

The Pioneer Anomaly

or

Do We Really Understand the Physics Within the Solar System?

Claus Lämmerzahl



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University Bremen

thanks to J. Anderson, H. Dittus, E. Hackmann, V. Kagramanova, J. Kunz, T. Morley, O. Preuss, P. Richter, S. Solanki, S. Turyshev and the Pioneer Exploration Collaboration

Utrecht, 26.5.2008



Bremen campus



Campus of University of Bremen



Bremen Drop Tower of ZARM



Tower 146 m

drop tube
110 m

free fall time
 $= 4.7 \text{ s}$

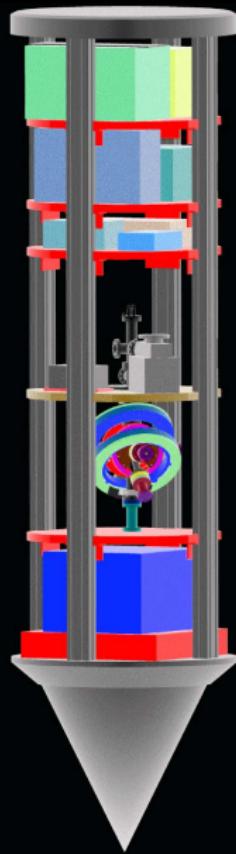
deceleration
 $\sim 30 \text{ g}$



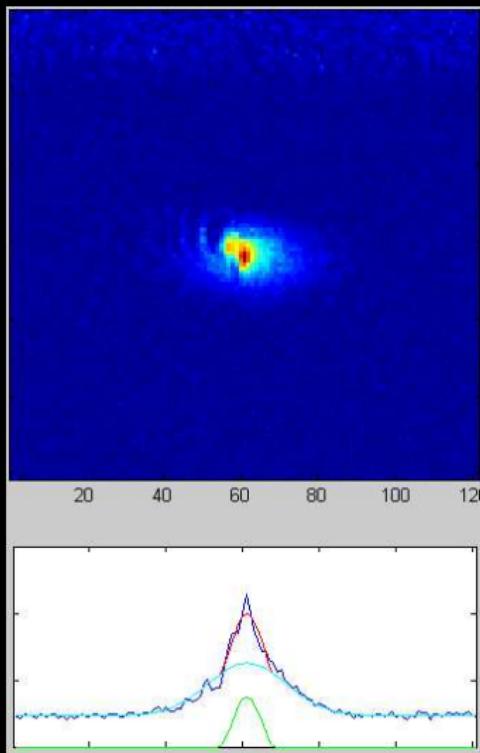
Bremen Drop Tower of ZARM



First BEC in microgravity / extended free fall



First BEC in microgravity / extended free fall



BEC in free fall may
become colder than today's
500 fK (Ketterle)

today's free expansion /
free fall time > 1 s

aim 1 fK

First BEC in microgravity / extended free fall



Outline

1 The situation of standard physics

- The status
- Problems?

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The present situation

All predictions of General Relativity are experimentally well tested and confirmed

Foundations

The Einstein Equivalence Principle

- Universality of Free Fall
- Universality of Gravitational Redshift
- Local Lorentz Invariance



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Implication

Gravity is a metrical theory

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Gravity is a metrical theory

Predictions for metrical theories

- Solar system effects
 - Perihelion shift
 - Gravitational redshift
 - Deflection of light
 - Gravitational time delay
 - Lense–Thirring effect
 - Schiff effect
- Strong gravitational fields
 - Binary systems
 - Black holes
- Gravitational waves



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General Relativity

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But: Dark clouds over General Relativity?

Unexplained observations



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Needed to describe galactic rotation curves, lensing, structure formation



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Quadrupole and octupole of CMB are correlated with Solar system ecliptic



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Gravity at large distances? Weak gravity? Small accelerations?



