

# The shadow of a black hole.

## Theory and prospects of observations

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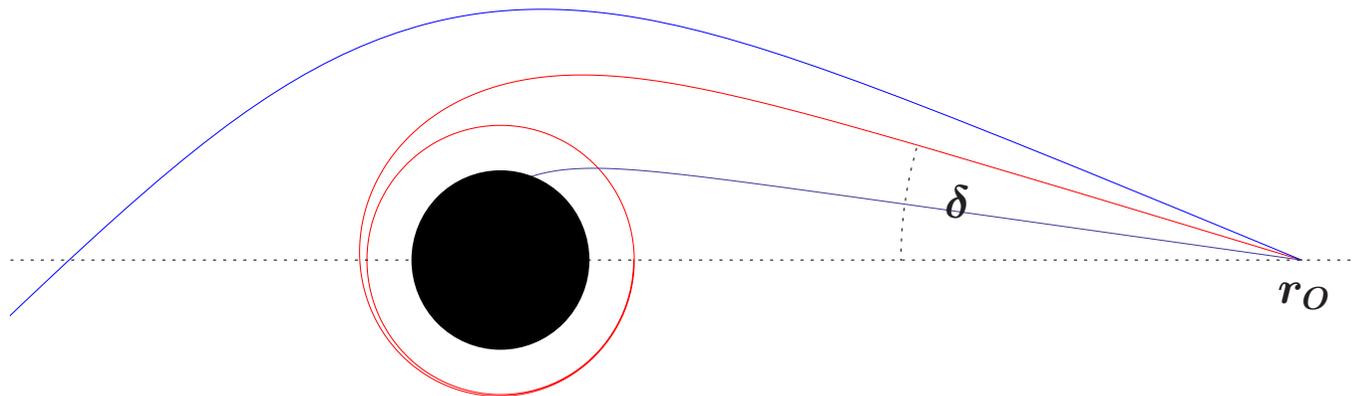
Topics to be discussed:

- Photon regions in generalized Kerr spacetimes
- Shadow of rotating black holes
- Observability of the shadow

VP: "Gravitational Lensing from a Spacetime Perspective", Living Rev. Relativity 7, (2004), <http://www.livingreviews.org/lrr-2004-9>

Moscow, Research Space Institute, 3 March 2015  
(Financially supported by the Dynasty Foundation)

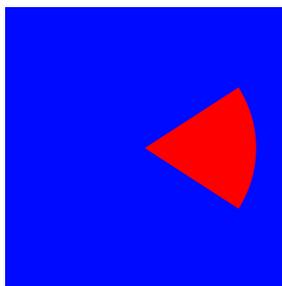
# Recall: Shadow of spherically symmetric and static black hole



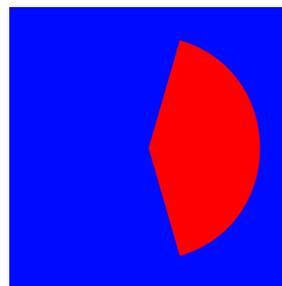
Angular radius  $\delta$  of the “shadow” of a Schwarzschild black hole:

$$\sin^2 \delta = \frac{27 r_S^2 (r_O - r_S)}{4 r_O^3}$$

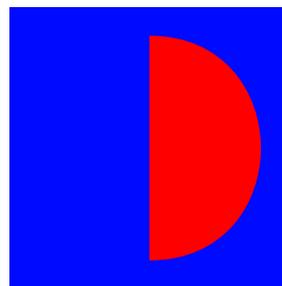
J. L. Synge, Mon. Not. R. Astr. Soc. 131, 463 (1966)



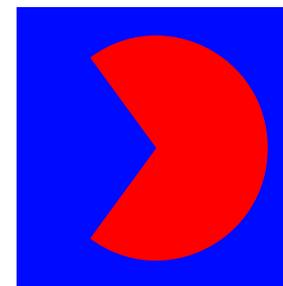
$$r_O = 1.05 r_S$$



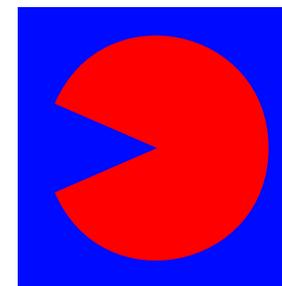
$$r_O = 1.3 r_S$$



$$r_O = 3 r_S / 2$$



$$r_O = 2.5 r_S$$



$$r_O = 6 r_S$$

## Rotating black holes:

Shadow no longer circular

Shape of shadow can be used for discriminating between different black holes

### Shadow of Kerr black hole:

J. Bardeen in C. DeWitt and B. DeWitt (eds.): “Black Holes”  
Gordon & Breach (1973)

### Shadow in Plebański spacetimes:

A. Grenzebach, V.P. C. Lämmerzahl: Phys. Rev. D 89, 124004  
(2014)

Plebański metric in Boyer–Lindquist coordinates  $(r, \vartheta, \varphi, t)$ :

$$g_{\mu\nu}dx^\mu dx^\nu = \Sigma\left(\frac{1}{\Delta_r}dr^2 + \frac{1}{\Delta_\vartheta}d\vartheta^2\right) + \frac{1}{\Sigma}\left((\Sigma + a\chi)^2\Delta_\vartheta \sin^2 \vartheta - \Delta_r\chi^2\right)d\varphi^2 \\ + \frac{2}{\Sigma}\left(\Delta_r\chi - a(\Sigma + a\chi)\Delta_\vartheta \sin^2 \vartheta\right)dt d\varphi - \frac{1}{\Sigma}\left(\Delta_r - a^2\Delta_\vartheta \sin^2 \vartheta\right)dt^2$$

$$\Sigma = r^2 + (\ell + a \cos \vartheta)^2$$

$$\chi = a \sin^2 \vartheta - 2\ell \cos \vartheta$$

$$\Delta_r = r^2 - 2mr + a^2 - \ell^2 + q_e^2 + q_m^2 \\ - \Lambda\left((a^2 - \ell^2)\ell^2 + \left(\frac{1}{3}a^2 + 2\ell^2\right)r^2 + \frac{1}{3}r^4\right)$$

$$\Delta_\vartheta = 1 + \Lambda\left(\frac{4}{3}a\ell \cos \vartheta + \frac{1}{3}a^2 \cos^2 \vartheta\right)$$

