The shadow of a black hole. Theory and prospects of observations

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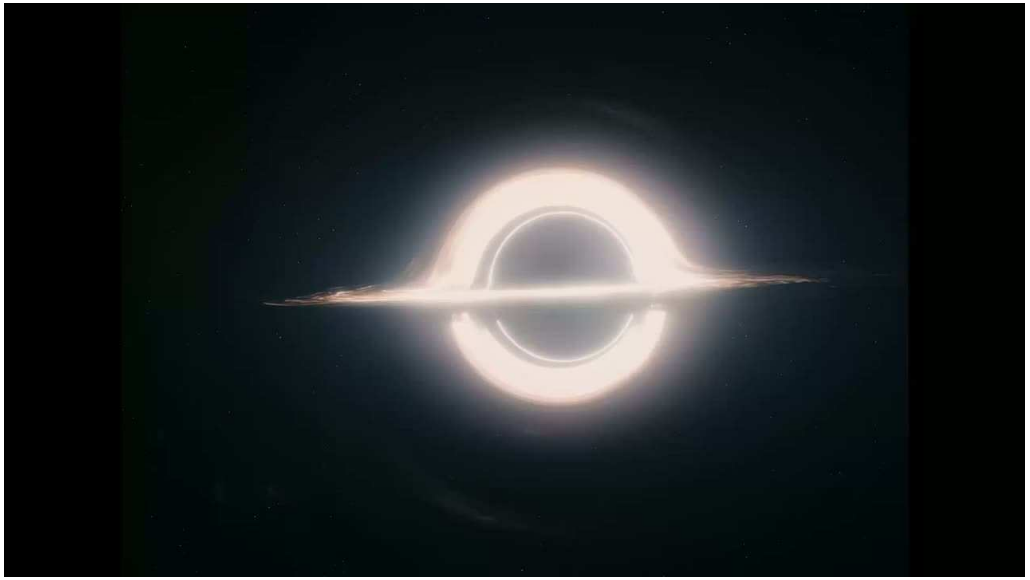


from the Movie “Interstellar” (2014)

Mexico City, UAM, Physics Department Seminar, 19 February 2016

Outline of the talk:

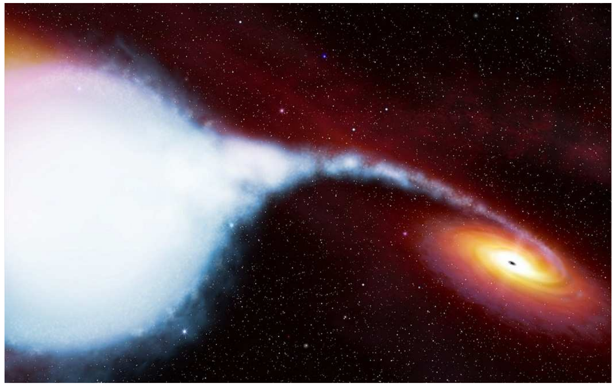
- Evidence for the existence of black holes
  - Stellar black holes (1 to 100 Solar masses)
  - Supermassive black holes (10^6 to 10^{11} Solar masses)
- Theoretical construction of the shadow
  - Shadow of a non-rotating black hole (Schwarzschild)
  - Shadow of a rotating black hole (Kerr)
- Perspectives of observing the shadow
  - Event Horizon Telescope / BlackHoleCam
  - Millimetron

For background material on the theory see

VP: "Gravitational Lensing from a Spacetime Perspective",
Living Rev. Relativity 7, (2004),
http://www.livingreviews.org/lrr-2004-9

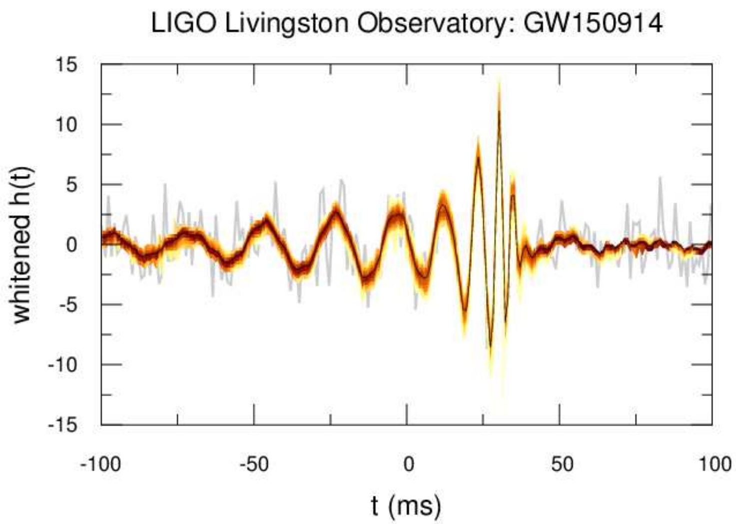
Evidence for stellar black holes:

X-ray binaries: Cygnus X-1 ($15M_\odot$)



Artist's impression (from Wikipedia)