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Ranging

Time and distance

Constancy of speed of light $c \Rightarrow$ distance \Leftrightarrow time

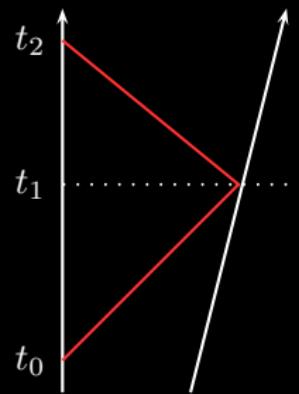
$$x = ct \quad \Leftrightarrow \quad t = \frac{x}{c}$$

Distance of satellite

- electromagnetic (radio) signal from Earth to satellite
- measured distance:

$$r = \frac{c}{2}(t_2 - t_0)$$

- needs only good clock on Earth



Earth satellite



Doppler tracking: The principal method of observation

Two way Doppler tracking

Sender at rest, moving receiver

- received frequency

$$\nu' = \frac{1}{\sqrt{1 - v^2/c^2}} \left(1 - \frac{v}{c}\right) \nu$$

sender

receiver



Doppler tracking: The principal method of observation

Two way Doppler tracking

Sender at rest, moving receiver

- received frequency

$$\nu' = \frac{1}{\sqrt{1 - v^2/c^2}} \left(1 - \frac{v}{c}\right) \nu$$

- frequency sent back

$$\nu'' = \frac{1}{\sqrt{1 - v^2/c^2}} \left(1 - \frac{v}{c}\right) \nu'$$

sender

receiver



Doppler tracking: The principal method of observation

Two way Doppler tracking

Sender at rest, moving receiver

- received frequency

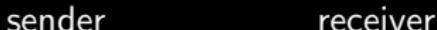
$$\nu' = \frac{1}{\sqrt{1 - v^2/c^2}} \left(1 - \frac{v}{c}\right) \nu$$

- frequency sent back

$$\nu'' = \frac{1}{\sqrt{1-v^2/c^2}} \left(1 - \frac{v}{c}\right) \nu'$$

- can extract velocity

$$\frac{\nu'' - \nu}{\nu} = -2 \frac{v/c}{1 + v/c} \approx -2 \frac{v}{c}$$



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The Pioneer anomaly

Observation

- Anomalous acceleration of Pioneer 10 and 11 spacecraft of

$$a_{\text{Pioneer}} = (8.74 \pm 1.33) \cdot 10^{-10} \text{ m/s}^2$$

toward the Sun (Anderson et al, PRL 1998, PRD 2002)

- Acceleration is constant starting with last flyby

Remark / Question

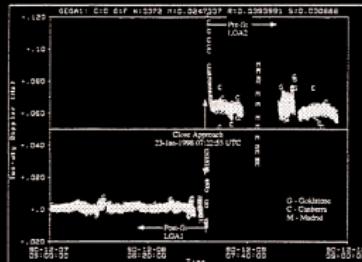
- Systematic effect?
- Influence from cosmic expansion?
- New physics?

The flyby anomaly

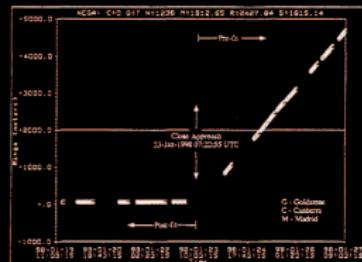
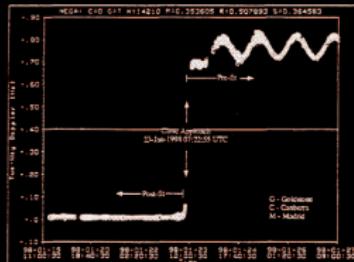
Observation

After satellite flybys at Earth the velocity is too large by a few mm/s ([Antreasian & Guinn 1998](#), [Anderson & Williams 2001](#), [Morley 2005](#), [CL et al 2007](#))