$m$ : mass

$a$ : spin

$q_e$ : el. charge

$q_m$ : magn.charge

$\ell$ : NUT parameter

$\Lambda$ : cosmol. constant

Lightlike geodesics:

$$\Sigma^2 \dot{\vartheta}^2 = \Delta_{\vartheta} K - \frac{(\chi E - L_z)^2}{\sin^2 \vartheta} =: \Theta(\vartheta)$$

$$\Sigma^2 \dot{r}^2 = ((\Sigma + a\chi)E - aL_z)^2 - \Delta_r K =: R(r)$$

$E$  = energy,  $L_z$  = angular momentum,  $K$  = Carter constant

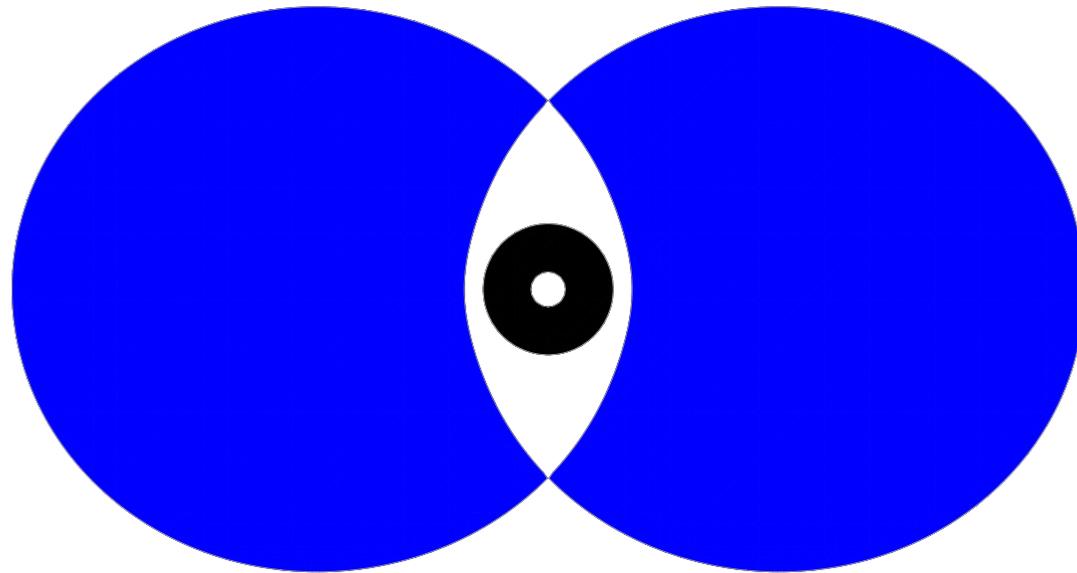
Spherical lightlike geodesics exist in the region where

$$R(r) = 0, \quad R'(r) = 0, \quad \Theta(\vartheta) \geq 0.$$

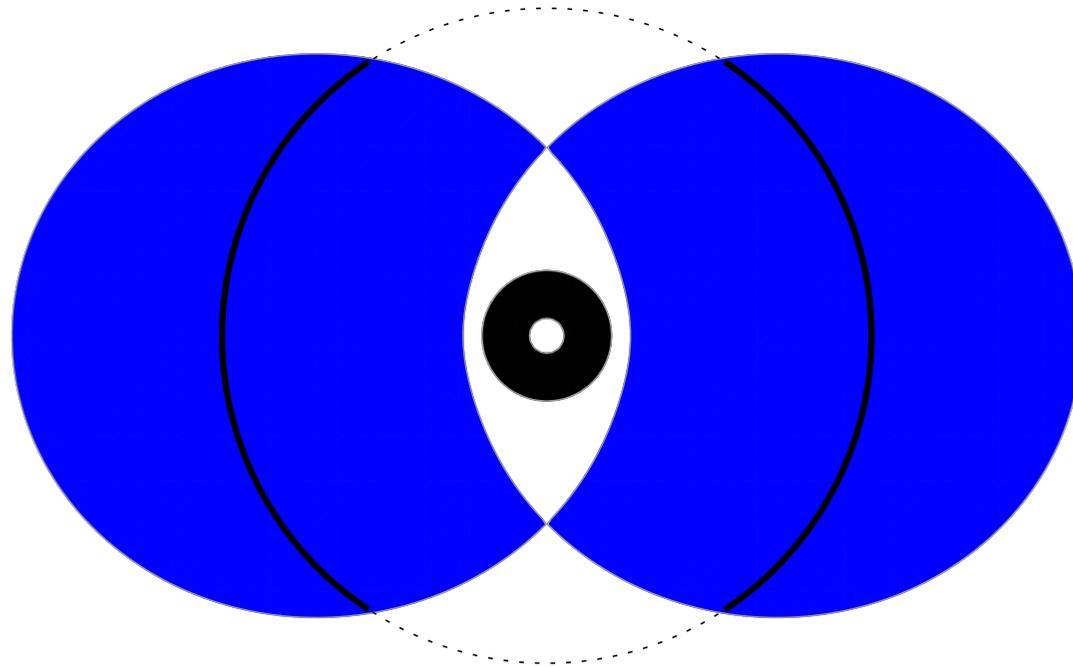
$$(4r\Delta_r - \Sigma\Delta'_r)^2 \leq 16a^2r^2\Delta_r\Delta_{\vartheta}\sin^2\vartheta \quad (\text{“photon region”})$$

