

Gravitational lensing by compact objects

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Part I

- **Historic introduction**
- **Observations**
- **The weak-field formalism of lensing**

Part II

- **Lensing beyond the weak-field formalism**
- **Spherically symmetric black holes**
- **Black hole impostors**

J. Wambsganss: "Gravitational Lensing in Astronomy",

<http://www.livingreviews.org/Irr-1998-12>

VP: "Gravitational Lensing from a Spacetime Perspective",

<http://www.livingreviews.org/Irr-2004-9>

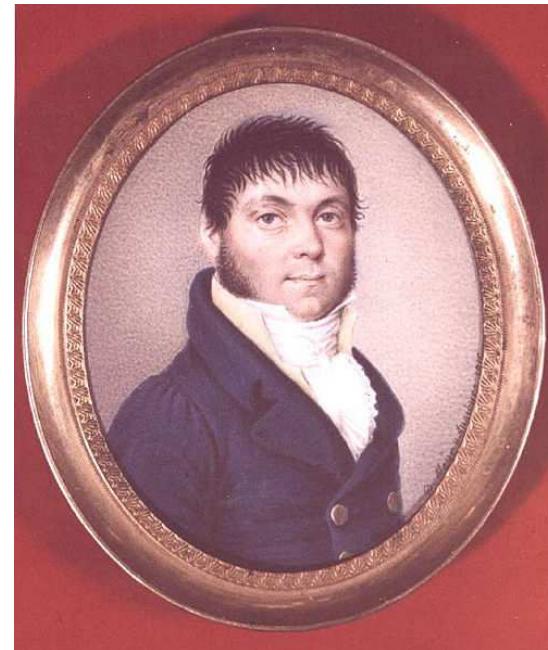
Part I

Newtonian light deflection:

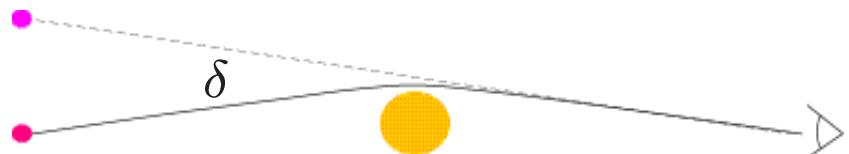
Henry Cavendish 1786, Johann v. Soldner 1801



Henry Cavendish
(1731 – 1810)



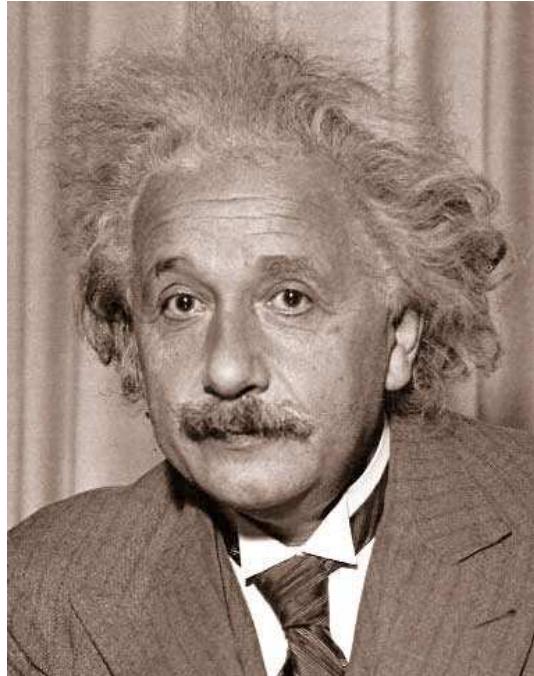
Johann v. Soldner
(1776 – 1833)



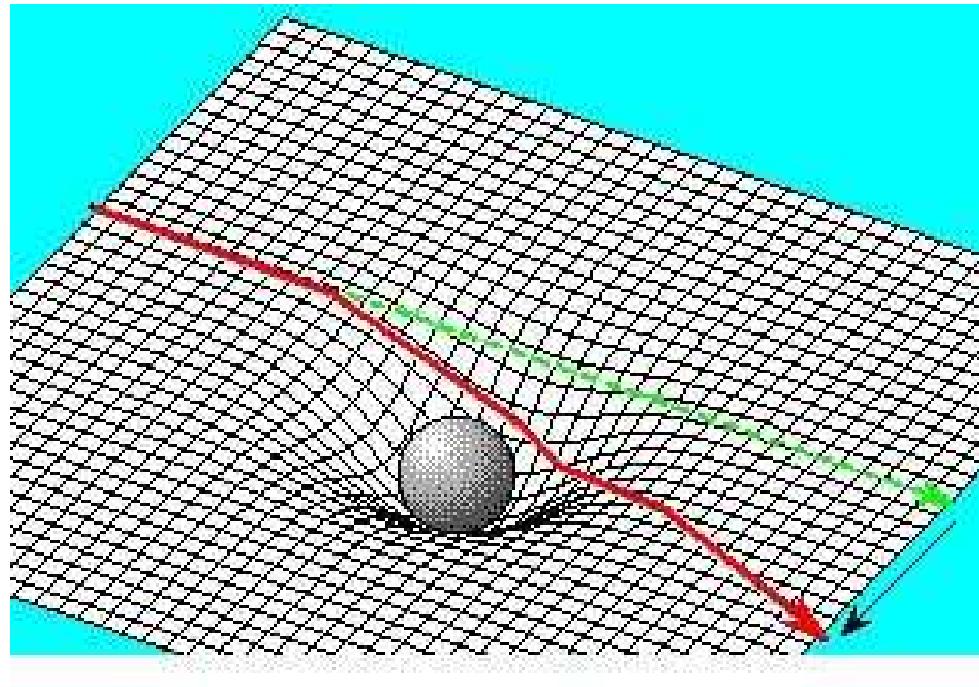
$$\delta = \frac{2 G M}{c^2 R}, \quad \delta_{\odot} = 0.87''$$

Einsteinian light deflection:

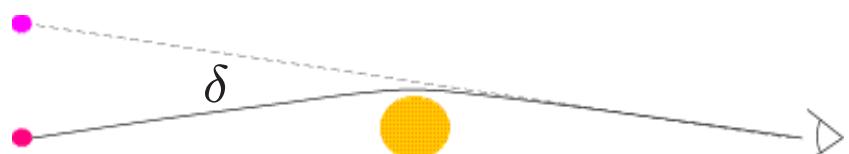
Albert Einstein 1915



Albert Einstein
(1879-1955)



curved spacetime



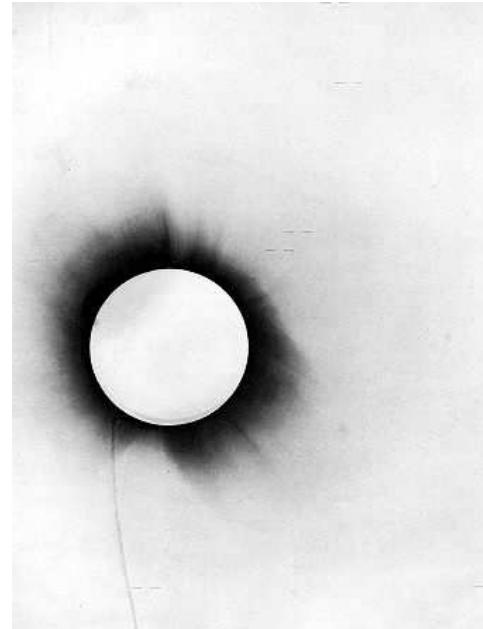
$$\delta = \frac{4 G M}{c^2 R}, \quad \delta_{\odot} = 1.75''$$

Confirmation of Einsteinian light deflection:

Arthur S. Eddington 1919



Arthur S. Eddington
(1882 – 1944)

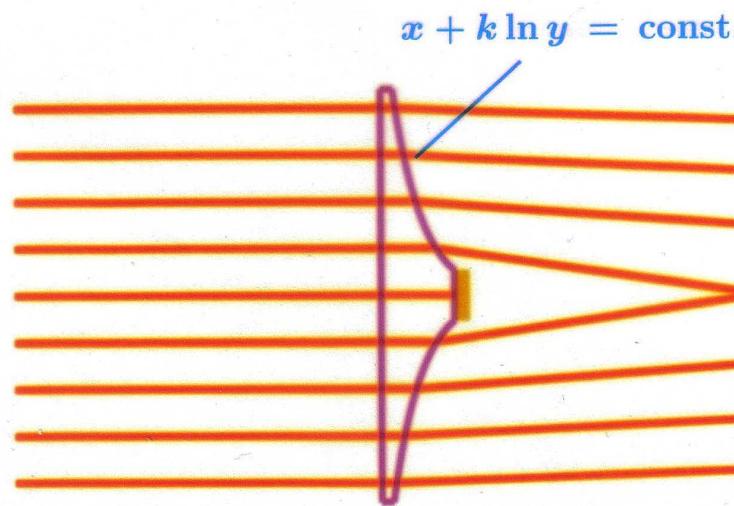
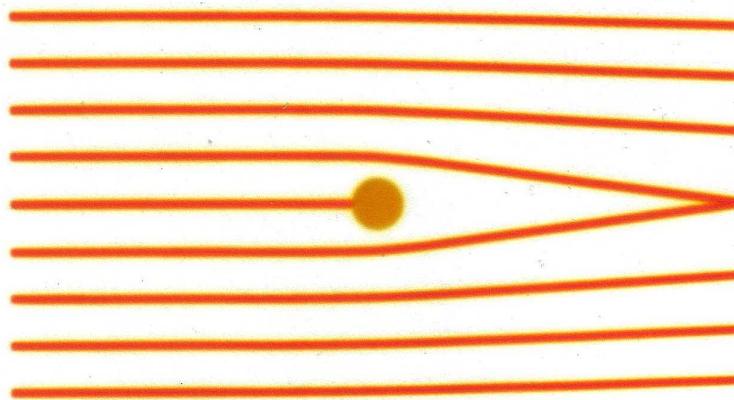