GEA1:
Two-way S-band
Doppler residuals &
range residuals



NEGA:
Two-way X-band
Doppler residuals &
range residuals



A stone thrown up with v comes back with $v + \Delta v$

The flyby anomaly

Observations

- Earth flybys

Mission	agency	year	pericentre	v_∞ [km/s]	e	Δv [mm/s]
Galileo	NASA	Dec 1990	959.9 km	8.848	2.47	3.92 ± 0.08
Galileo	NASA	Dec 1992	303.1 km	8.877	2.32	— ^a
NEAR	NASA	Jan 1998	538.8 km	6.851	1.81	13.46 ± 0.13
Cassini	NASA	Aug 1999	1173 km	16.01	5.8	0.11
Stardust	NASA	Jan 2001	5950 km	??	??	— ^b
Rosetta	ESA	Mar 2005	1954 km	3.863	1.327	1.82 ± 0.05
Hayabusa	Japan	May 2004	3725 km	??	??	— ^c
MESSENGER ^d	private	Aug 2005	2347 km	4.056	1.36	~ 0

^a no reliable data: too low orbit with too large atmospheric drag

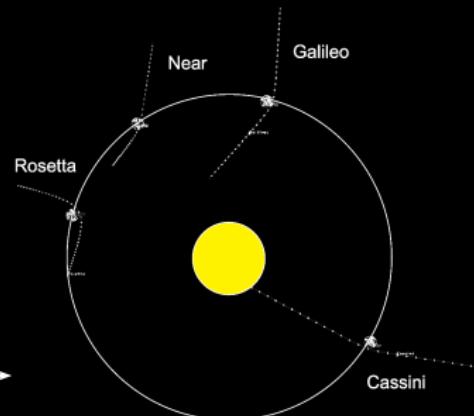
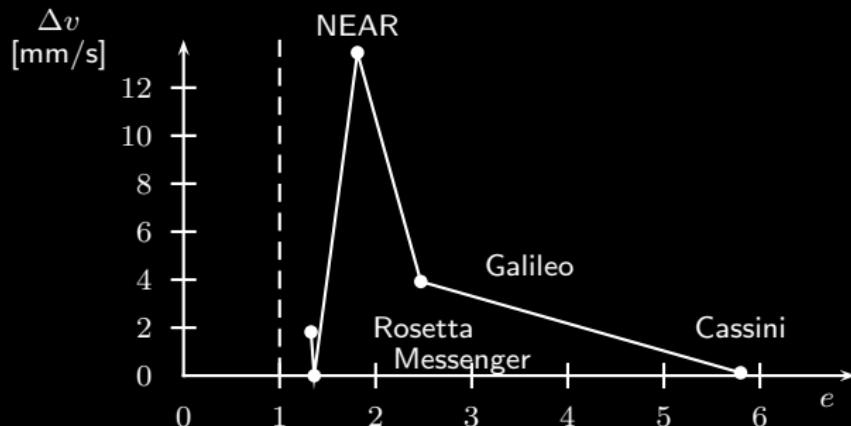
^b no reliable data: thruster activities

^c no data available

^d US spacecraft operated by a private company KinetX

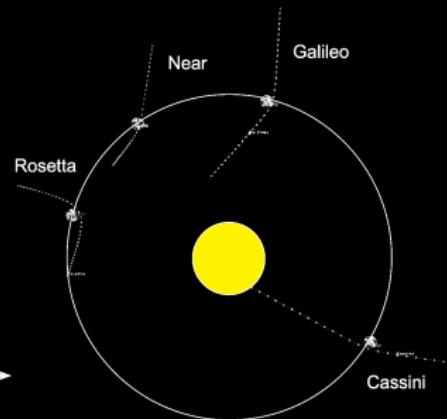
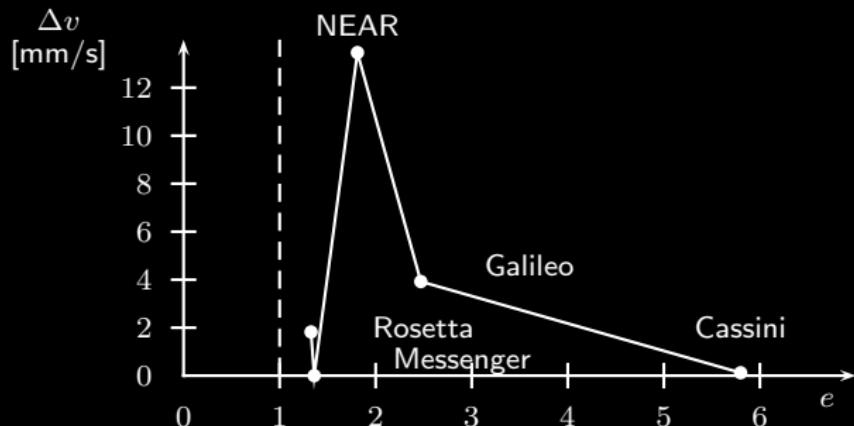
- Anomaly seems to exist also for flybys at other planets.