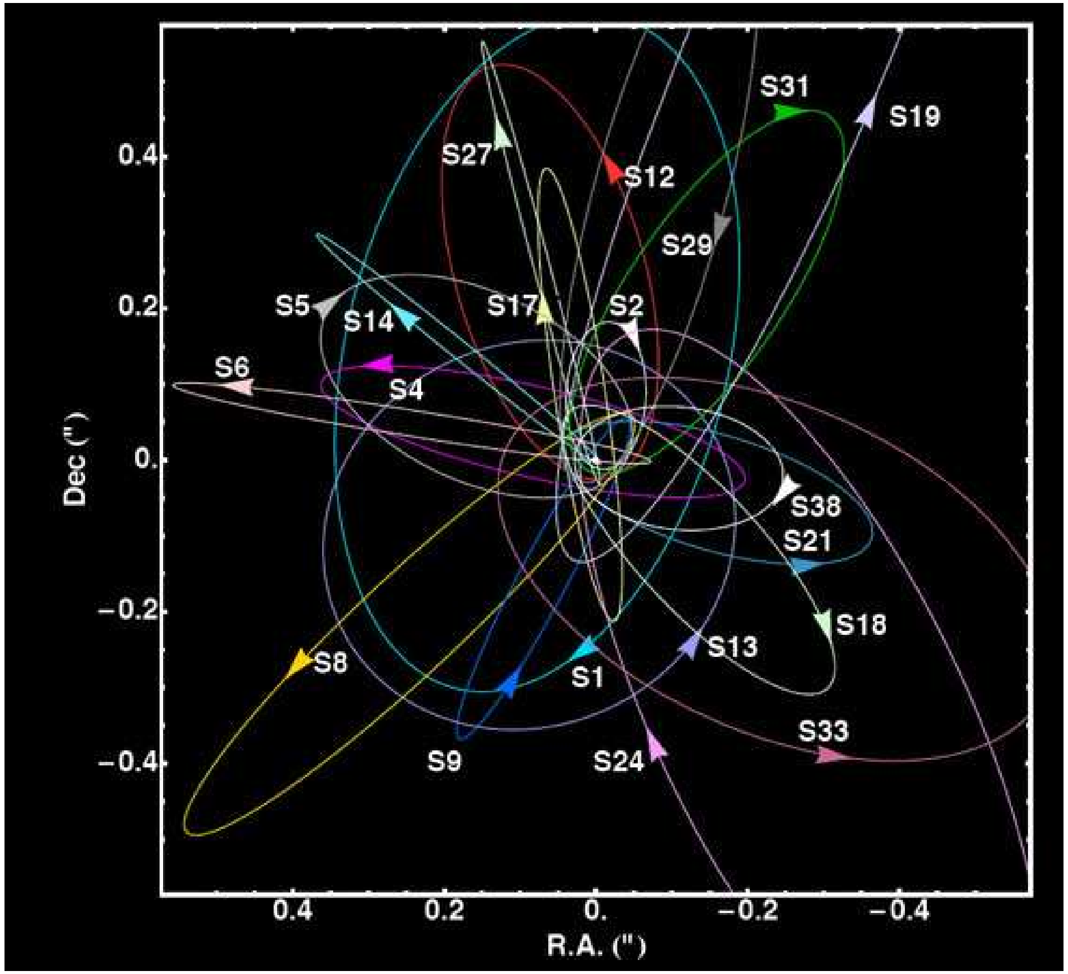
Gravitational waves: GW150914 ($60M_\odot$)



Signal detected by the LIGO instruments

Evidence for supermassive black holes:

Object at the centre of our galaxy ($4.3 \times 10^6\, M_{\odot}$)

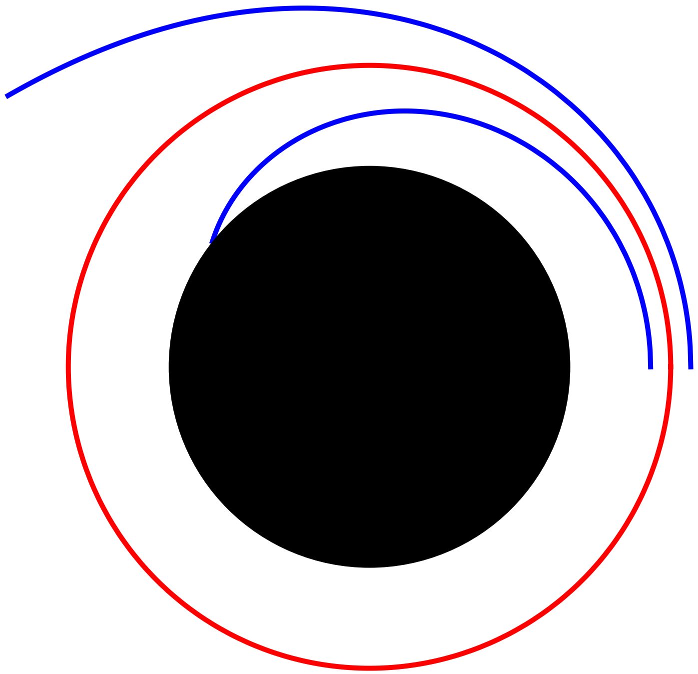


S-stars (source: Gillessen et al.)

Object at the centre of M87 ($3 \times 10^9 M_{\odot}$) . . .

Non-rotating black holes:

Schwarzschild metric (sphericaly symmetric and static)

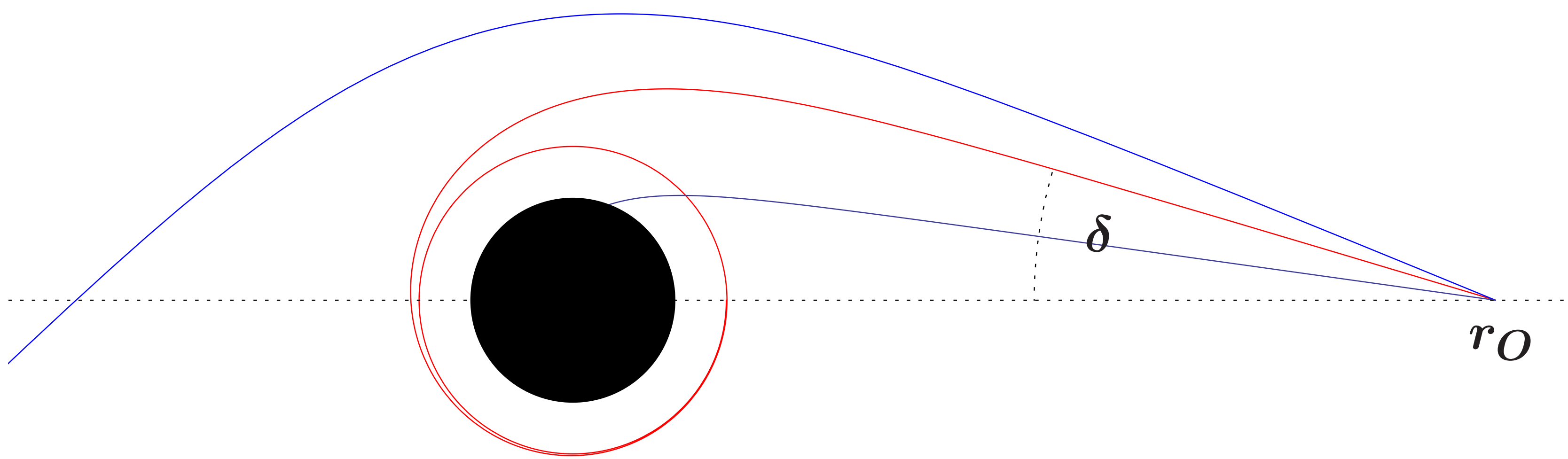


Horizon:

$$r_S = \frac{2GM}{c^2} = 2m$$

Light sphere (photon sphere)

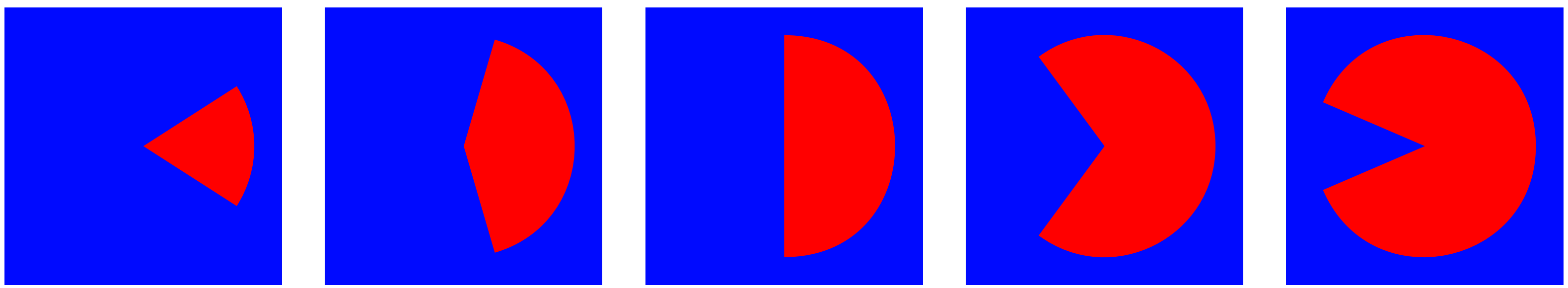
$$\frac{3}{2} r_S = \frac{3GM}{c^2} = 3m$$



Angular radius δ of the "shadow" of a Schwarzschild black hole:

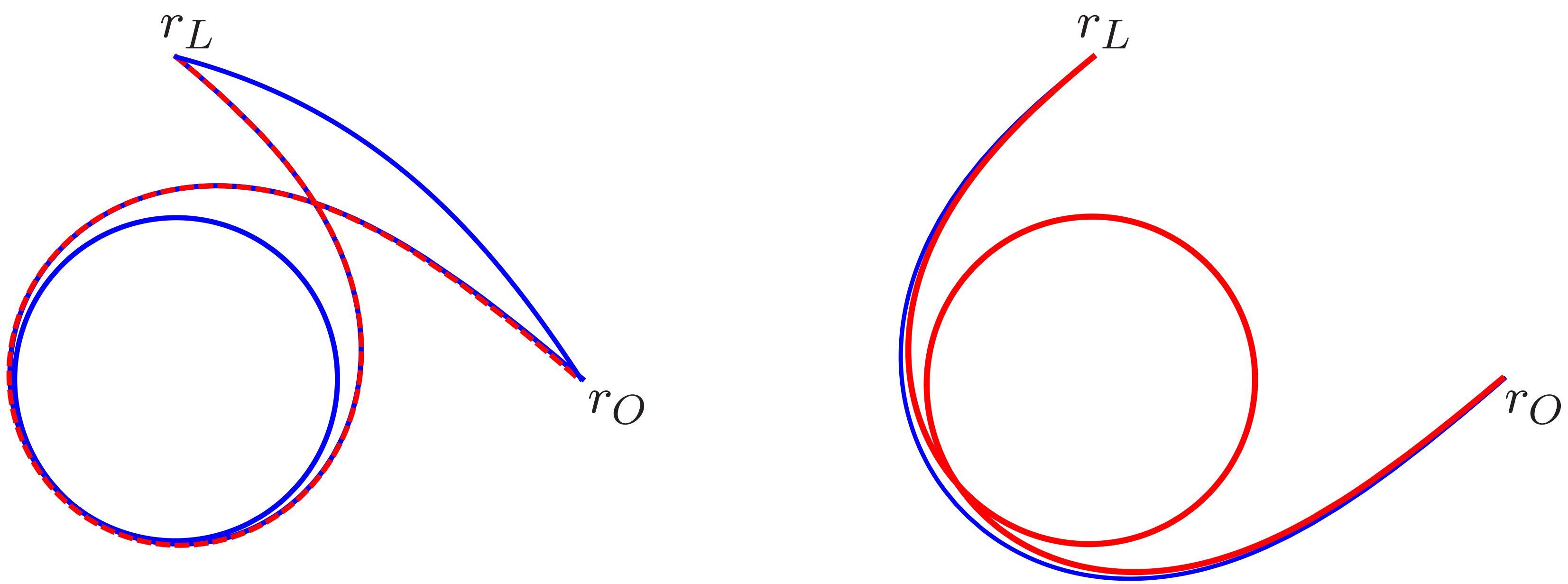
$$\sin^2\delta = \frac{27\, r_S^2\,(r_O - r_S)}{4\, r_O^3} = \frac{27\, m^2}{r_O^2}\left(1 - \frac{2m}{r_O}\right)$$