(unstable if  $R''(r) \geq 0$ )

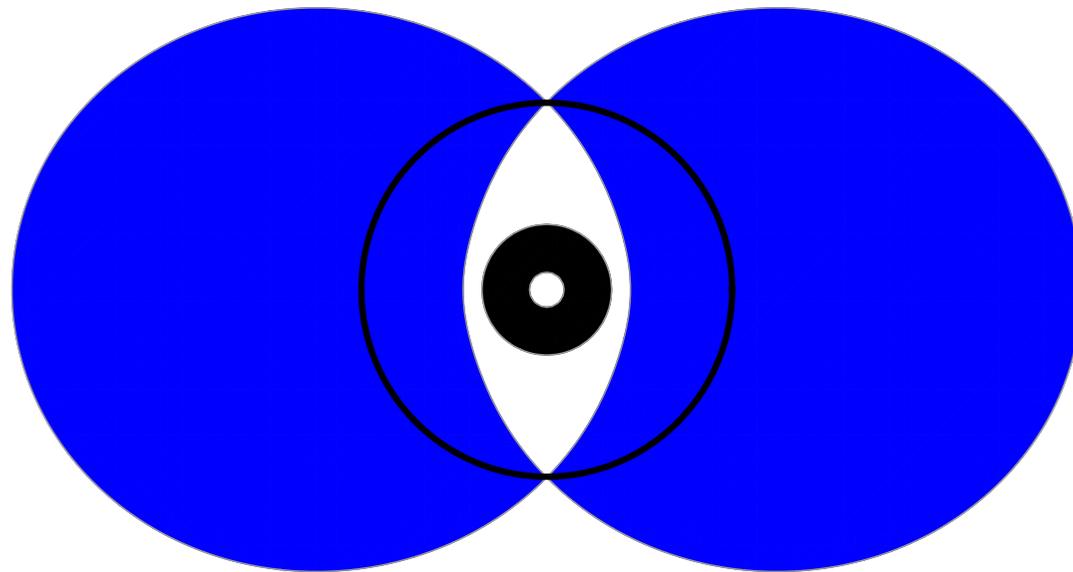
Photon region for Kerr black hole with  $a = 0.75 m$



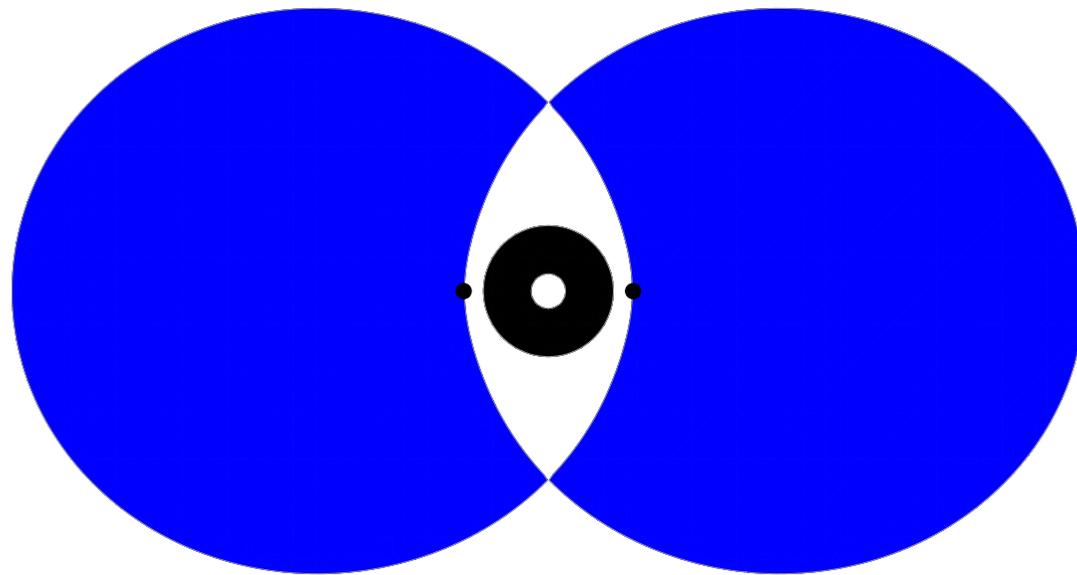
Photon region for Kerr black hole with  $a = 0.75 m$



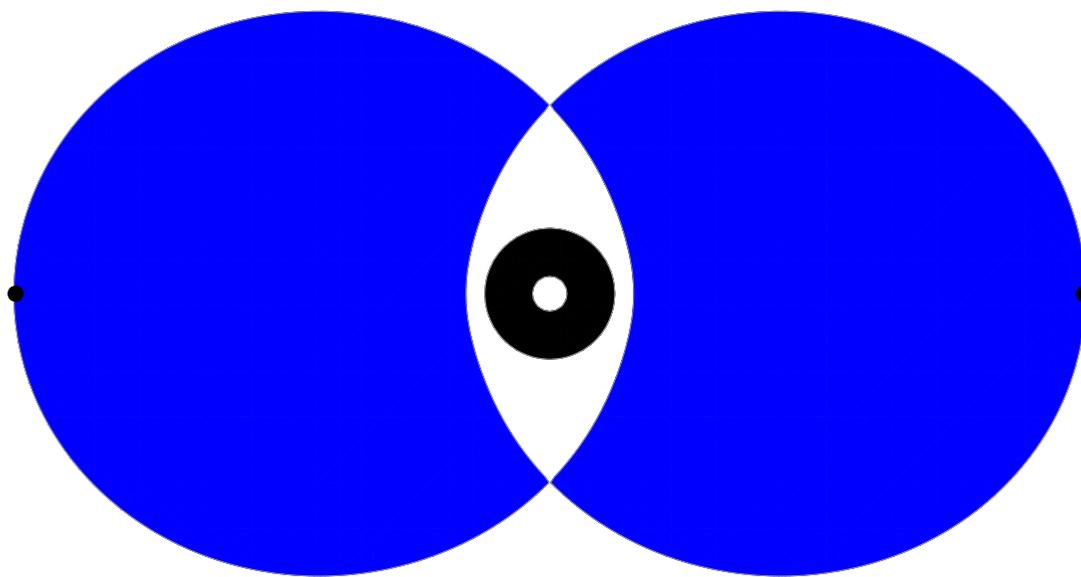
Photon region for Kerr black hole with  $a = 0.75 m$