The flyby anomaly



Δv as function of e . Earth flybys in barycentric coordinates (Campbell 2006)

The flyby anomaly



Δv as function of e . Earth flybys in barycentric coordinates (Campbell 2006)

Remarks / Questions

- Violation of energy conservation?
- More systematic study needed: Dependence on height, velocity, inclination, excentricity, ...?
- Effect on **escape orbits** only

The flyby anomaly

First analysis

Velocity increase is related to an anomalous acceleration of

$$a_{\text{flyby}} \sim 2 \cdot 10^{-4} \text{ m s}^{-2}, \quad \frac{a_{\text{flyby}}}{a_{\text{Newton}}} \sim 10^{-5}$$

First error analysis

- Atmospheric drag
- Errors in Earth gravity model
- Ocean tides
- Earth solid tides
- Charging of spacecraft
- Magnetic moment of spacecraft
- Further non-gravitational forces (trapped air, ion plasma drag, ...)

Nothing could explain the flyby-anomaly

The flyby anomaly

Attempts at explanation by "new physics" (not published)

- Non-conservative potential energy
- Non-Newtonian gravity (e.g. Moffat, PPN, Yukawa, ...)
- Torsion (*eps2 model*); fit the data, but not compatible with stability of planetary orbits

Nothing could explain the flyby-anomaly

Tasks

- Better observation of future flybys
- Flybys at other planets (Mars, ...)
- Theory (dynamics of extended objects: gravity induced rotations)



Increase of Astronomical Unit

Observation

Various groups reported

- Krasinsky & Brumberg 2005: 15 ± 4 m/cy
- Pitjeva in Standish 2005: 7 ± 1 m/cy

Remarks / Question

- \dot{G} excluded by LLR
- Mass loss of Sun leads only to 1 m/cy
- Influence of cosmic expansion by many orders of magnitude too small
- Increase of Solar wind plasma on long timescales?
- Drift of clocks $t \rightarrow t + \frac{1}{2}\alpha t^2$ with $\alpha \sim 10^{-19} \text{ s}^{-1}$?
-



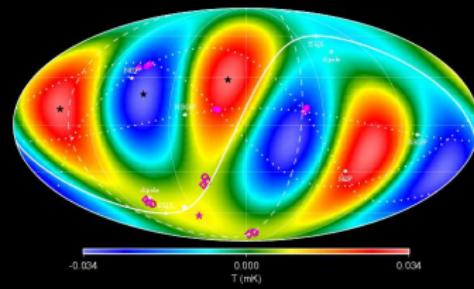
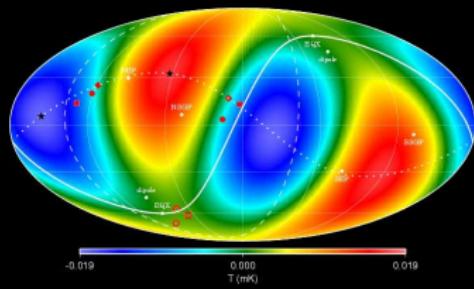
Quadrupole / Octopole anomaly

Observation

Oliveira-Costa et al, PRD 2004, Schwarz et al, PRL 2005

Anomalous behavior of low l contributions of CMB:

- Quadrupole and octopole are aligned to $> 99.87\%$ C.L.
- Quadrupole and octopole aligned to ecliptic to $> 99\%$ C.L.
- No correlation with the galactic plane



Remarks / Question

- Influence of Solar system on CMB observations?
- Systematics?