J. L. Synge, Mon. Not. R. Astr. Soc. 131, 463 (1966)

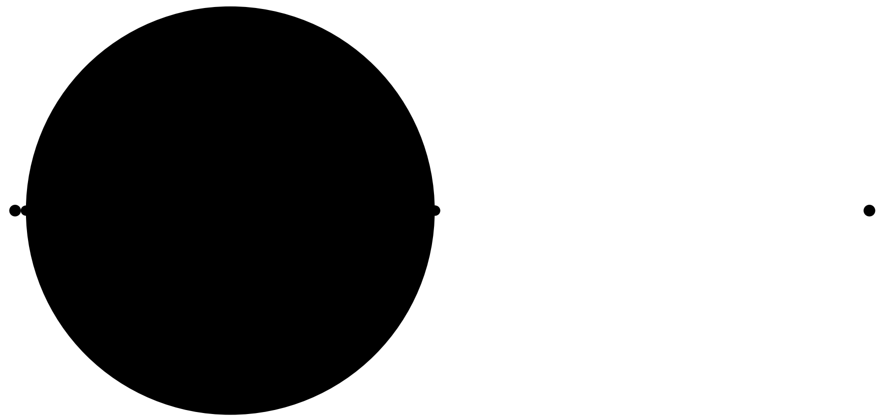


$r_O = 1.05\, r_S$ $\quad$ $r_O = 1.3\, r_S$ $\quad$ $r_O = 3\, r_S/2$ $\quad$ $r_O = 2.5\, r_S$ $\quad$ $r_O = 6\, r_S$

Schwarzschild black hole produces infinitely many images:



Imaging of a point source by a Schwarzschild black hole



Rotating black holes:

The shadow is no longer circular.

The shape of the shadow depends on the spin and can be used for discriminating between different black holes

Shape of the shadow of a Kerr black hole:

J. Bardeen in C. DeWitt and B. DeWitt (eds.): "Black Holes" Gordon & Breach (1973)

Shape and size of the shadow of Plebański-Demiański black holes:

A. Grenzebach, VP, C. Lämmerzahl:
Phys. Rev. D 89, 124004 (2014)
Int. J. Mod. Phys. D 24, 154024 (2015)

Kerr black hole is characterised by two parameters:

mass parameter $m = \frac{GM}{c^2}$

spin parameter $a = \frac{J}{Mc}$

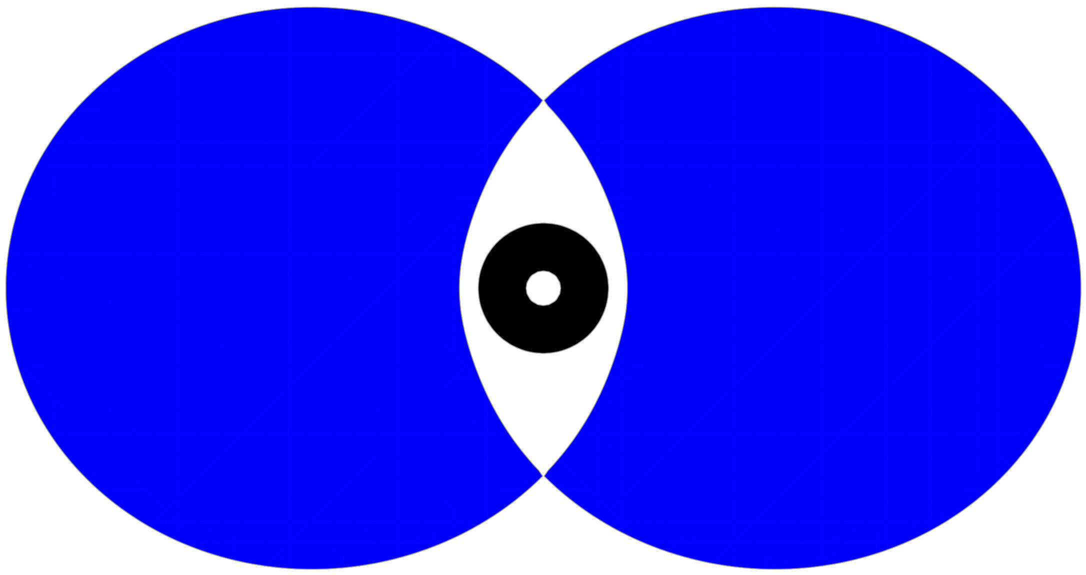
$$0 \leq a^2 \leq m^2 : \quad \text{black hole}$$

$$m^2 < a^2 : \quad \text{naked singularity}$$

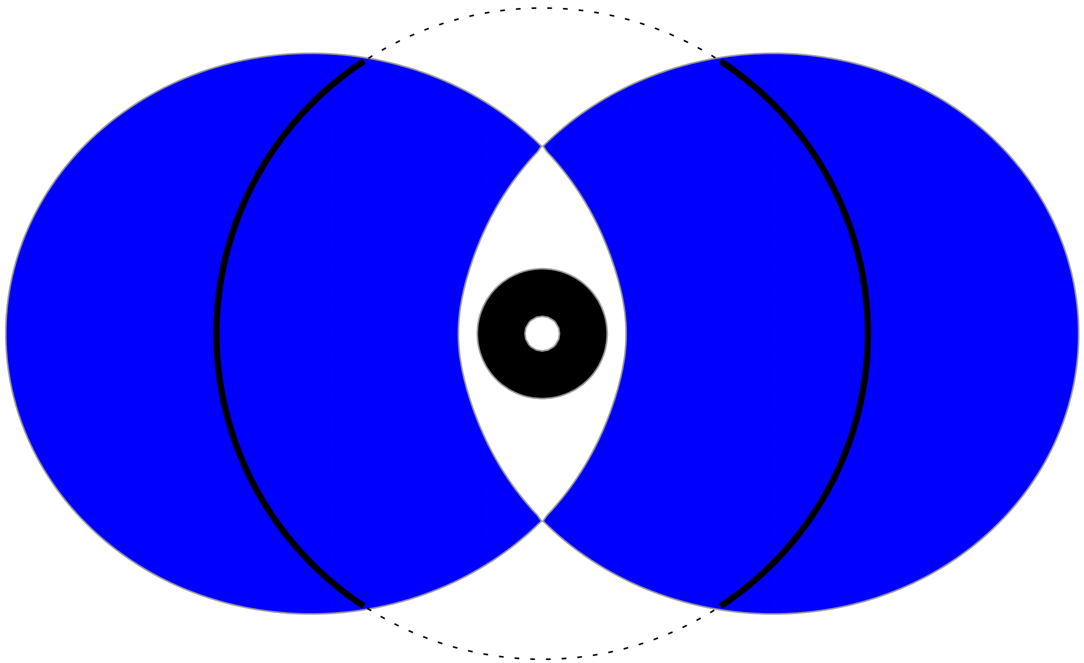
Plebański-Demiański black hole has additional parameters:

$q_e, q_m, \Lambda, \ell, \alpha$

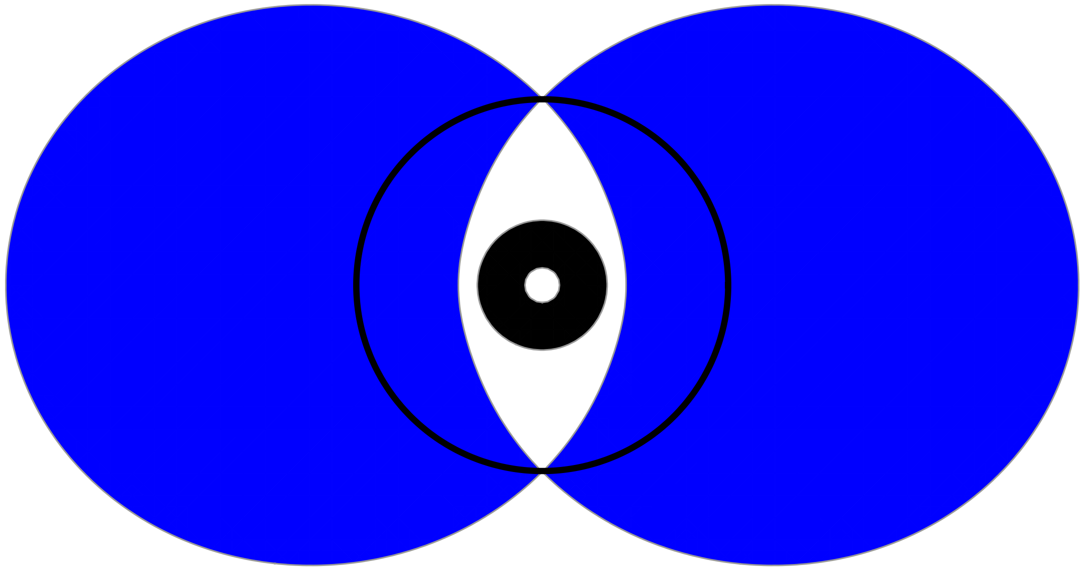
Photon region for Kerr black hole with $a = 0.75\,m$



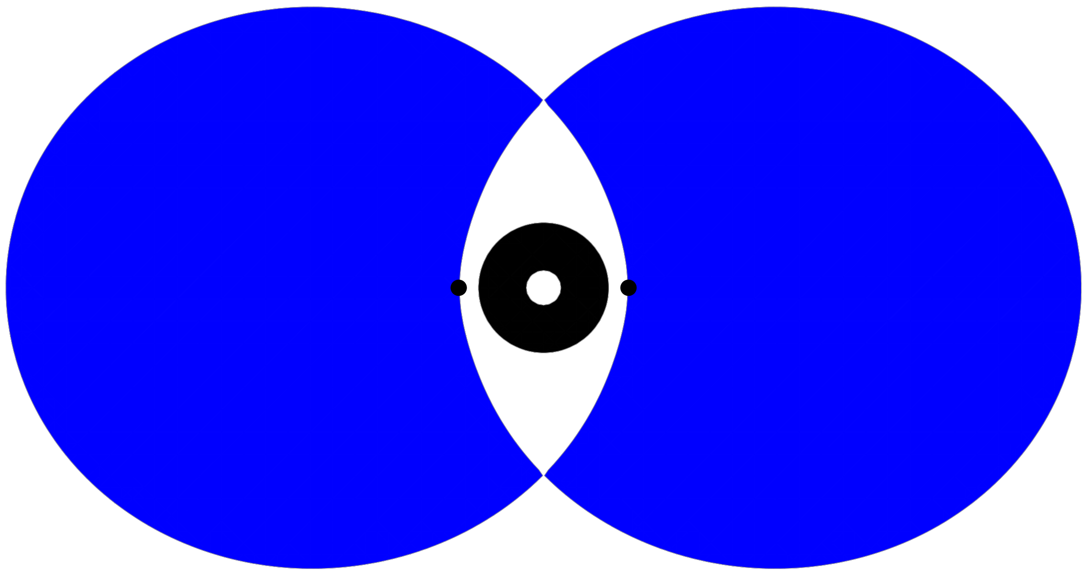
Photon region for Kerr black hole with $a = 0.75\,m$