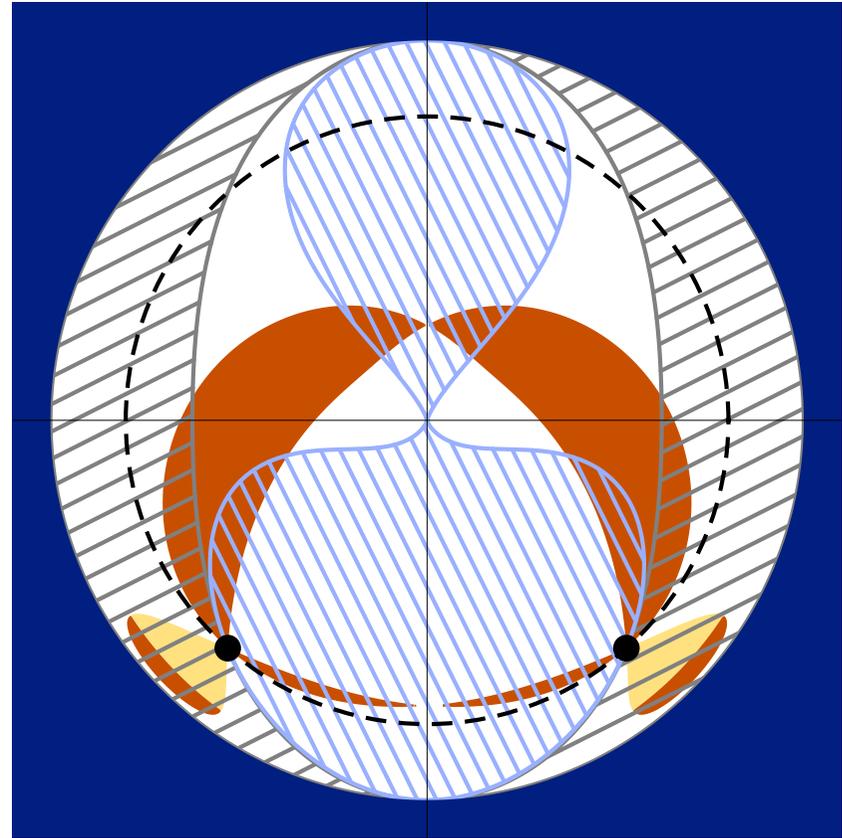
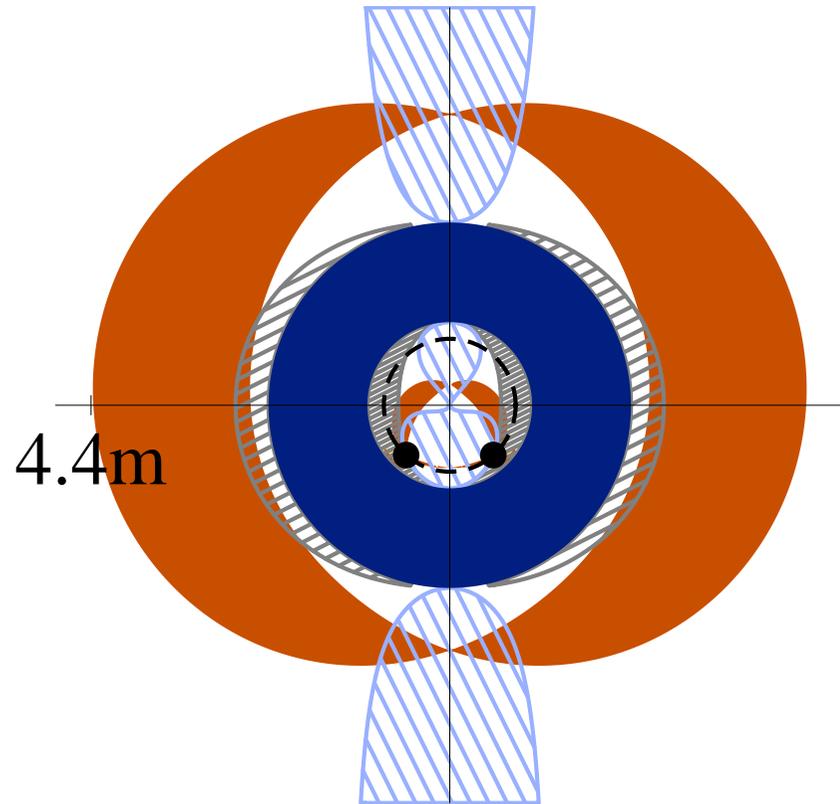
Photon region for Kerr black hole with  $a = 0.75 m$



Photon region for Kerr black hole with  $a = 0.75 m$



... with NUT parameter, charge, cosmological constant



The shadow is determined by light rays that approach an unstable spherical lightlike geodesic.