Status of Anomalies

The status of these anomalies

Anomaly	Observational status	Interpretation
Dark Energy, Dark Matter	well established	under discussion
Flyby anomaly, Pioneer Anomaly	quite well established	unclear
quadrupole anom., increase of AU	unclear	unclear

It is worth to discuss these anomalies

- Try to find conventional explanations
- Try to find relations between anomalies (anomalies very probably are no isolated phenomena). Do some effects have the same cause?
- Influence of cosmic expansion or dark matter? ($a_{\text{Pioneer}} \sim cH \sim a_{\text{MOND}}$)
- Are there similar influences in other gravitating systems, e.g., binary systems, stars moving around the black hole in the center of our galaxy, ...?
- Something wrong with weak gravity?

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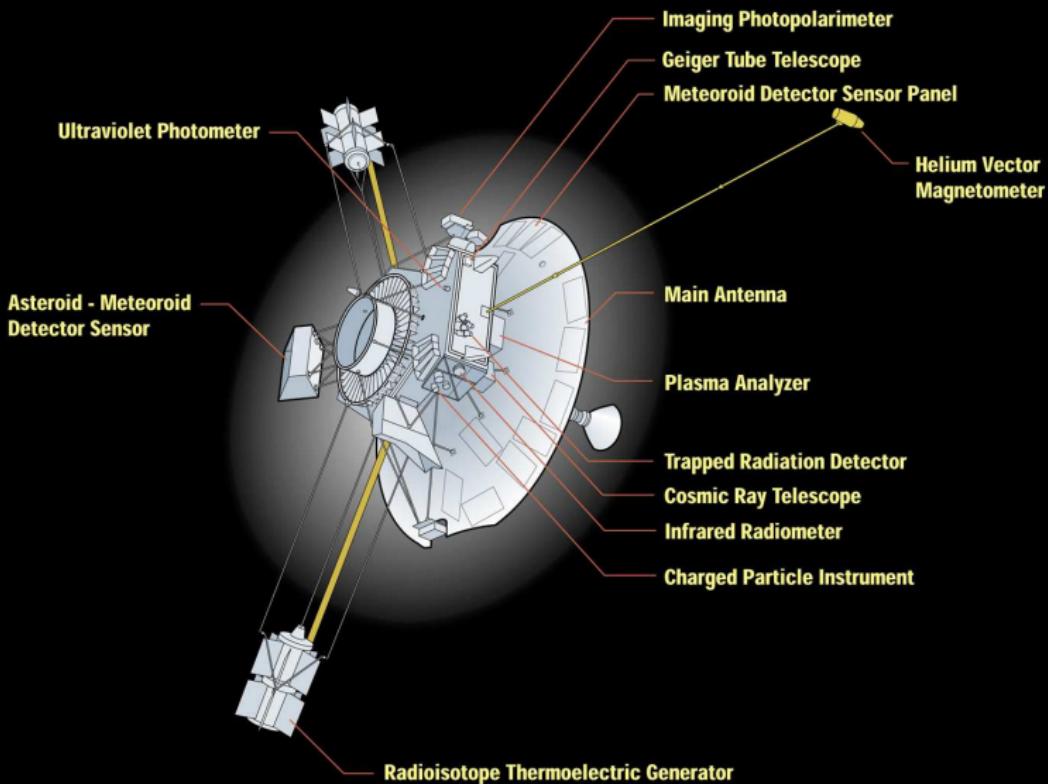
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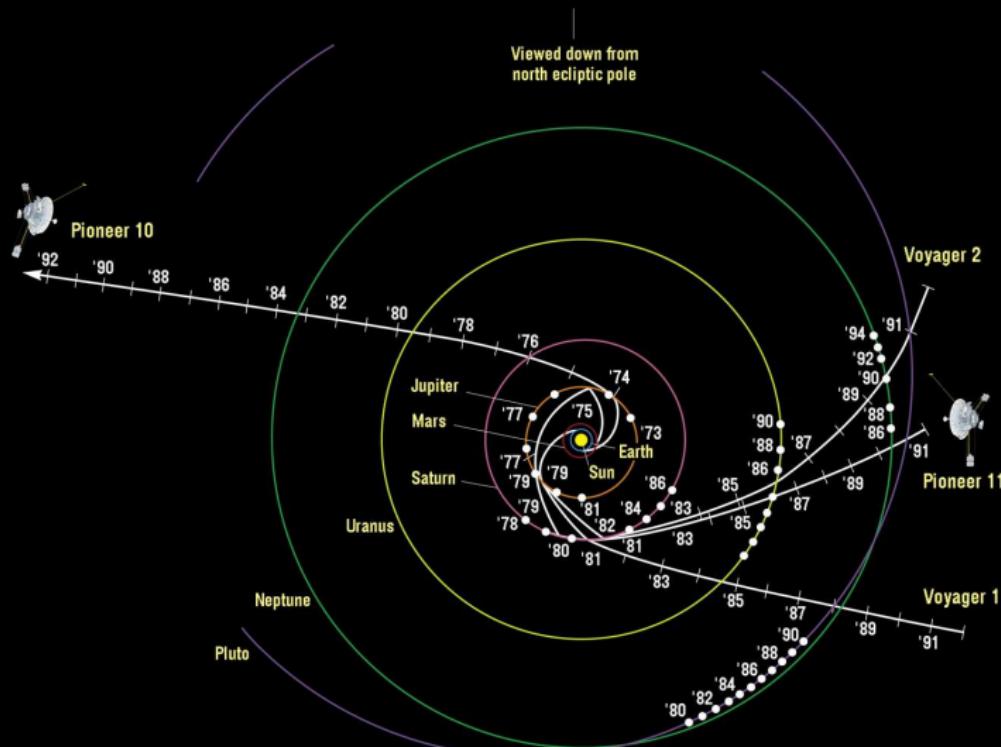
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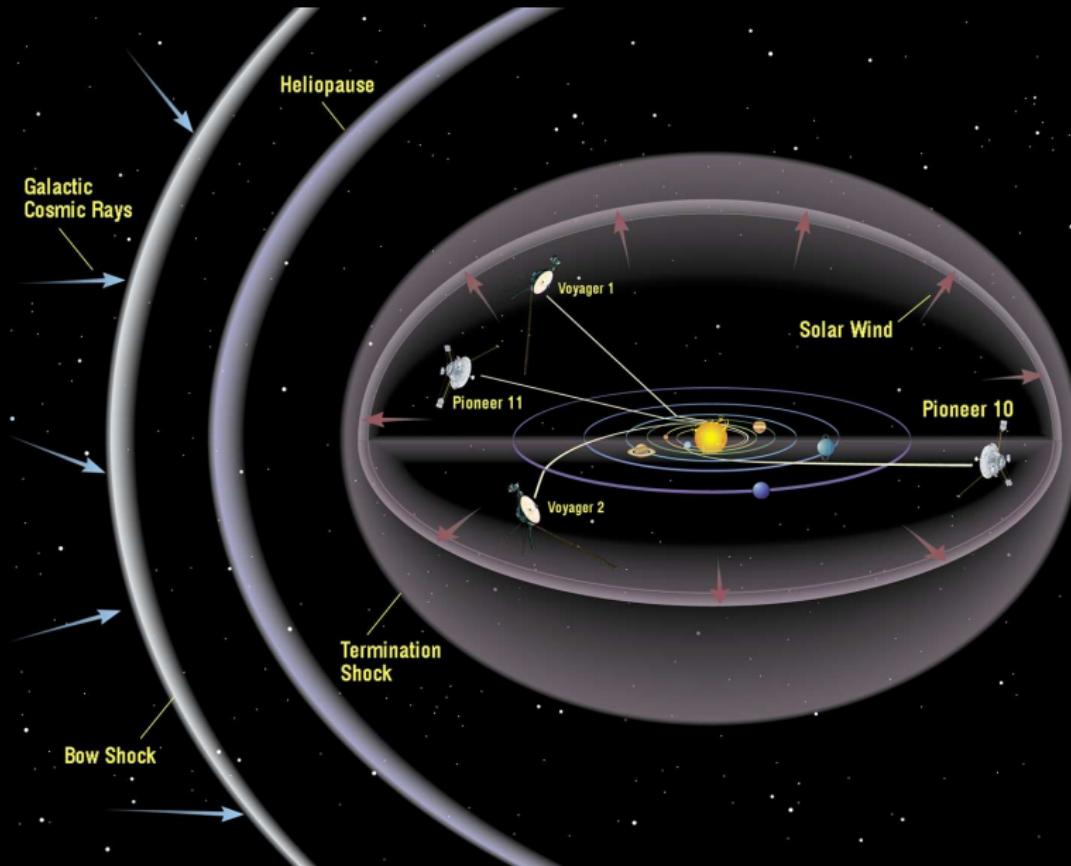
The Pioneer spacecraft