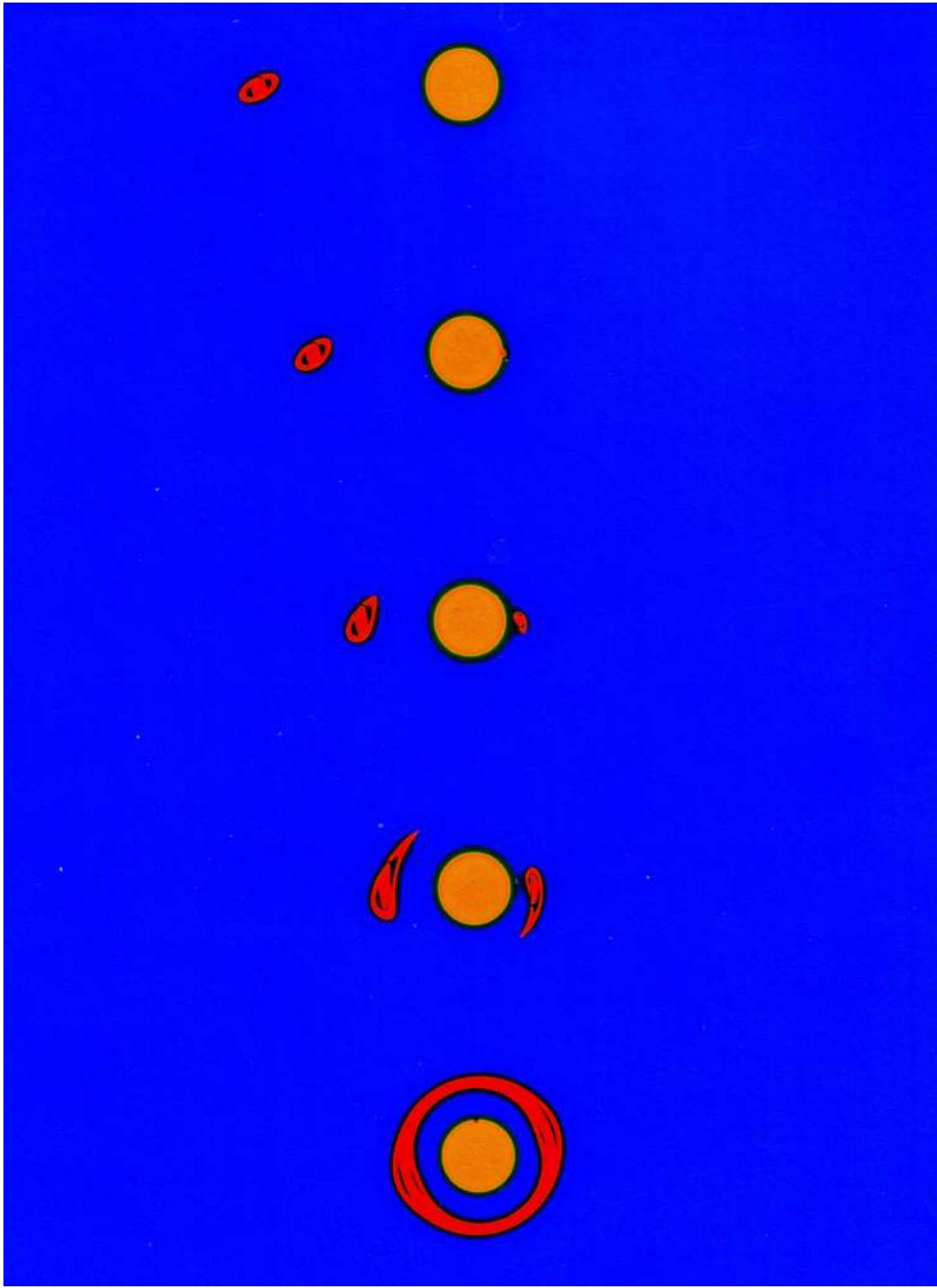
Sun eclipse 1919

Principle: $\delta = 1.61'' \pm 0.40''$, Sobral: $\delta = 1.98'' \pm 0.16''$

D. Lebach et al. (1995):
$$\left| \frac{\delta - \delta_{\text{Einstein}}}{\delta_{\text{Einstein}}} \right| \leq 0.02 \%$$

Simulation of light deflection with a plastic lens:





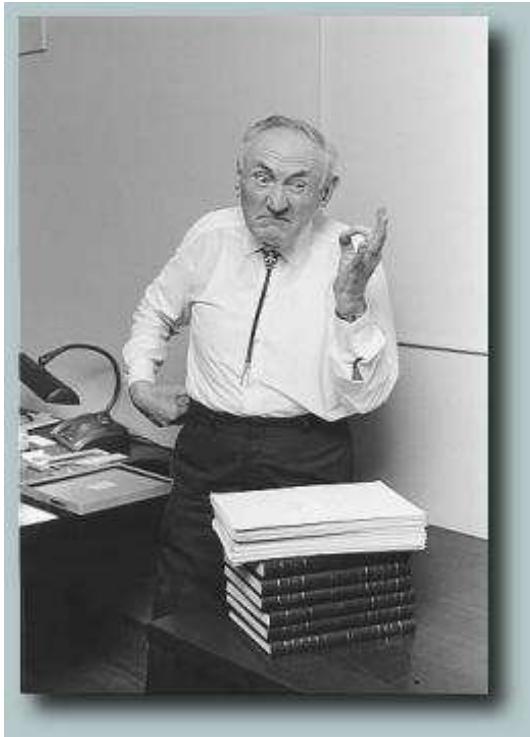
Are multiple images and Einstein rings observable?

star “lensed” by star

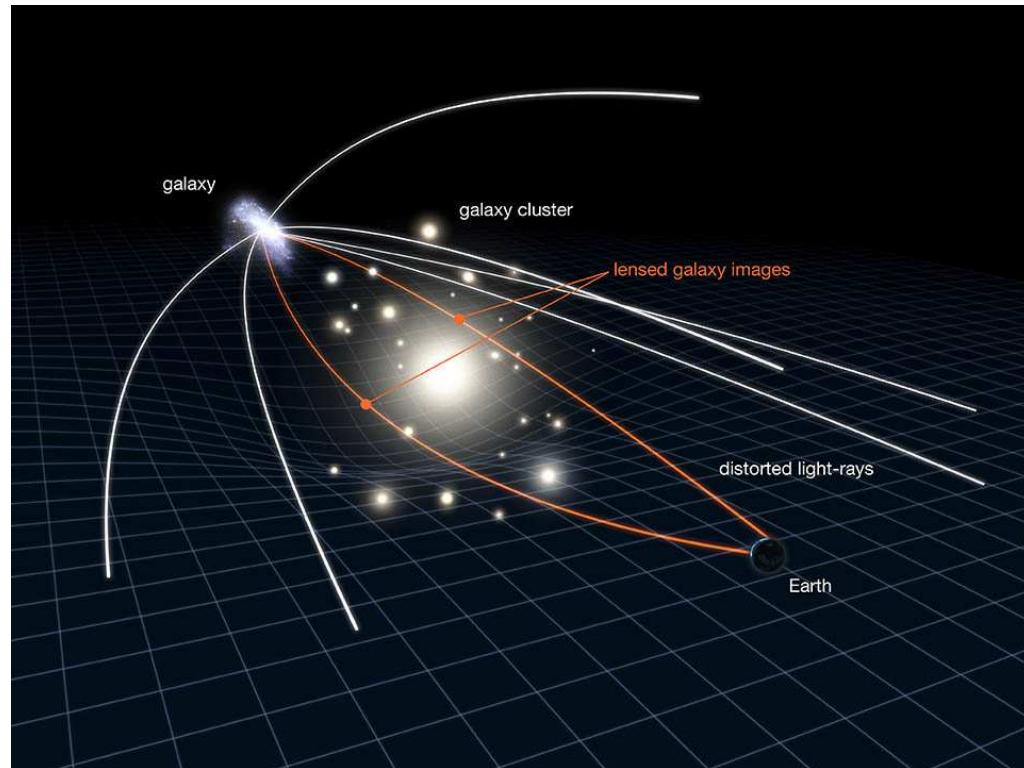
Albert Einstein (1936):
not observable

galaxy “lensed” by galaxy

Fritz Zwicky (1937):
probably observable

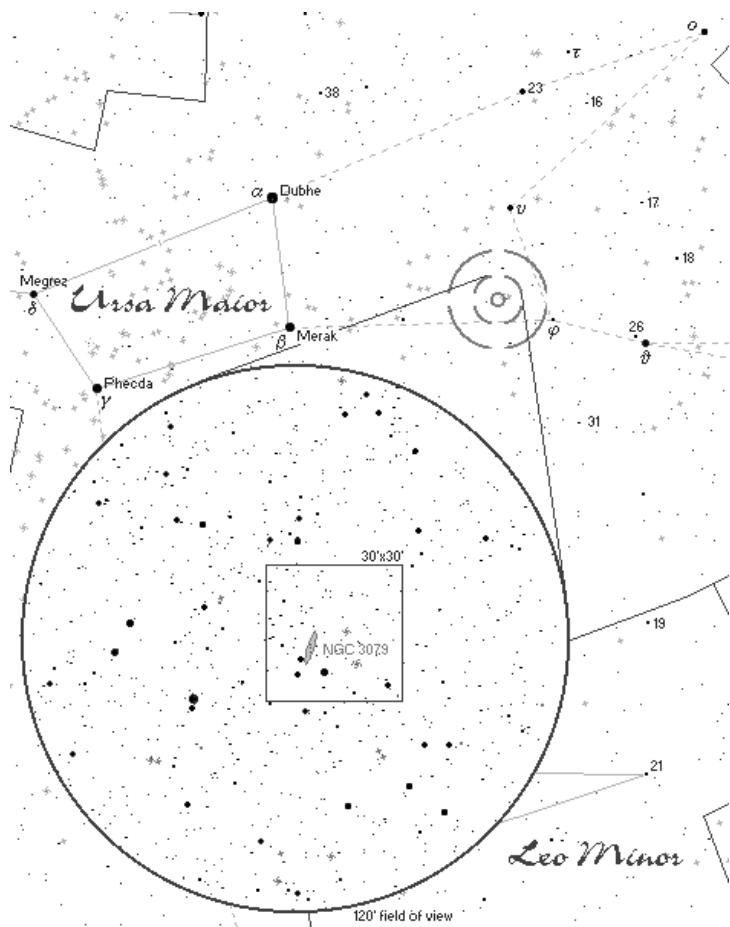


Fritz Zwicky
(1898 – 1974)