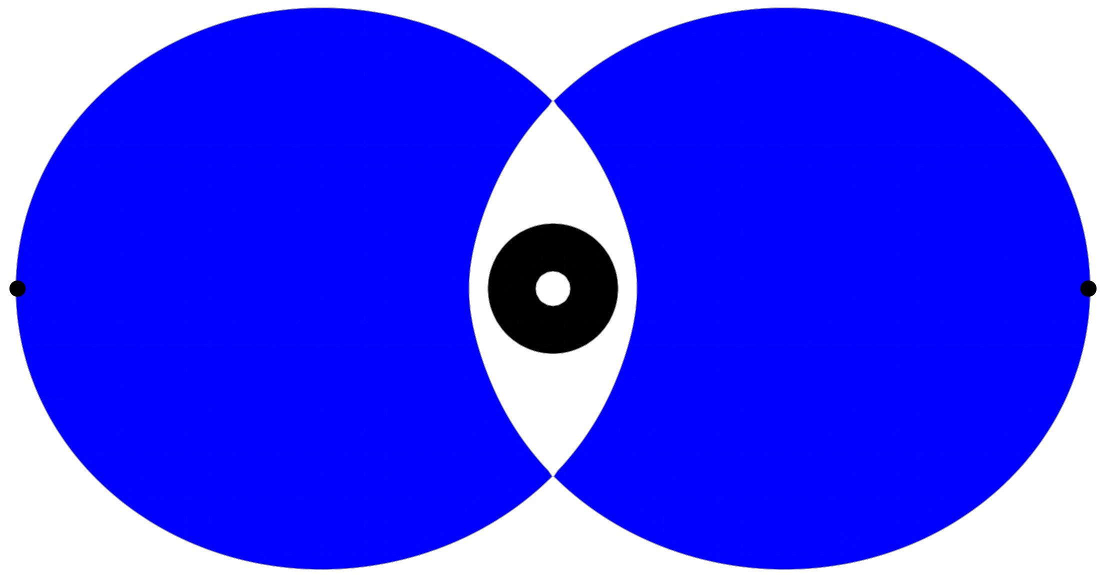
Photon region for Kerr black hole with $a = 0.75\,m$

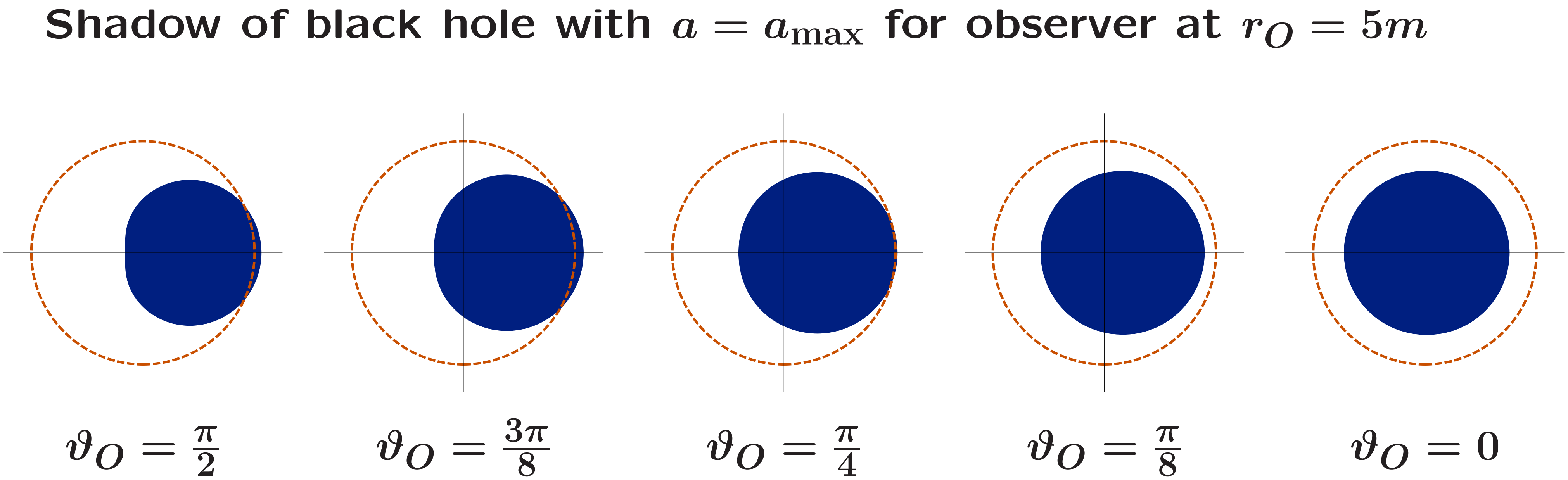


Photon region for Kerr black hole with $a = 0.75\,m$



Photon region for Kerr black hole with $a = 0.75\,m$




Shadow of black hole with $a = a_{\max}$ for observer at $r_O = 5m$
$\vartheta_O = \frac{\pi}{2}$
$\vartheta_O = \frac{3\pi}{8}$
$\vartheta_O = \frac{\pi}{4}$
$\vartheta_O = \frac{\pi}{8}$
$\vartheta_O = 0$

Vertical angular radius δ_v of the shadow $\big(\vartheta_O = \pi/2\big)$:

$$\sin^2 \delta_v$$

$$= \frac{27m^2 r_O^2 \big(r_O^2 - 2mr_O + a^2\big)}{r_O^6 + 6a^2 r_O^4 - 3a^2(9m^2 - 4a^2) r_O^2 + 8a^6}$$

$$= \frac{27m^2}{r_O^2} \Big(1 + O(m/r_O)\Big)$$

Up to terms of order $O(m/r_O)$, Synge's formula is still correct for the vertical diameter of the shadow

Perspectives of observations

Object at the centre of our galaxy:

Mass $= 4 \times 10^6 \, M_\odot$

Distance $= 8 \, \mathrm{kpc}$

Angular diameter of the shadow by Synge's formula $\approx 54\mu$as

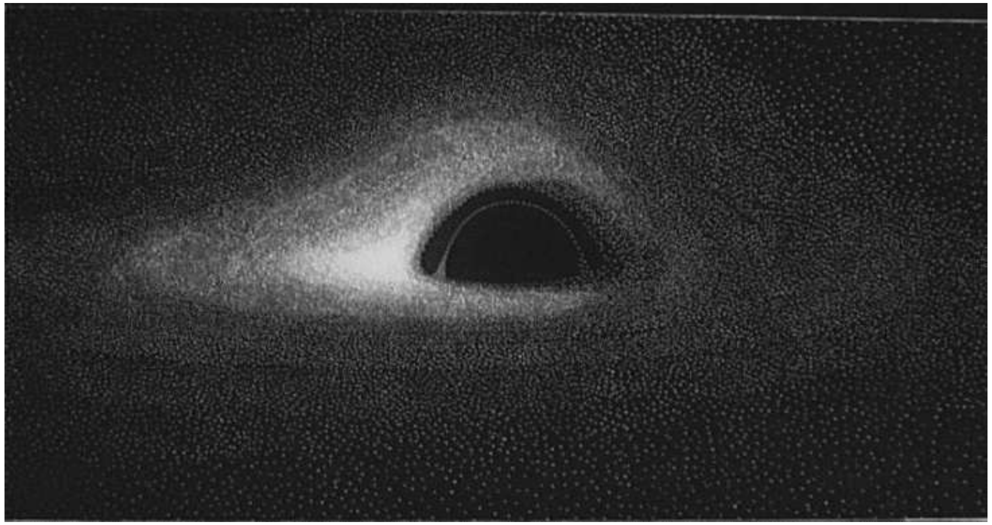
(corresponds to a grapefruit on the moon)

Object at the centre of M87:

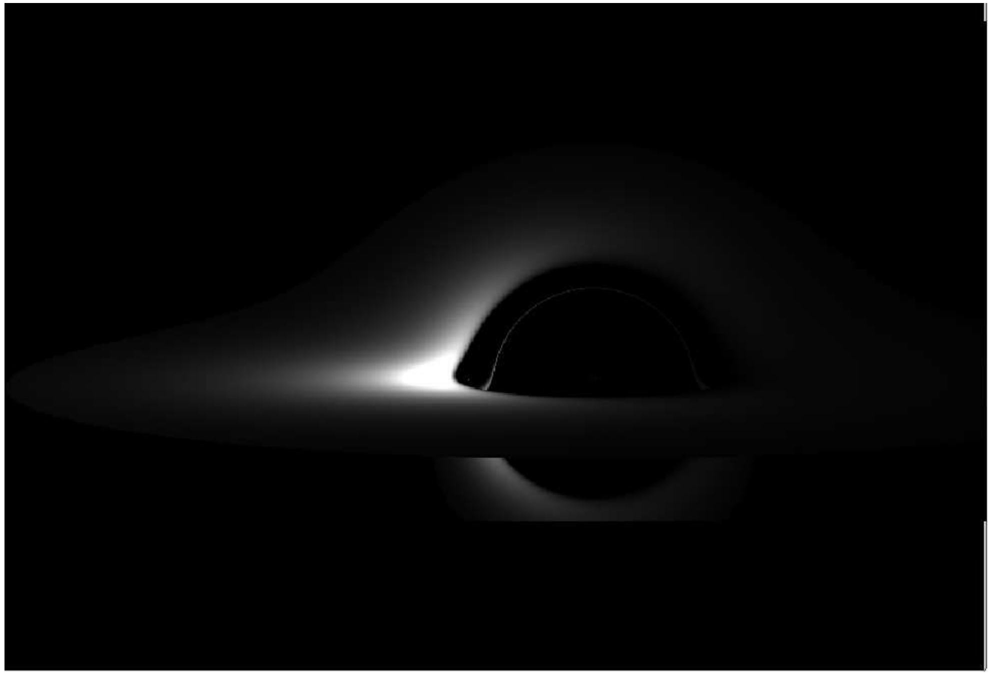
Mass $= 3 \times 10^9 \, M_\odot$

Distance $= 16 \, \mathrm{Mpc}$

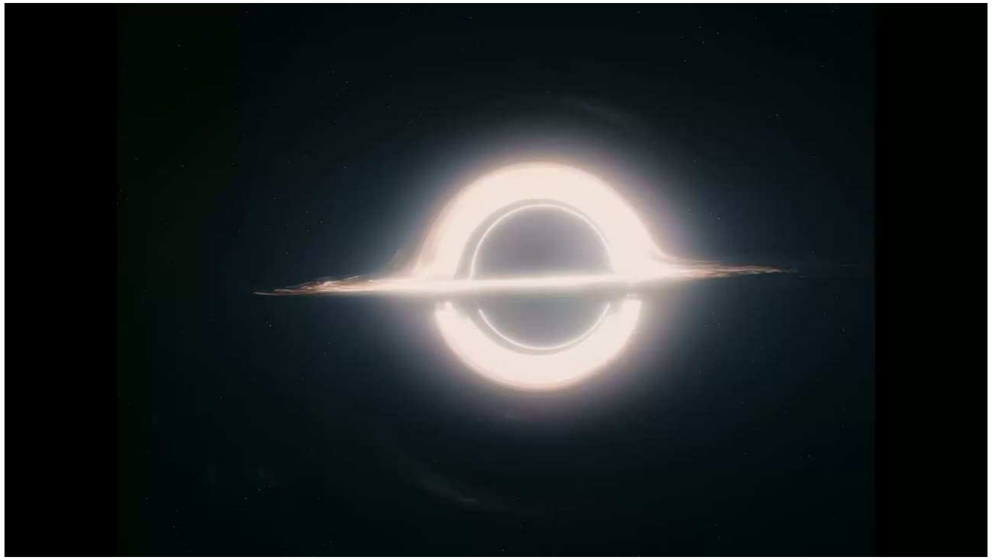
Angular diameter of the shadow by Synge's formula $\approx 20\mu$as



J.-P. Luminet (1979)