The orbits



The environment



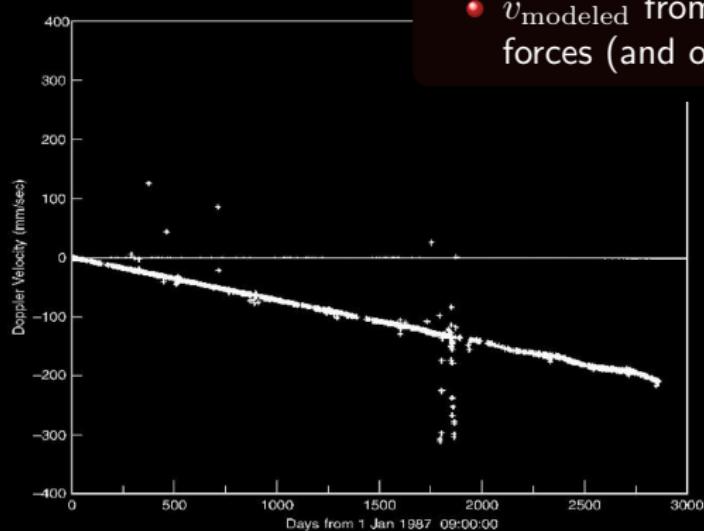
The result

Result from Doppler tracking of the Pioneers

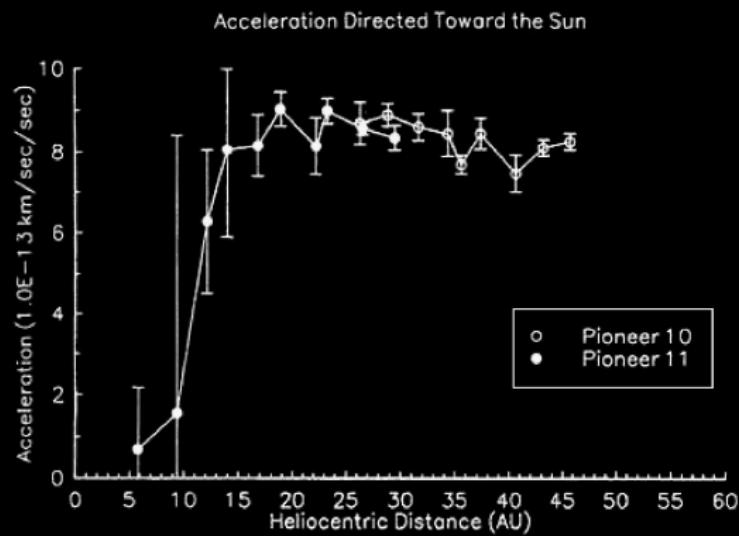
$$v_{\text{observed}} - v_{\text{modeled}} = -a_{\text{Pioneer}} t$$

with

- v_{observed} from Doppler tracking
- v_{modeled} from calculations due to gravitational forces (and others)