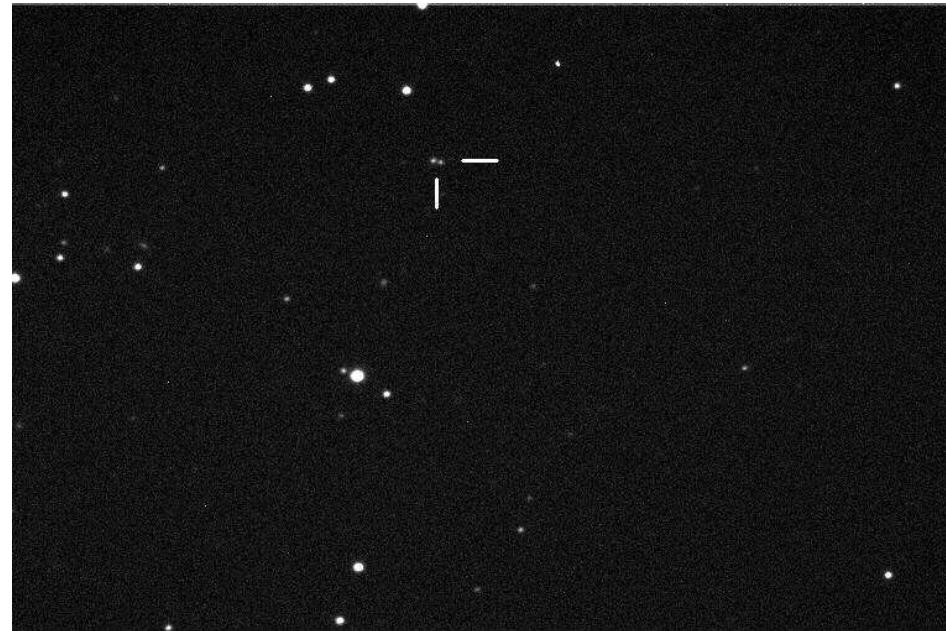


Detection of first case of multiple imaging:

D. Walsh, R. Carlswell, R. Weyman (1979)



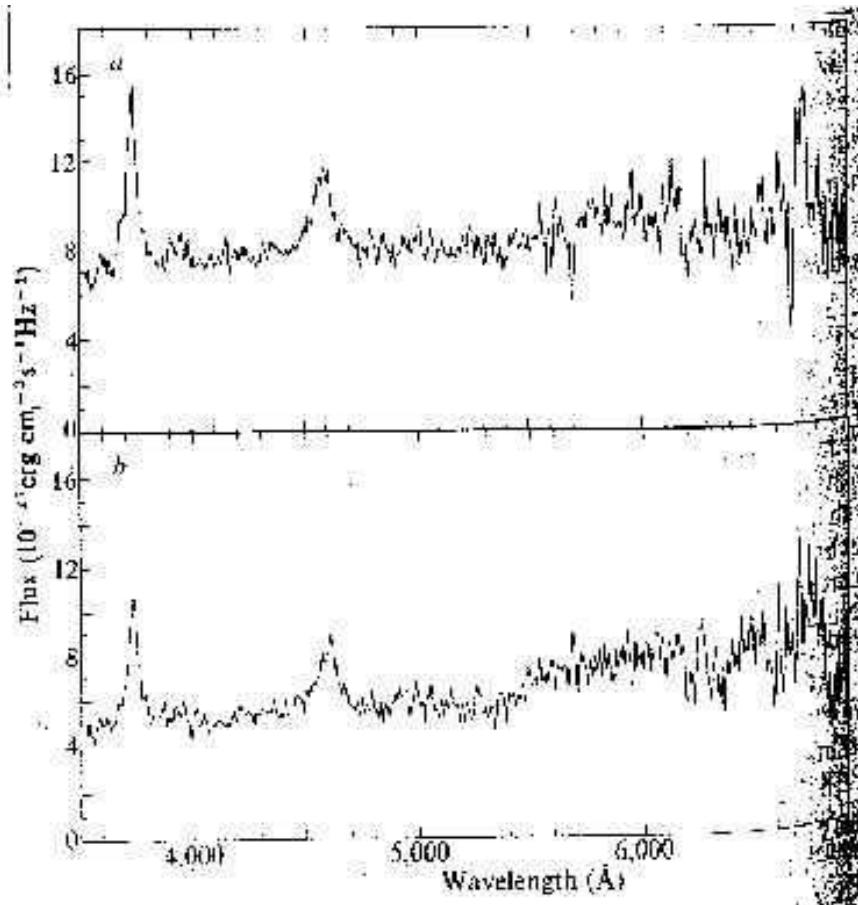
Double quasar QS0 0957 +561



angular separation = 6''

magnitude = 17^m

Spectra

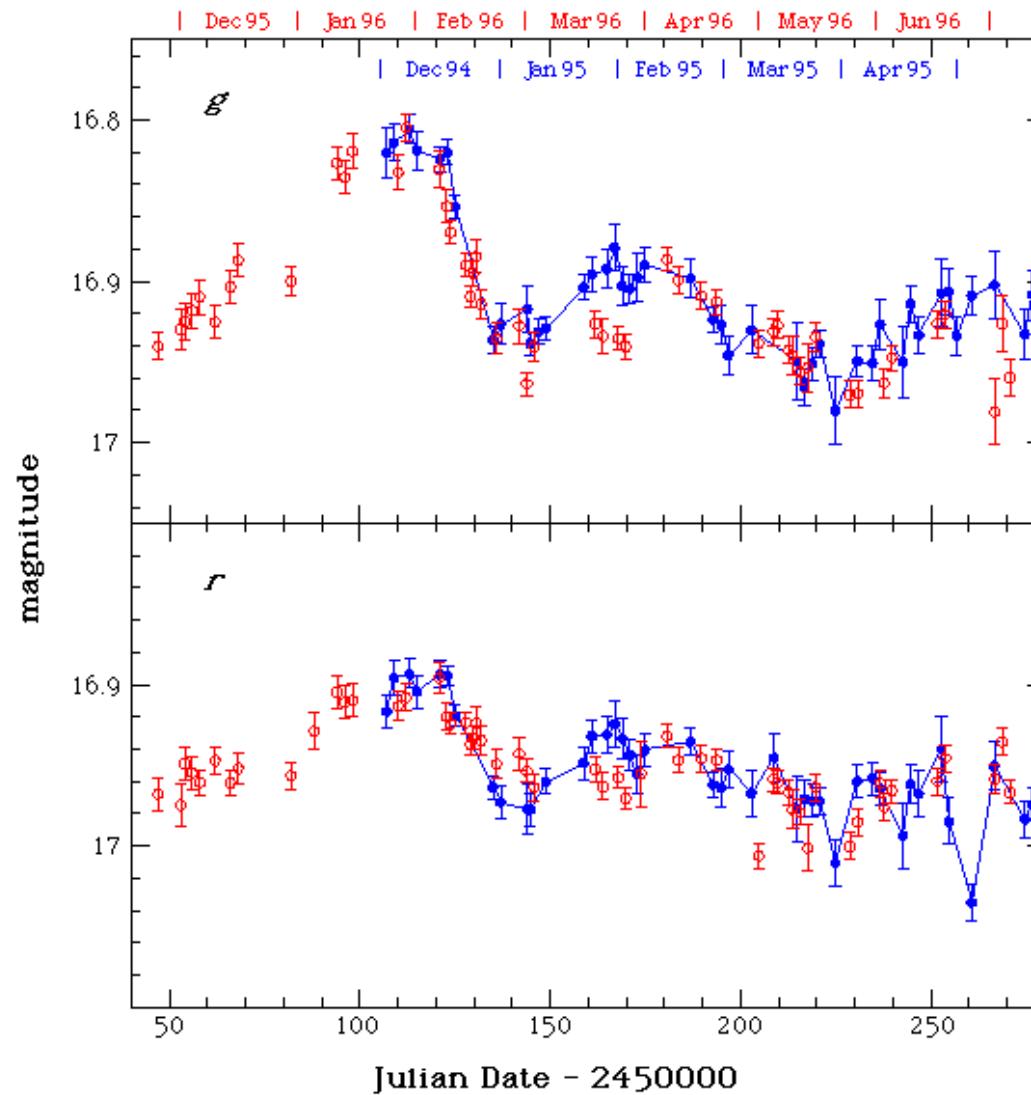


Red shift:

$$z_{\text{quasar}} = 1.4$$

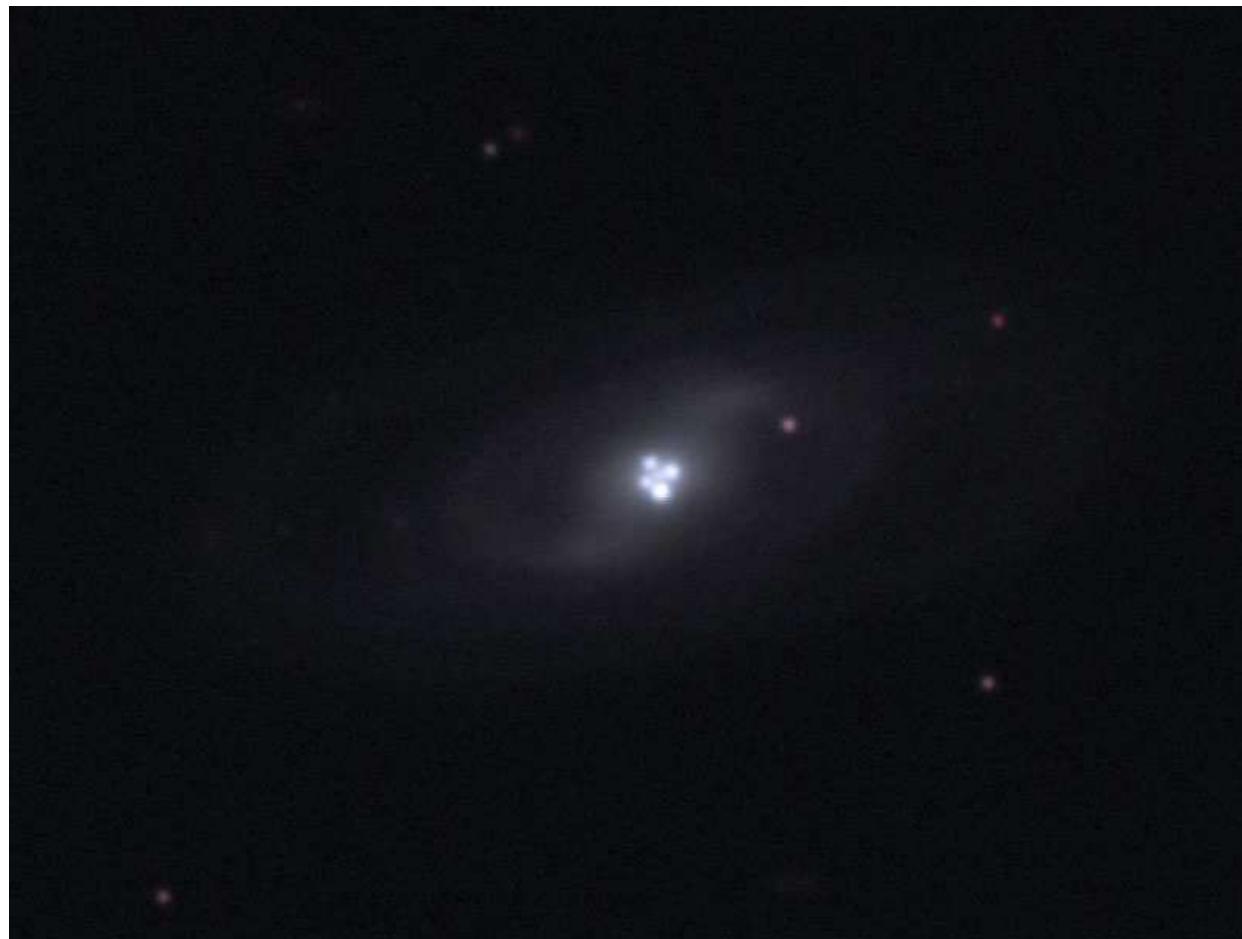
$$z_{\text{galaxy}} = 0.4$$

Light curves



time delay = 417 Tage

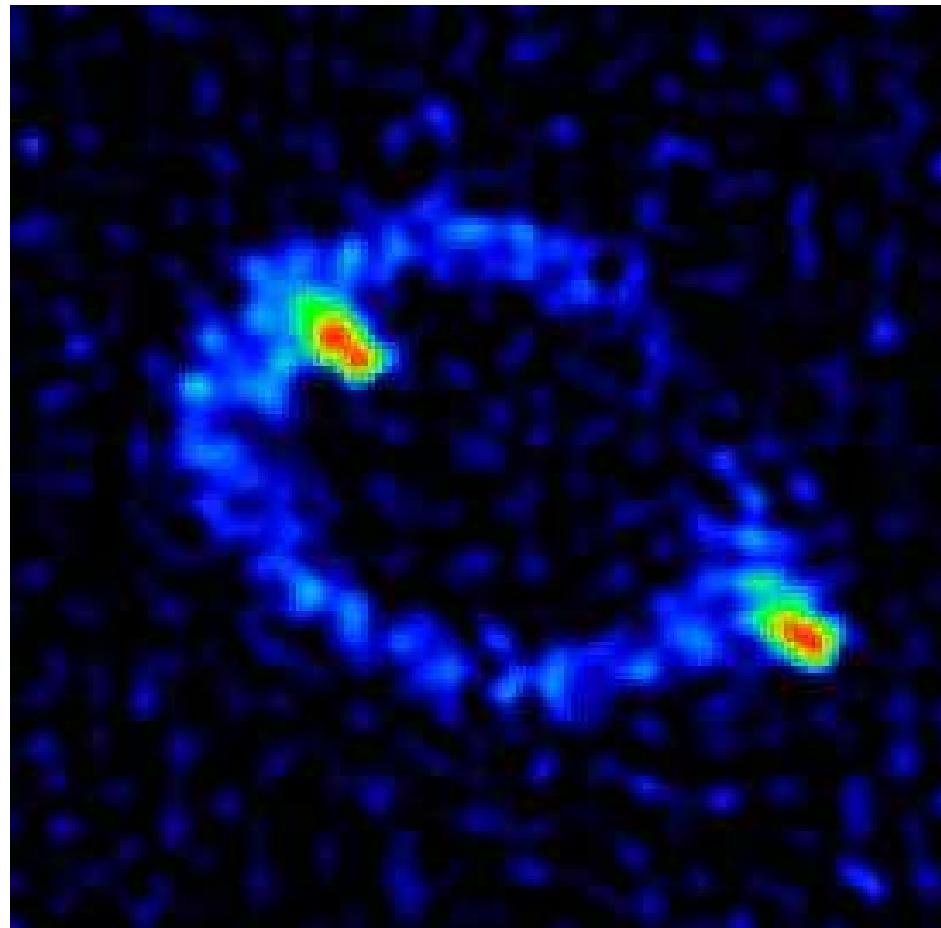
Other example of multiple imaging:



Einstein Cross

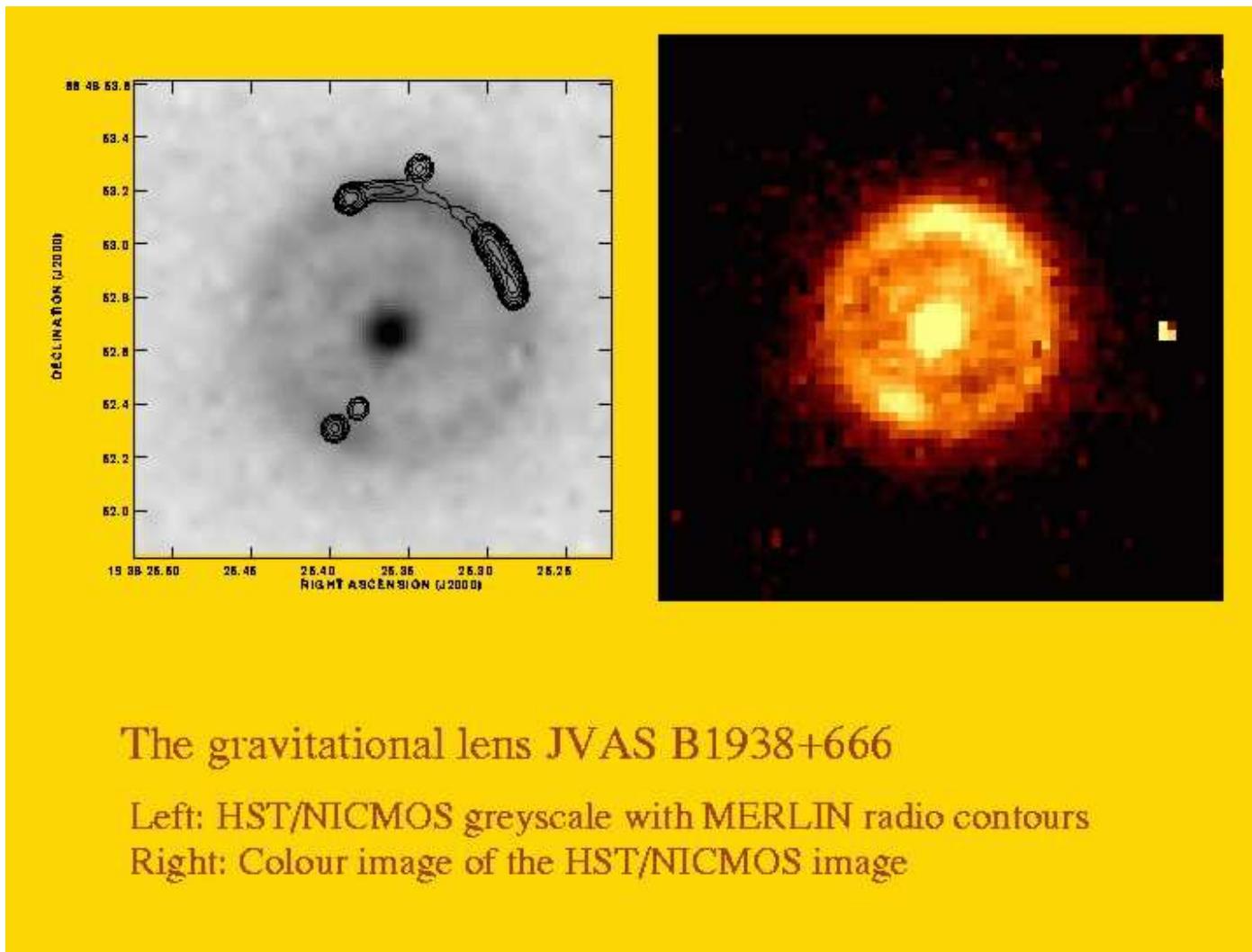
First Einstein ring observed:

Jacqueline Hewitt et al. (1988)



MG1131+0456

Other example of Einstein ring:



Discovery of “Giant Luminous Arcs”:

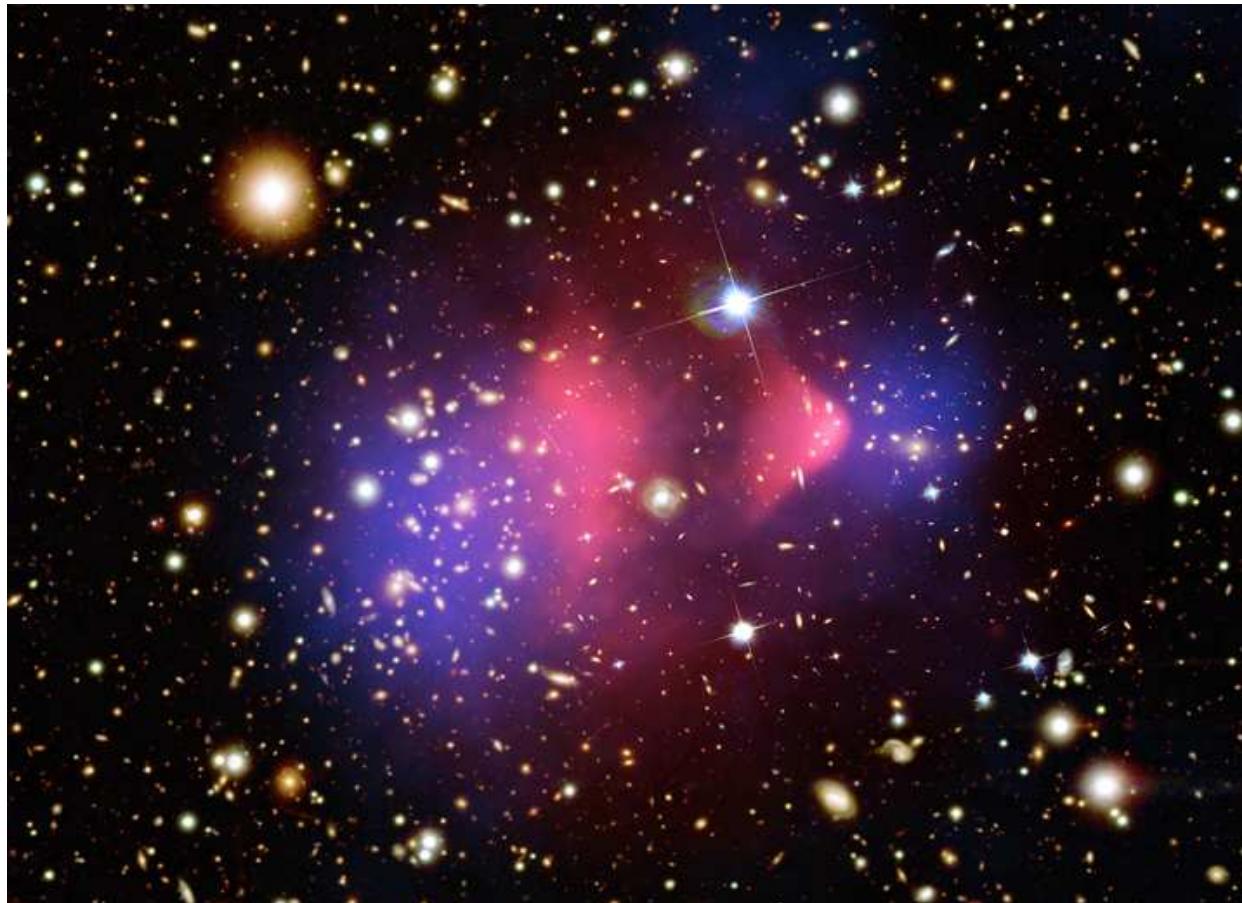
R. Lynds, V. Petrosian (1986), G. Soucail et al. (1987)



Abell 370

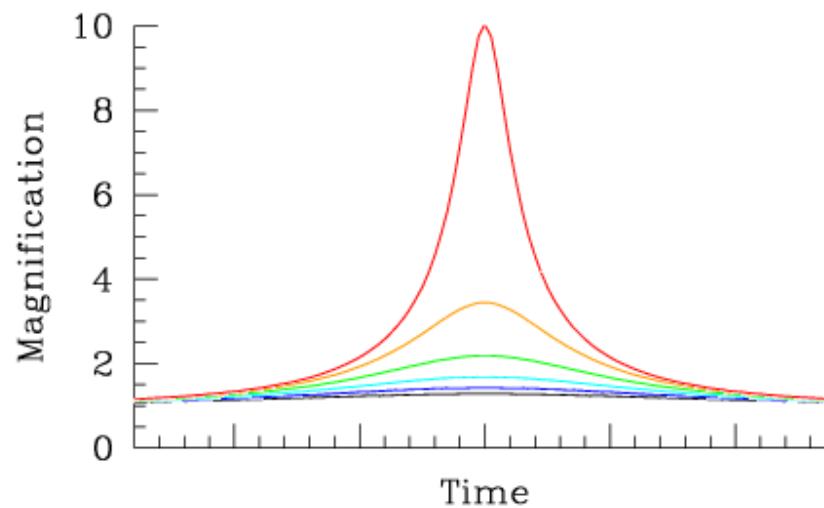
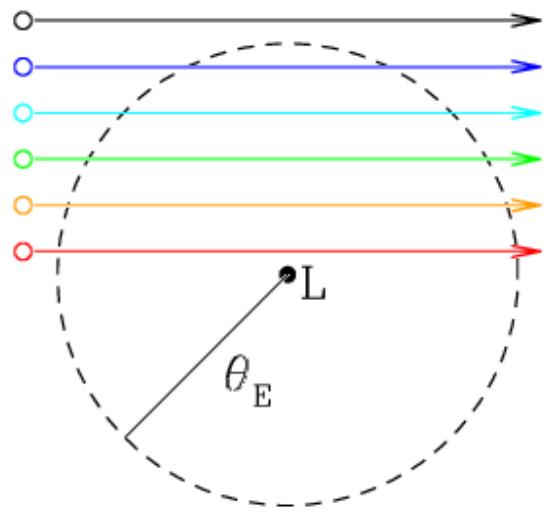
Gravitational lensing as a tool:

(a) Detecting dark matter in galaxy clusters



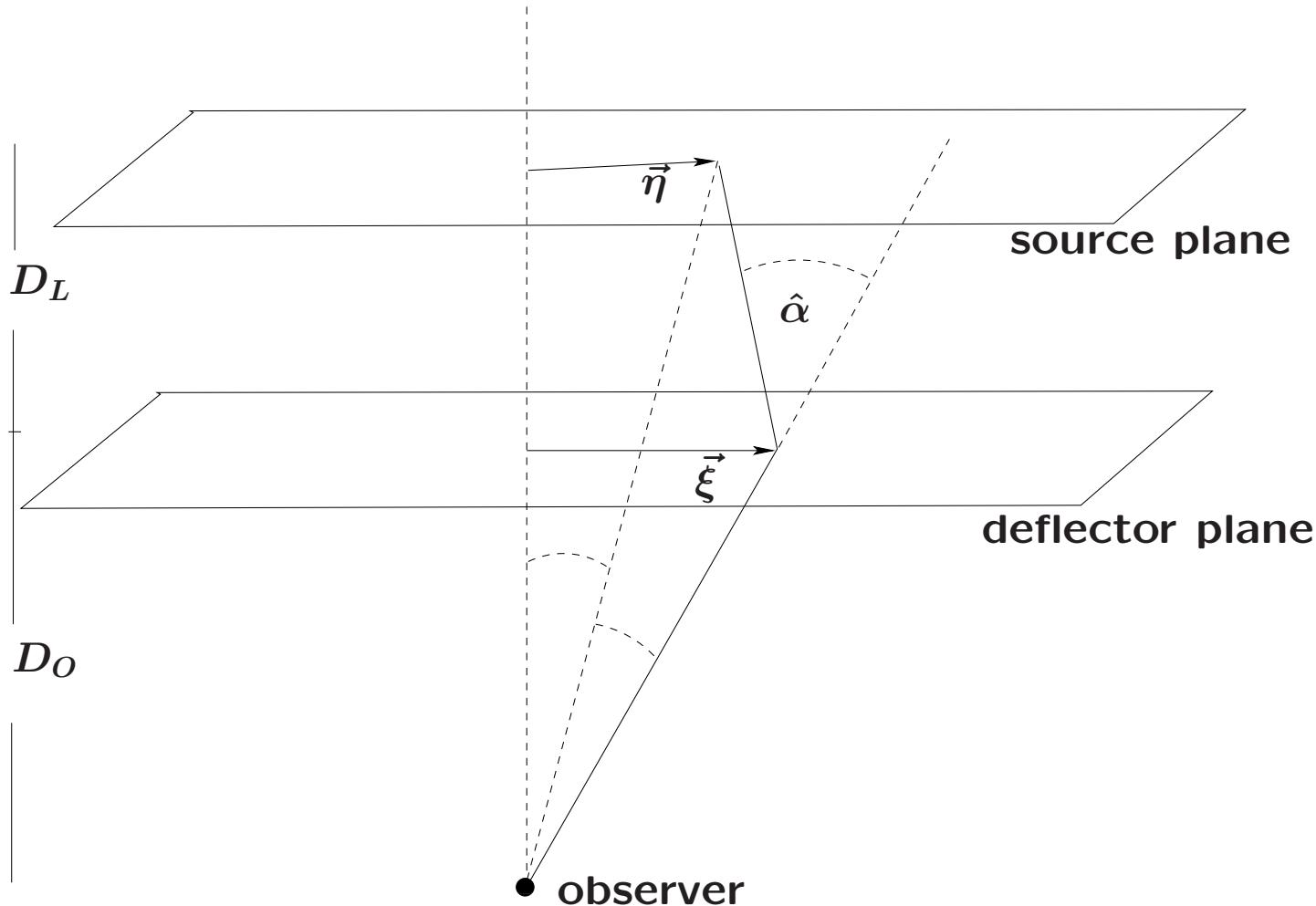
Bullet Cluster

(b) Detecting Brown Dwarfs, Exoplanets, etc.