Choose observer at  $r_O$  and  $\vartheta_O$

Choose tetrad

$$e_0 = \frac{(\Sigma + a\chi)\partial_t + a\partial_\varphi}{\sqrt{\Sigma\Delta_r}} \Big|_{(r_O, \vartheta_O)}$$

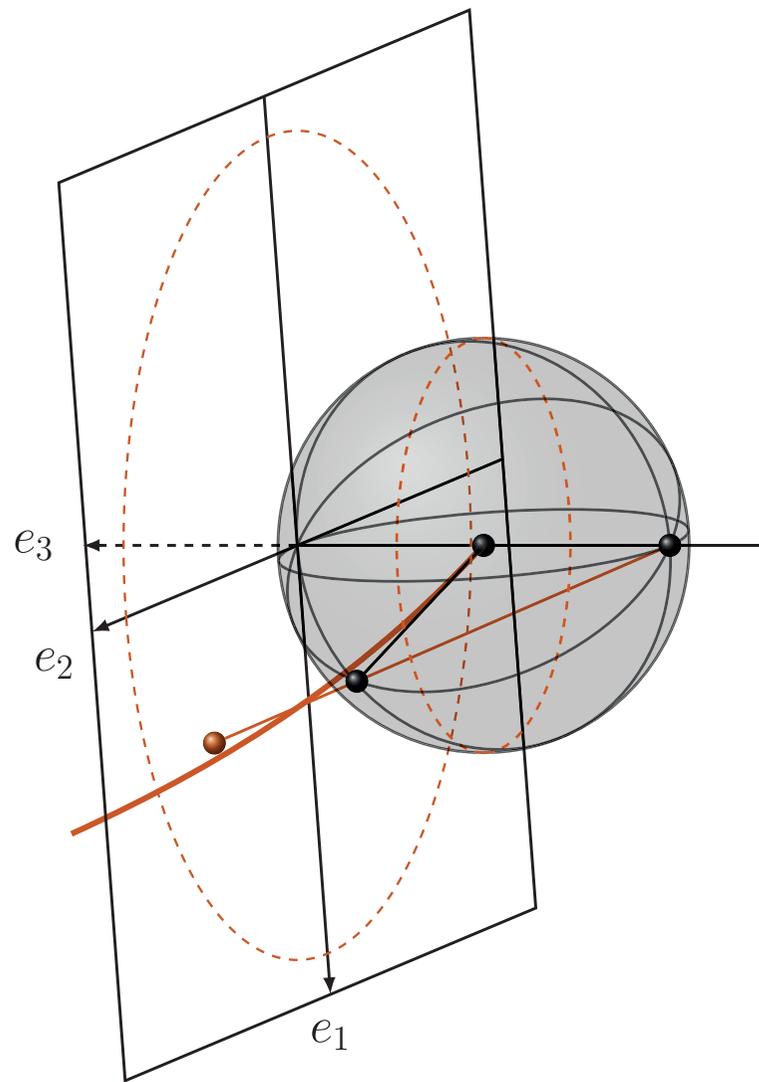
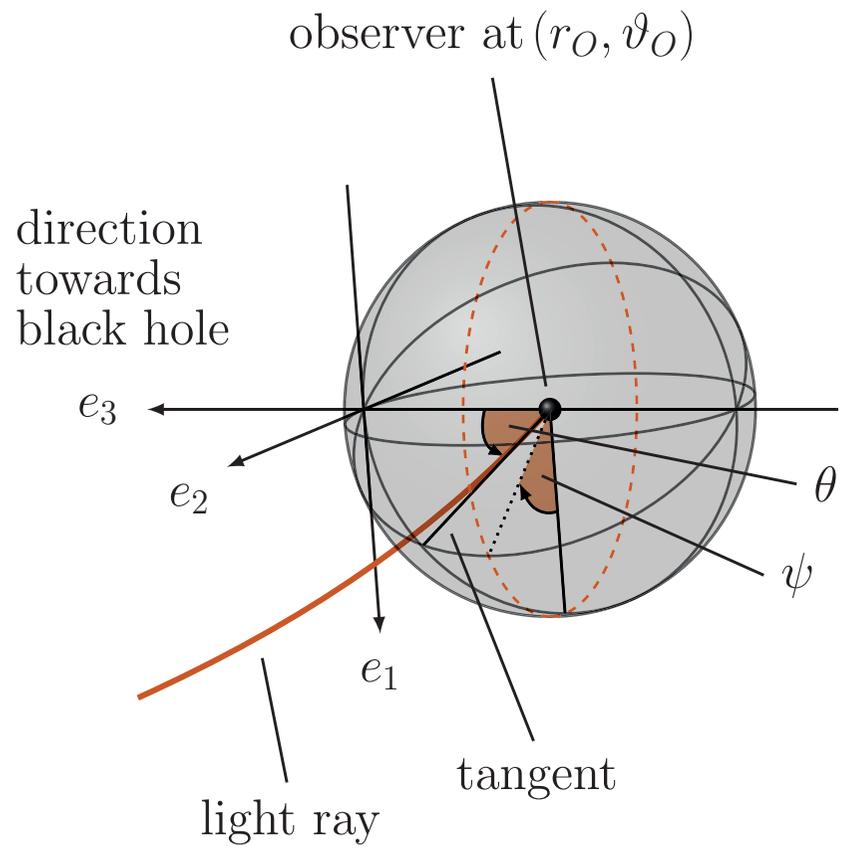
$$e_1 = \sqrt{\frac{\Delta_\vartheta}{\Sigma}} \partial_\vartheta \Big|_{(r_O, \vartheta_O)}$$

$$e_2 = -\frac{(\partial_\varphi + \chi\partial_t)}{\sqrt{\Sigma\Delta_\vartheta} \sin \vartheta} \Big|_{(r_O, \vartheta_O)}$$

