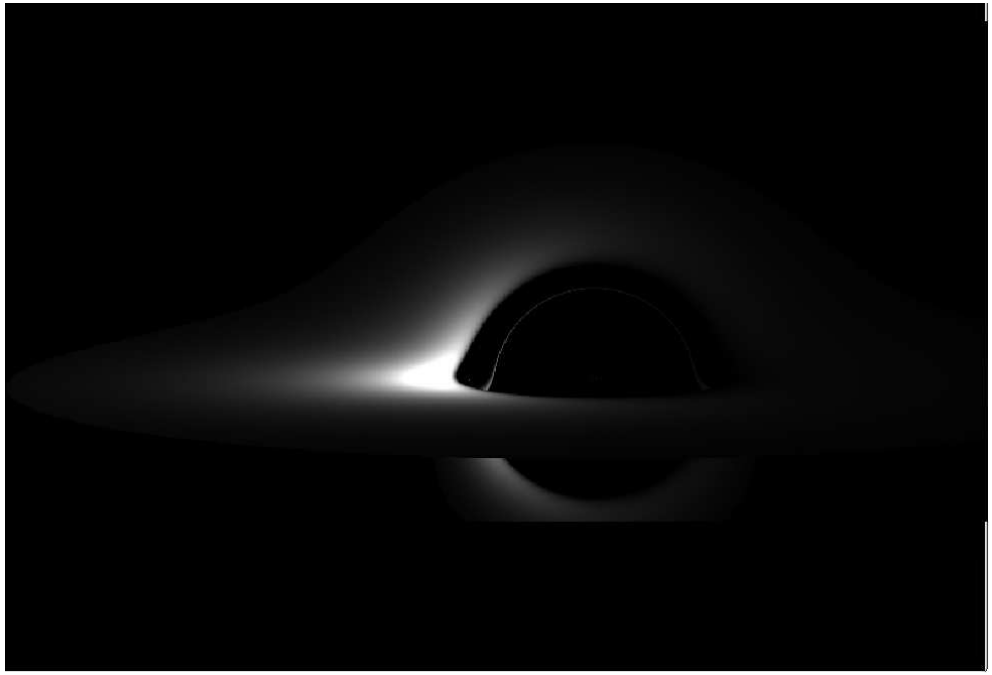


H. Falcke, F. Melia, E. Agol: Astrophys. J. 528, L13 (2000)

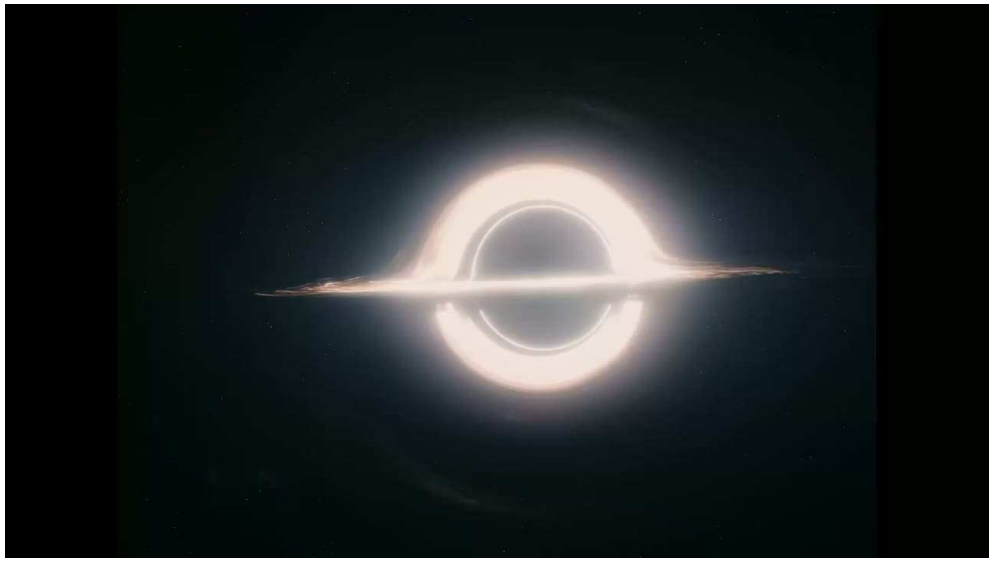
Observations should be done at sub-millimeter wavelengths



J.-P. Luminet (1979)