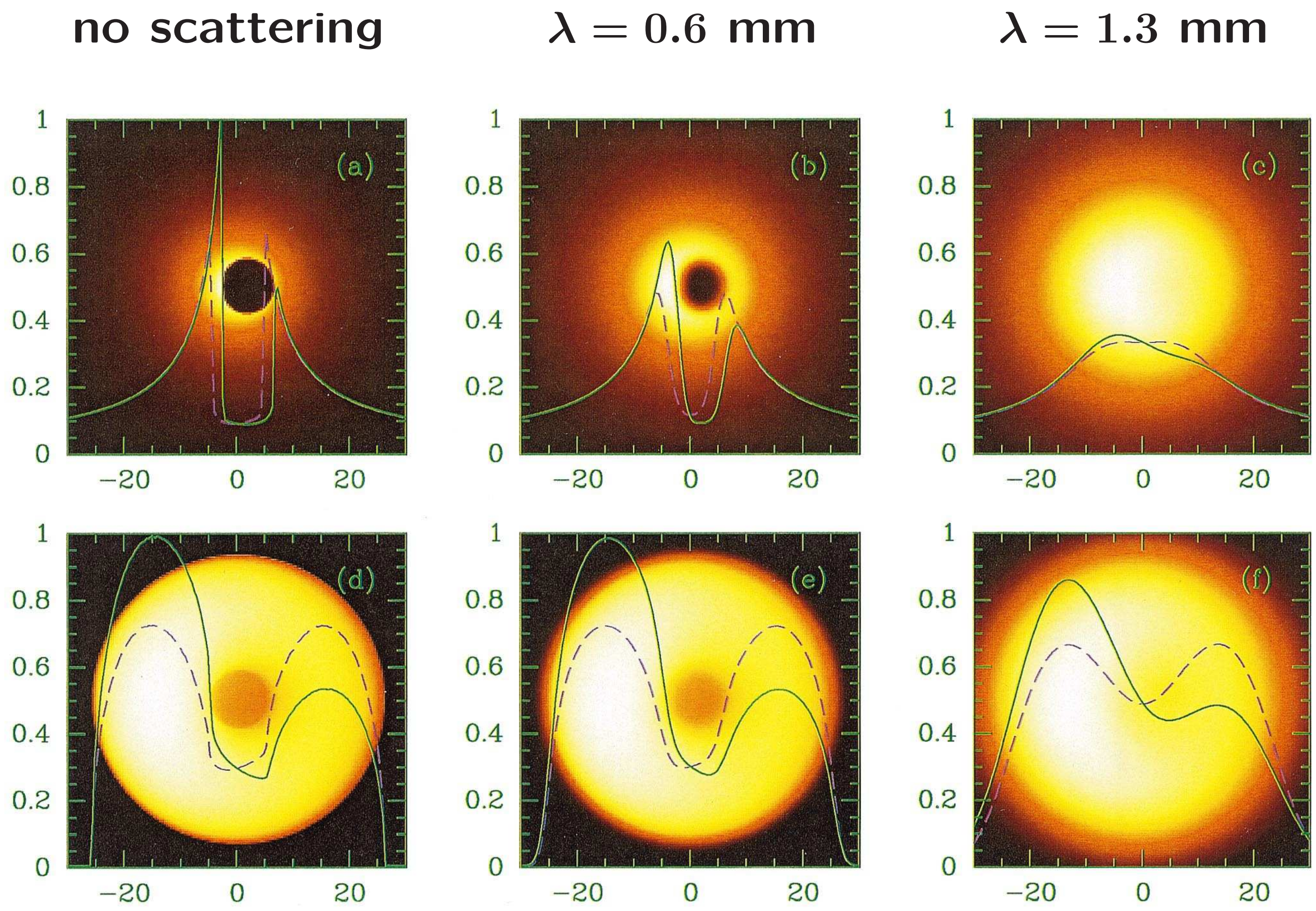


T. Müller (2012)



Interstellar (2014)

Kerr shadow with emission region and scattering taken into account:



H. Falcke, F. Melia, E. Agol: Astrophys. J. 528, L13 (2000)

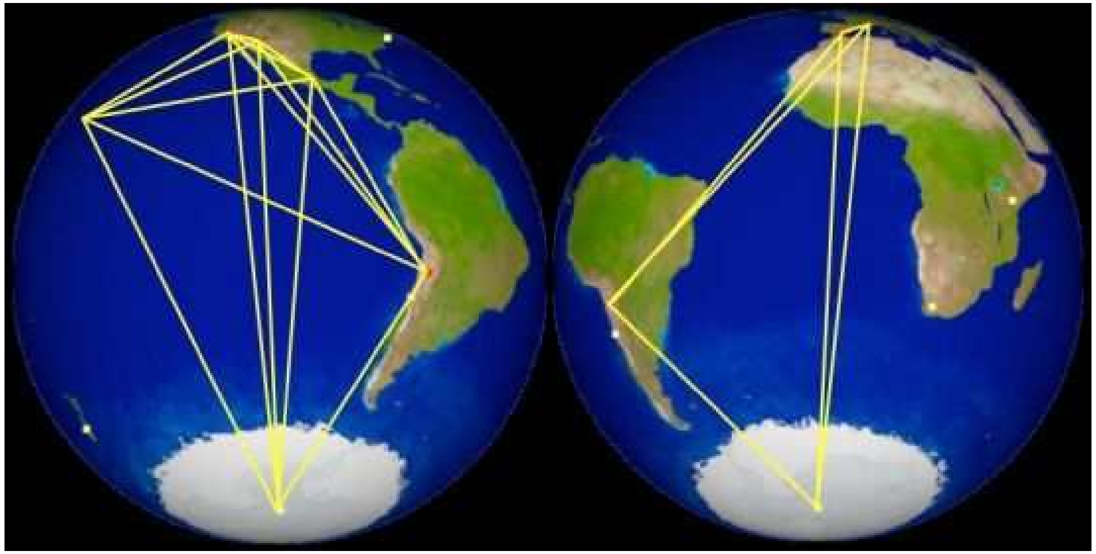
Observations should be done at (sub-)millimeter wavelength

Projects to view the shadow with (sub-)millimeter VLBI:

Event Horizon Telescope (EHT),

Using ALMA (Chile), LMT (Mexico), NOEMA (France), SMT (Arizona), CARMA (California), SMA (Hawaii), South Pole Telescope ...

LMT



EHT

BlackHoleCam

H. Falcke, L. Rezzolla, M. Kramer

Millimetron (≈ 2025)

Expectation:

Observation of the shadow by the EHT/BlackHoleCam soon

Further strong evidence for a black hole at the centre of our galaxy

Unequivocal determination of its spin