$$e_3 = -\sqrt{\frac{\Delta_r}{\Sigma}} \partial_r \Big|_{(r_O, \vartheta_O)}$$

Observer with other 4-velocity: Aberration

A. Grenzebach, to appear, arXiv:1502.02861



celestial coordinates at observer  $(\theta, \psi)$

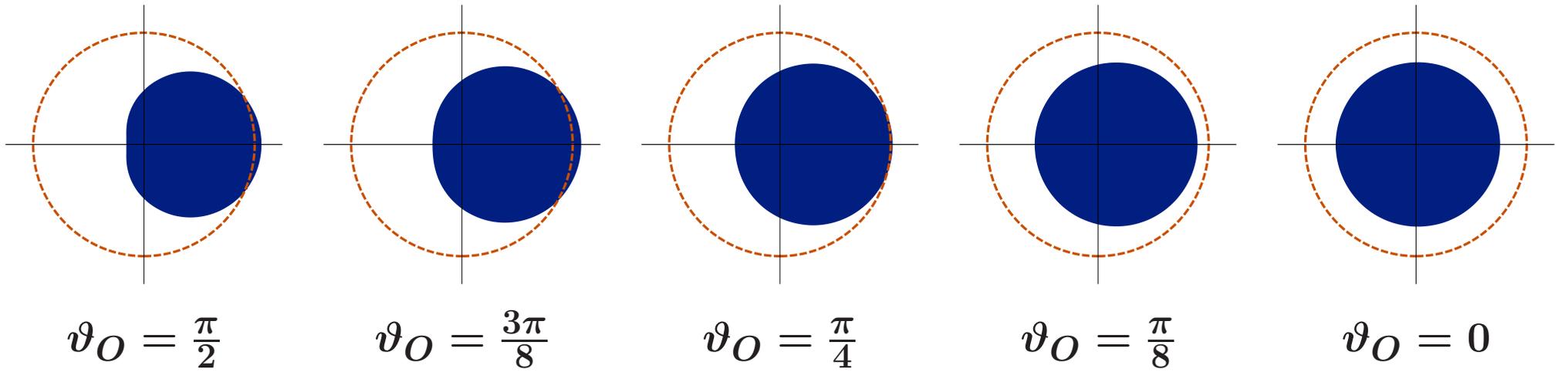
constants of motion  $\left(K_E = \frac{K}{E^2}, \tilde{L}_E = \frac{L_z}{E} - a\right)$

$$\sin \theta = \frac{\sqrt{\Delta_r K_E}}{r^2 + \ell^2 - a\tilde{L}_E} \Big|_{r=r_O}, \quad \sin \psi = \frac{\tilde{L}_E + a \cos^2 \vartheta + 2\ell \cos \vartheta}{\sqrt{\Delta_\vartheta K_E} \sin \vartheta} \Big|_{\vartheta=\vartheta_O}$$

$$K_E = \frac{16r^2 \Delta_r}{(\Delta'_r)^2} \Big|_{r=r_p}, \quad a\tilde{L}_E = \left(r^2 + \ell^2 - \frac{4r \Delta_r}{\Delta'_r}\right) \Big|_{r=r_p}$$

Analytic formula for shadow allows to extract parameters of the spacetime from the shape of the shadow

Shadow of black hole with  $a = a_{\max}$  for observer at  $r_O = 5m$



# Perspectives of observations

## Object at the centre of our galaxy:

$$\text{Mass} = 4 \times 10^6 M_{\odot}$$

$$\text{Distance} = 8 \text{ kpc}$$

If it is a Schwarzschild black hole, the diameter of the shadow should be  $\approx 54 \mu\text{as}$

(corresponds to a grapefruit on the moon)

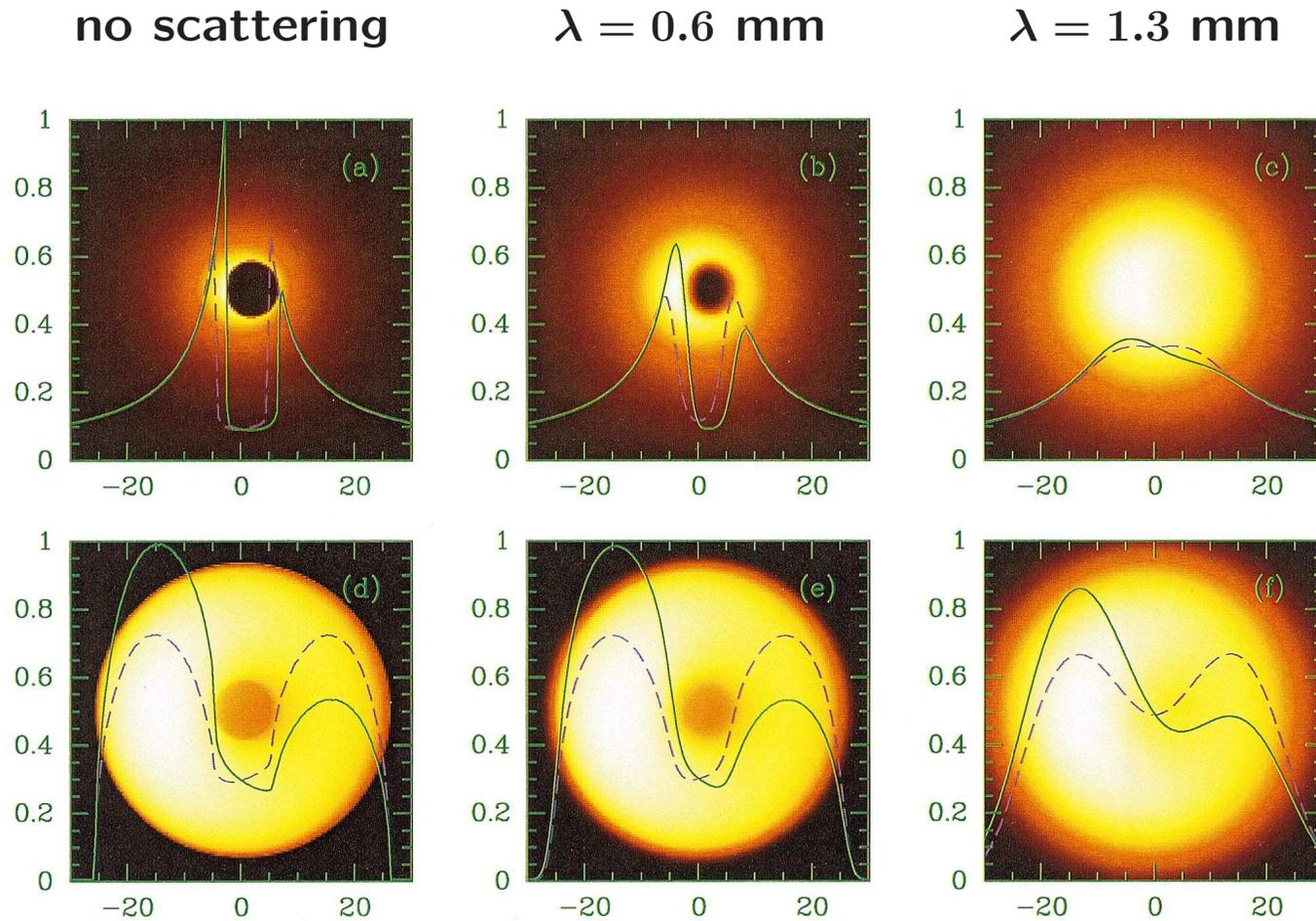
## Object at the centre of M87:

$$\text{Mass} = 3 \times 10^9 M_{\odot}$$

$$\text{Distance} = 16 \text{ Mpc}$$

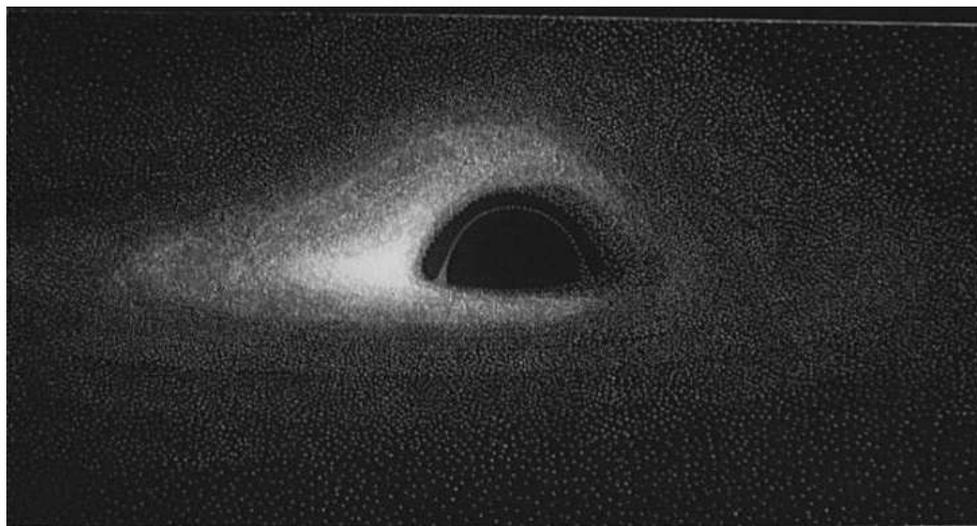
If it is Schwarzschild black hole, the diameter of the shadow should be  $\approx 9 \mu\text{as}$