T. Müller (2012)

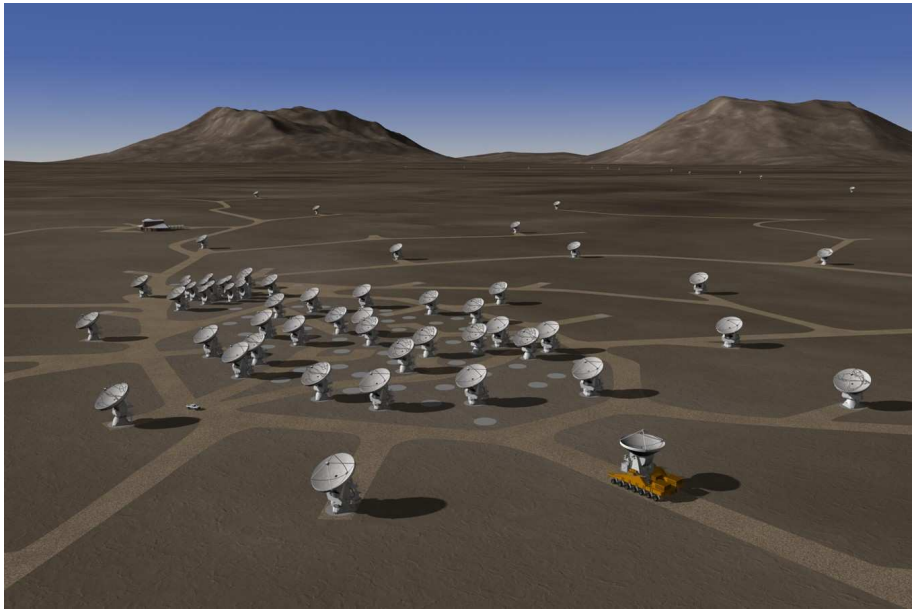


from the Movie “Interstellar” (2014)

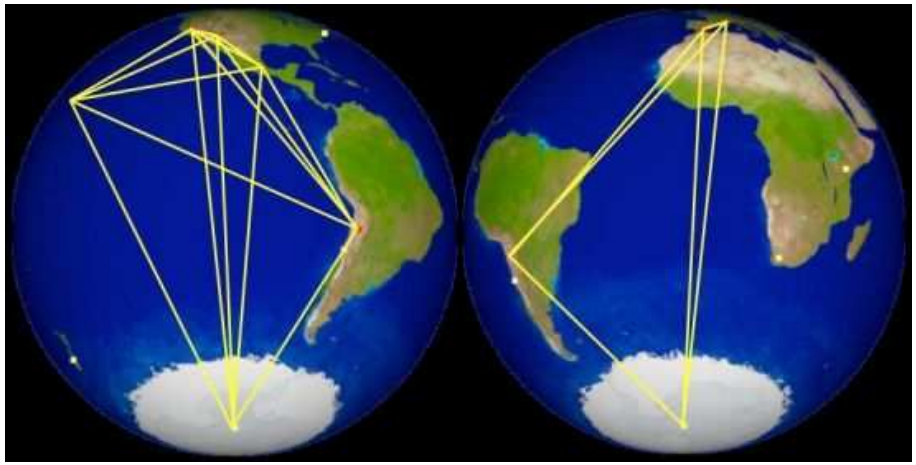
Projects to view the shadow with sub-millimeter VLBI:

Event Horizon Telescope (EHT),

Using ALMA, NOEMA, LMT, CARMA, South Pole Telescope ...



ALMA

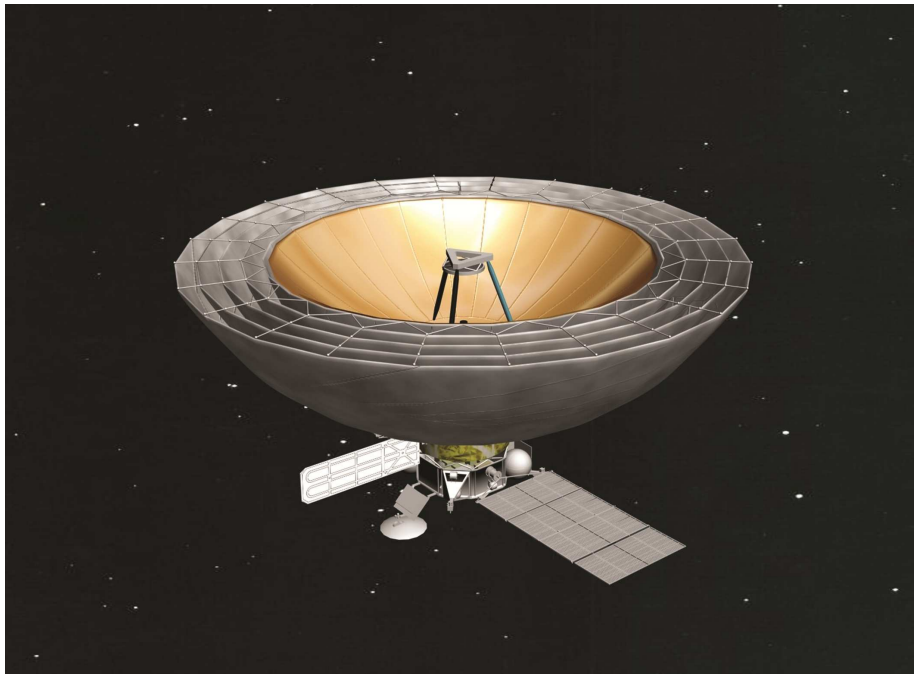


EHT

BlackHoleCam

H. Falcke, L. Rezzolla, M. Kramer

Millimetron (≈ 2025)



Good chance to see the shadow of the centre of our galaxy within a few years