“Microlensing”

Lens map of the weak-field formalism (S. Refsdal, 1963)



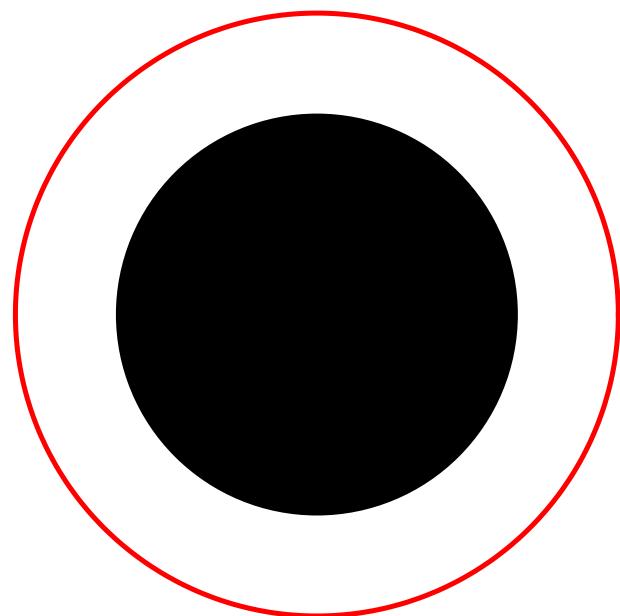
$$\vec{\eta} = \frac{D_L + D_o}{D_o} \vec{\xi} - D_L \vec{\alpha}, \quad \vec{\alpha} = \frac{4G}{c^2} \int_{\mathbb{R}^2} \frac{(\vec{\xi} - \vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2} \Sigma(\vec{\xi}') d^2 \vec{\xi}' .$$

Part II

Gravitational lensing by compact objects

Schwarzschild black hole

(spherically symmetric and static)



Horizon:

$$r_S = \frac{2GM}{c^2}$$

Light sphere (photon sphere)

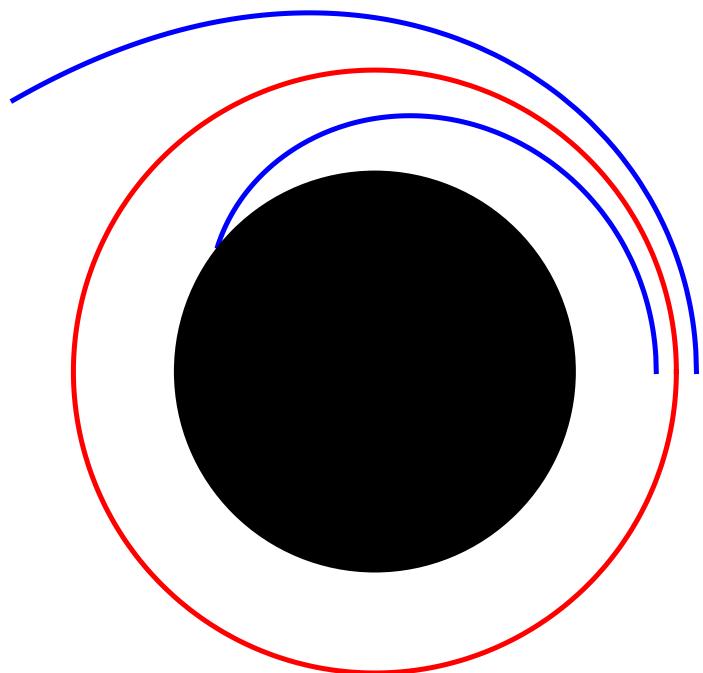
$$\frac{3}{2} r_S = \frac{3GM}{c^2}$$

Part II

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Schwarzschild black hole

(spherically symmetric and static)

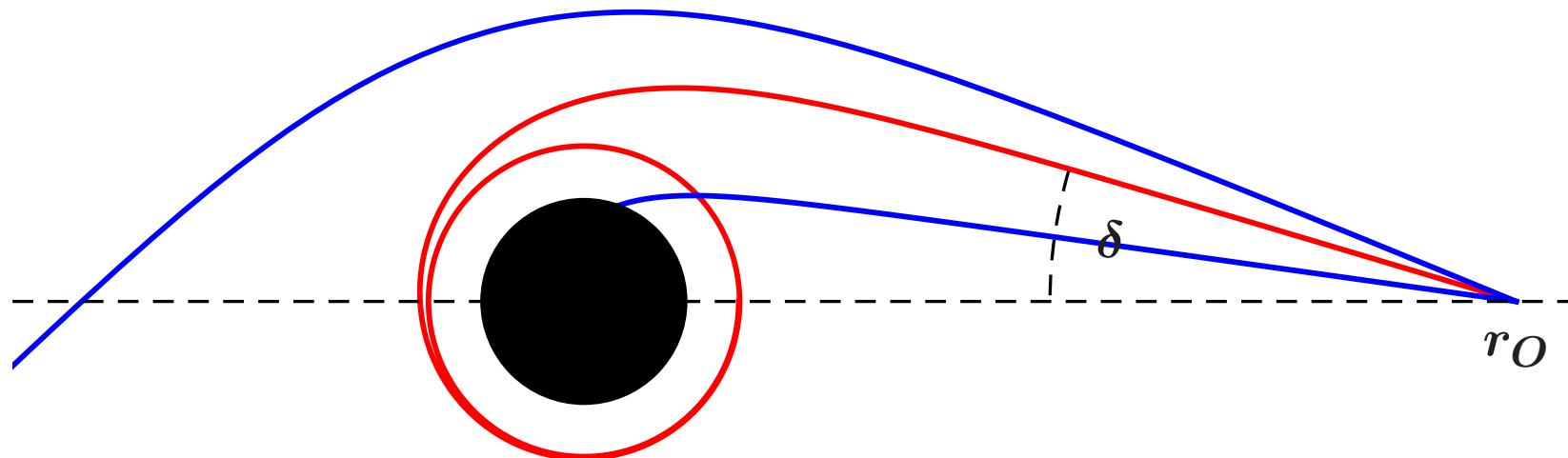


Horizon:

$$r_S = \frac{2GM}{c^2}$$

Light sphere (photon sphere)

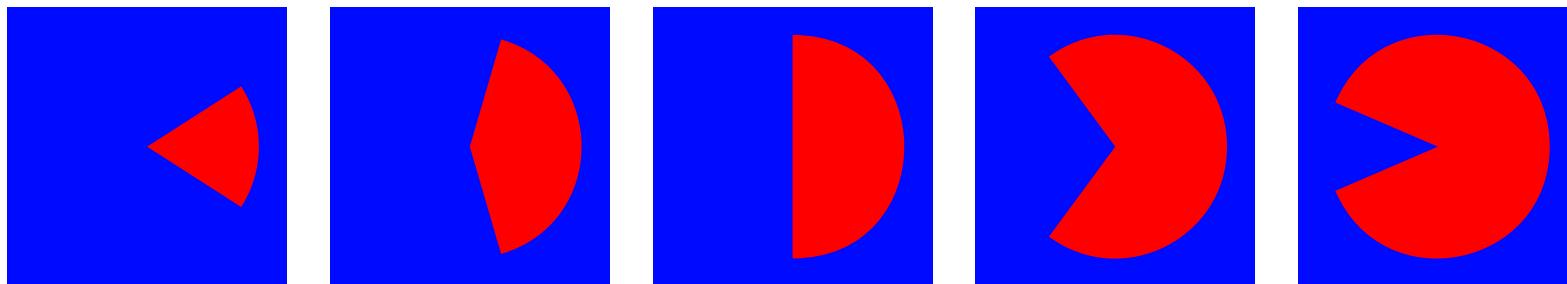
$$\frac{3}{2} r_S = \frac{3GM}{c^2}$$



Angular radius δ of the “shadow” of a Schwarzschild black hole:

$$\sin^2 \delta = \frac{27 r_S^2 (r_O - r_S)}{4 r_O^3}$$

J. L. Synge, Mon. Not. R. Astr. Soc. 131, 463 (1966)



$r_O = 1.05 r_S$

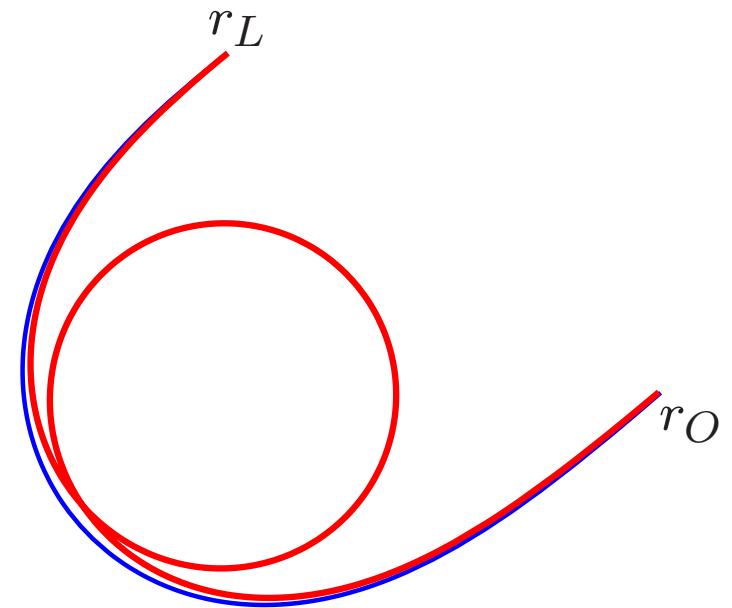
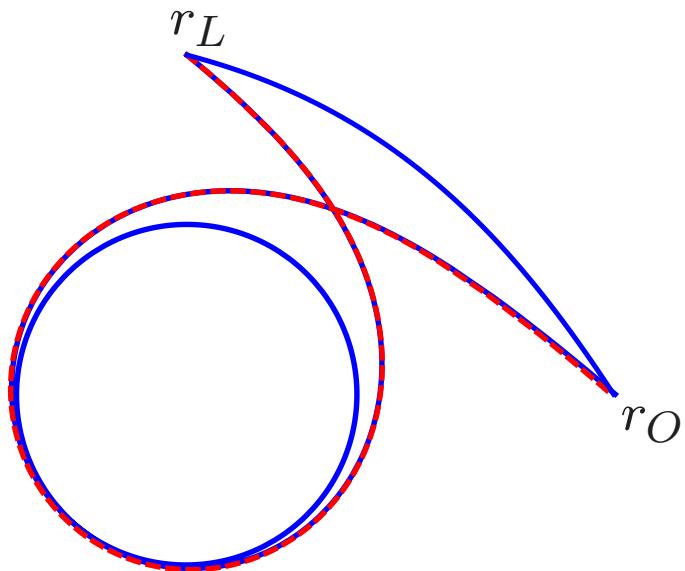
$r_O = 1.3 r_S$

$r_O = 3 r_S/2$

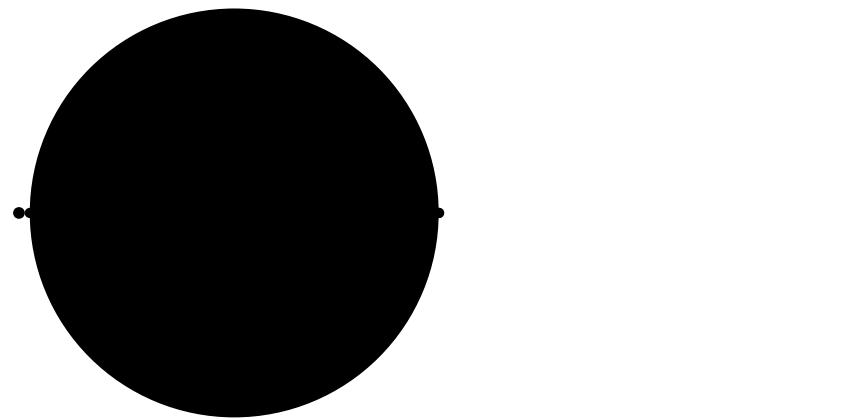
$r_O = 2.5 r_S$

$r_O = 6 r_S$

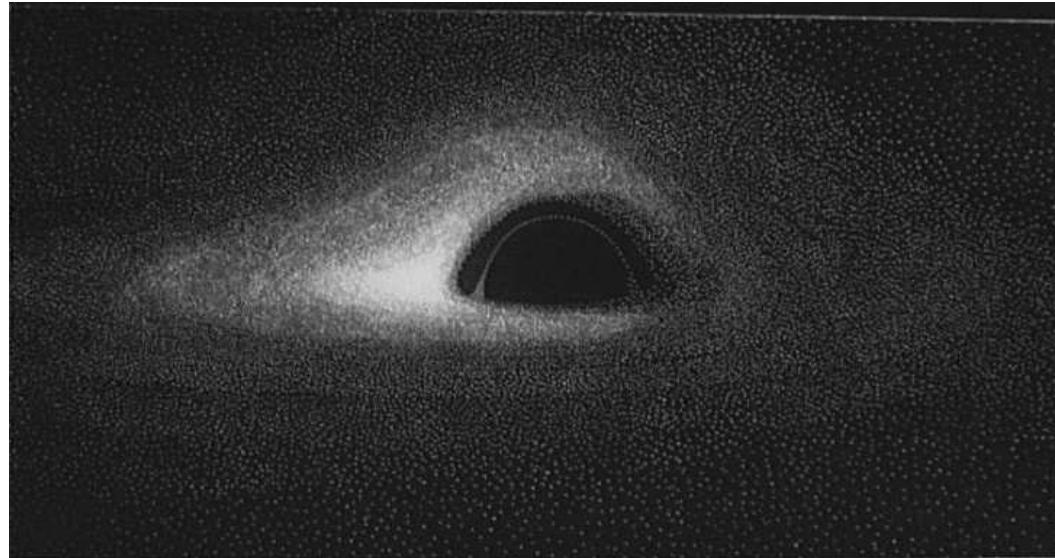
Schwarzschild black hole produces infinitely many images:



Visual appearance of a Schwarzschild black hole



Visual appearance of a Schwarzschild black hole with a thin accretion disc

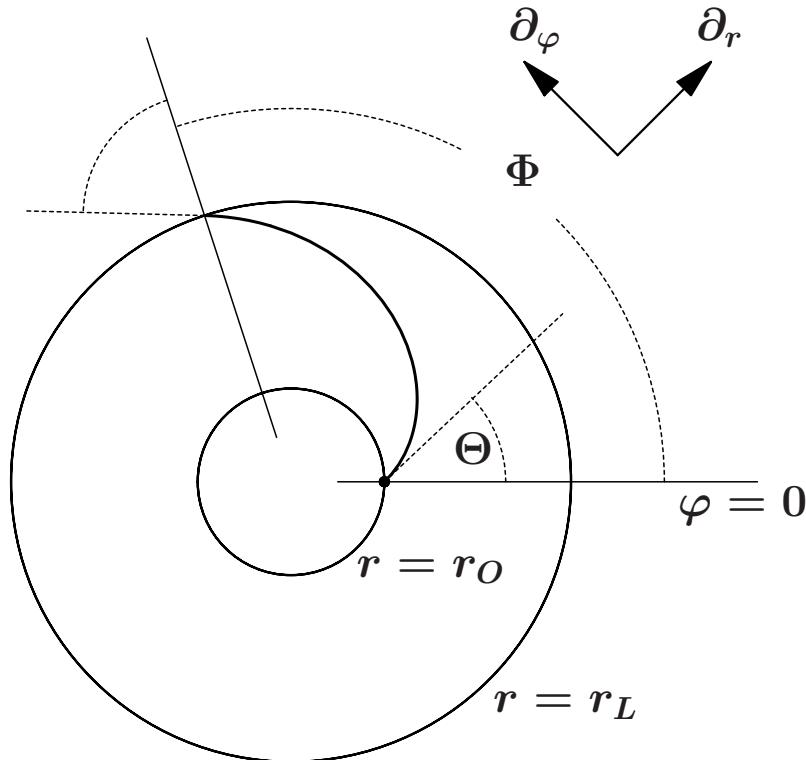


(picture from J.-P. Luminet, Astron. Astrophys. 75, 228 (1979))

Exact lens map for spherically symmetric and static spacetimes

VP: Phys. Rev. D 69, 064917 (2004)

$$g = e^{2f(r)} \left(-c^2 dt^2 + S(r)^2 dr^2 + R(r)^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2) \right)$$