# Kerr shadow with emission region and scattering taken into account:

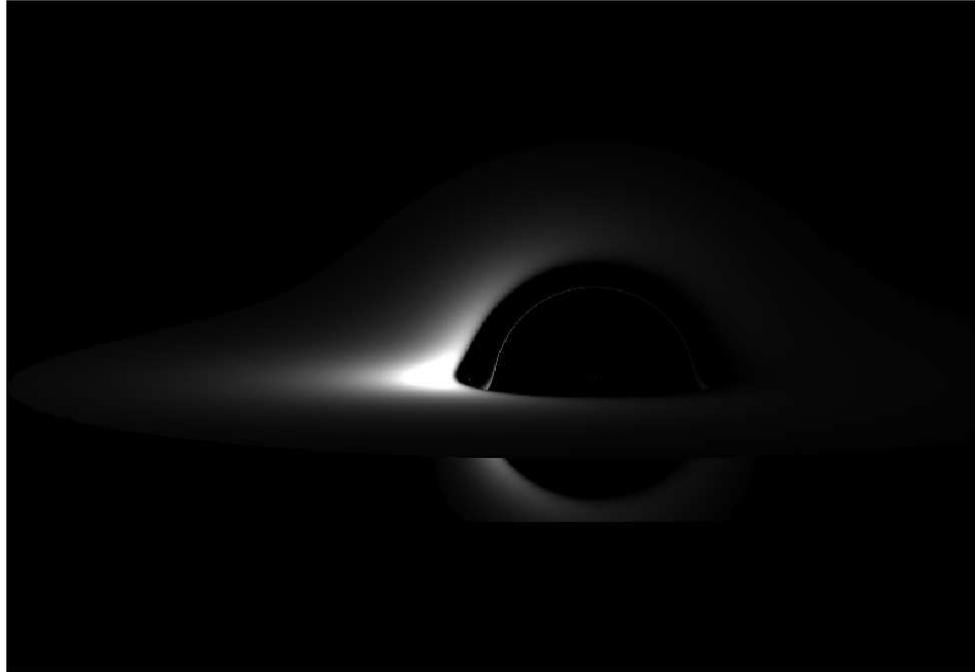


H. Falcke, F. Melia, E. Agol: *Astrophys. J.* 528, L13 (2000)

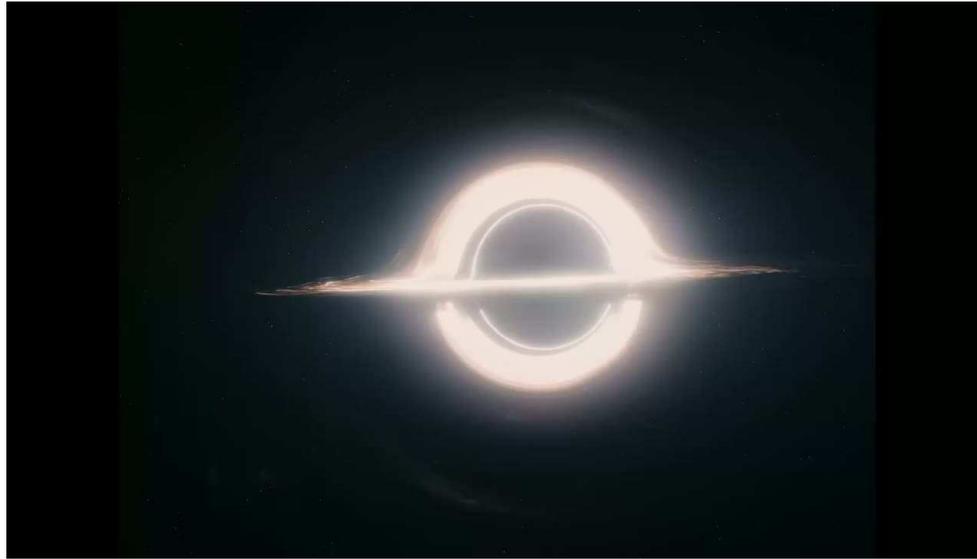
Observations should be done at sub-millimeter wavelength



**J.-P. Luminet (1979)**



T. Müller (2012)

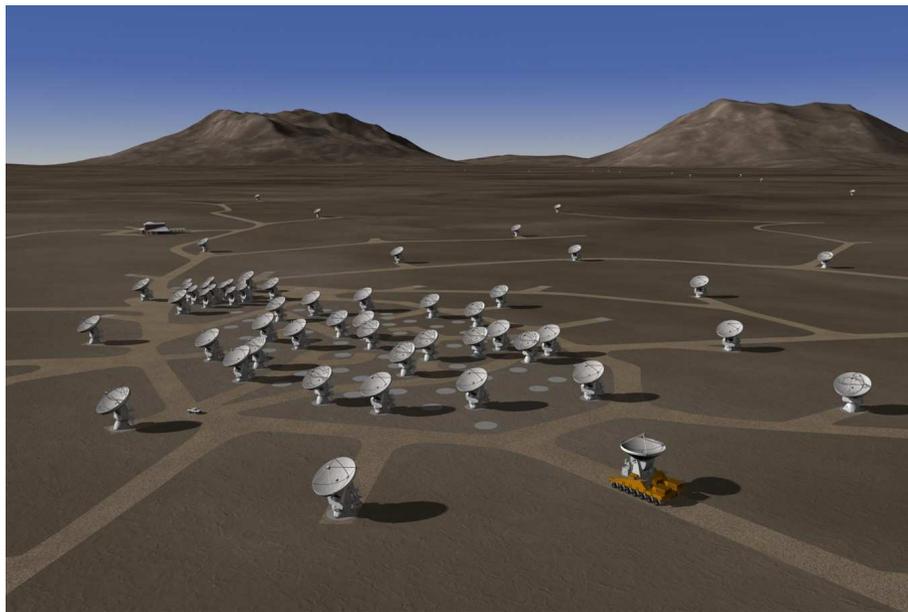


from the Movie “Interstellar” (2014)

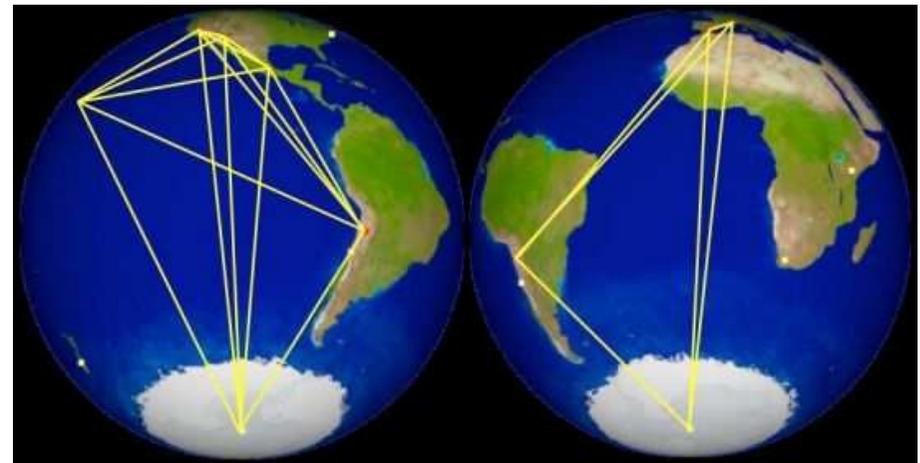
Projects to view the shadow with sub-millimeter VLBI:

Event Horizon Telescope (EHT),

Using ALMA, NOEMA, LMT, CARMA, South Pole Telescope ...



ALMA



EHT

## BlackHoleCam



H. Falcke, L. Rezzolla, M. Kramer

## Millimetron ( $\approx 2025$ )



Good chance to see the shadow of the centre of our galaxy within a few years