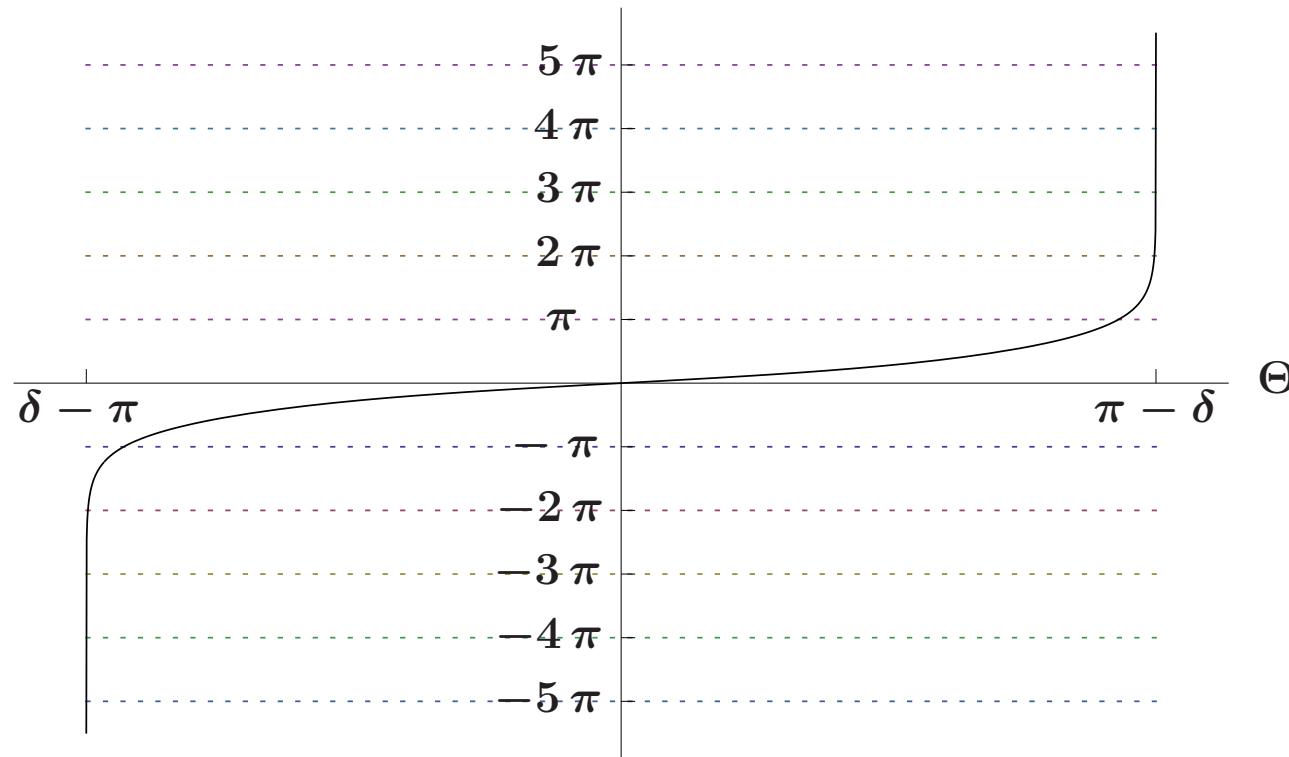


$$\Phi = R(r_O) \sin \Theta \int_{r_O}^{r_L} \frac{S(r) dr}{R(r) \sqrt{R(r)^2 - R(r_O)^2 \sin^2 \Theta}}$$

Schwarzschild spacetime

$$g = - \left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2) , \quad r_s = \frac{2GM}{c^2}$$

$$S(r)^{-1} = 1 - \frac{r_s}{r}, \quad R(r) = \frac{r}{\sqrt{1 - \frac{r_s}{r}}} \\ \Phi(\Theta)$$



Lens map $\Theta \mapsto \Phi$ for $r_O = 2.5 r_s$ and $r_L = 5 r_s$

Infinite sequences of images converge towards $\pm\delta$

Other black holes:

- Reissner-Nordström
- Janis-Newman-Winicour
- Newman-Unti-Tamburino (NUT)
- Black holes from nonlinear electrodynamics
- Black holes from higher dimensions, braneworld scenarios, ...

All of them have an unstable photon sphere \Rightarrow Qualitative lensing features are similar to Schwarzschild

Quantitative features (ratio of angular separations of images, ratio of fluxes of images) are different

V.Bozza: Phys. Rev. D 66, 103001 (2002)

If higher-order images are seen, we can distinguish a Schwarzschild black hole from other black holes

Black hole impostor: Ellis wormhole

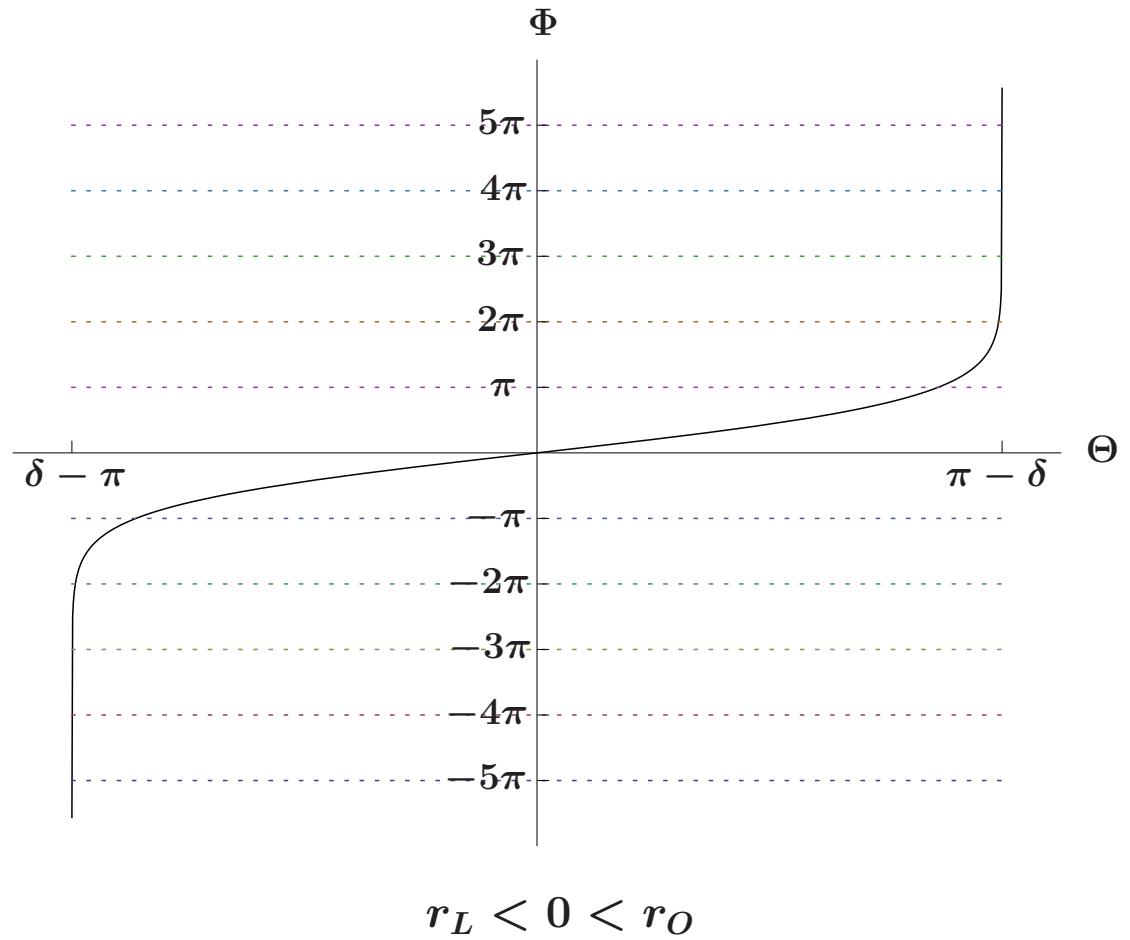
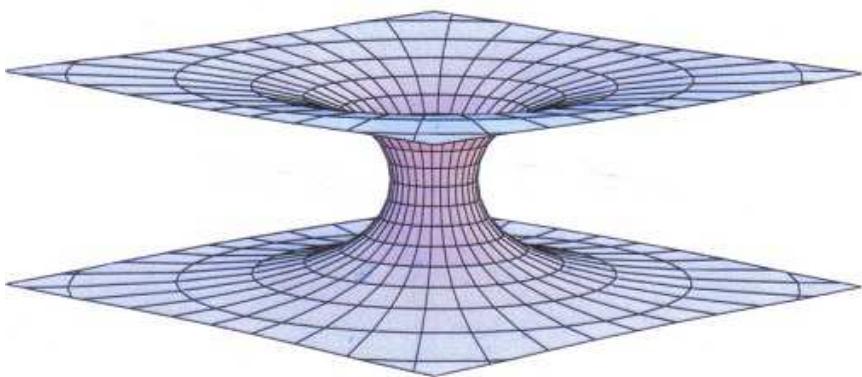
H. Ellis: J. Math. Phys. 14, 104 (1973)

$$g = -c^2 dt^2 + dr^2 + (r^2 + a^2) (d\vartheta^2 + \sin^2 \vartheta d\varphi^2)$$

$$S(r) = 1$$

$$R(r) = \sqrt{r^2 + a^2}$$

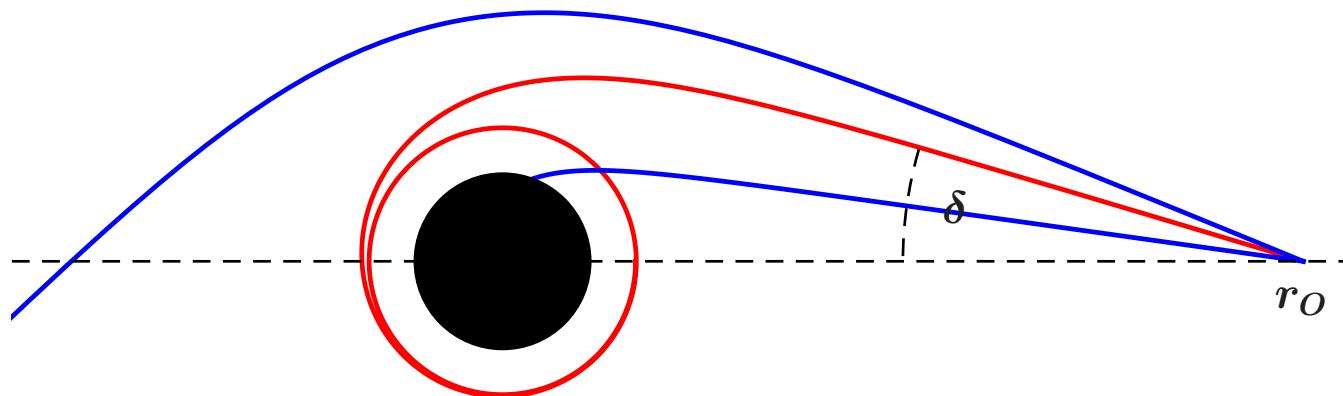
$$\sin^2 \delta = \frac{a^2}{r_O^2 + a^2}$$



Qualitatively the same, quantitatively different from black holes

Black hole impostor: Ultracompact star

Uncharged dark star with radius between $\frac{2GM}{c^2}$ and $\frac{3GM}{c^2}$



Lensing features indistinguishable from Schwarzschild black hole

Claim: ultracompact objects cannot exist

V. Cardoso, L. Crispino, C. Macedo, H. Okawa, P. Pani:

Phys. Rev. D 90, 044069 (2014)