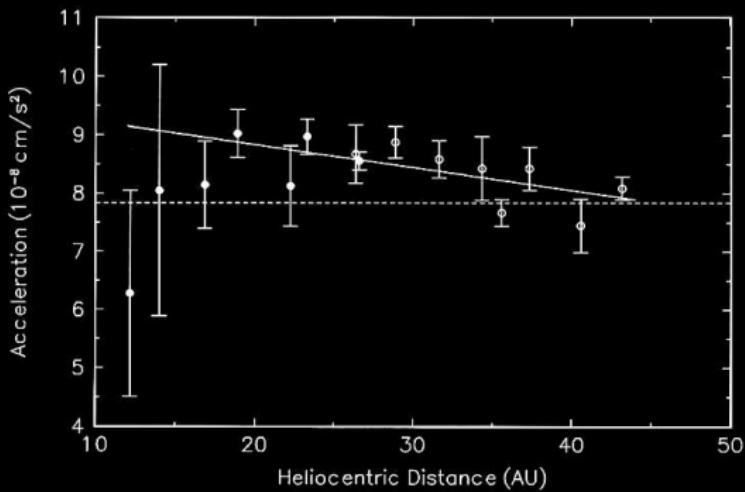


The result



The acceleration seems to appear just with the last flyby, that is, after entering an escape orbit.

The result



The acceleration seems to appear just with the last flyby, that is, after entering an escape orbit.

The basic observation

The observation (Anderson et al 1998, 2002)

- Measured anomalous, uniformly blue shifted Doppler frequency drift

$$\frac{d\nu}{dt} = (5.99 \pm 0.01) \cdot 10^{-9} \text{ Hz/s}$$

- Can be interpreted as a constant acceleration

$$a_{\text{Pioneer}} = (8.74 \pm 1.33) \cdot 10^{-10} \text{ m/s}^2$$

- Acceleration **constant** and **toward** the Sun
- Temporal and spatial variations less than 3%

Status

- No cause for the effect has been found
- All errors are much smaller than the effect
- $a_{\text{Pioneer}}/a_{\text{Newton}} \sim 10^{-5}$

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Orbit determination: Modeling

- Model of gravitational forces
 - Model of external non-gravitational forces
 - Model of internal non-gravitational forces
 - Model of observation stations
 - Model of signal propagation
 - Codes
- } \Rightarrow Orbit and velocity determination

Model of gravitational forces

Relativistic equations of motion for celestial bodies including order v^4/c^4

- Relativistic gravitational accelerations (EIH model) include:
Sun, Moon, 9 planets are point masses in isotropic, PPN, N -body metric
- Newtonian gravity from large asteroids
- Terrestrial and lunar figure effects
- Earth tides
- Lunar physical librations

Orbit determination: Modeling

Model of external non-gravitational forces

- Solar radiation and Solar wind pressure
- Drag from interplanetary dust

Model of internal non-gravitational forces

- Thermal radiation
- Attitude-control propulsive maneuvers and propellant (gas) leakage from propulsion system
- Torques produced from above forces

Modeling of signal propagation

- Relativistic model for light propagation including order v^2/c^2
- Dispersion

Orbit determination: Modeling

Model of observation stations

Orbit determination includes

- Precession, nutation, sidereal rotation, polar motion, tidal effects, and tectonic plates drift
- Informations on tidal deceleration, non-uniformity of rotation, Love numbers, and Chandler wobble from LLR, SLR, and VLBI (from ICRF)
- DSN antennae contributions to the spacecraft radio tracking data

Codes

Use of four independent codes

- JPL Orbit Determination Program (various generations from 1970 – 2001)
- The Aerospace Corporation code POEAS (during period 1995 – 2001)
- Goddard Space Flight Center conducted a study in 2003 (data from NSSDC)
- Code of University of Oslo

Next discussion: Errors

Orbit determination: Modeling

Model of observation stations

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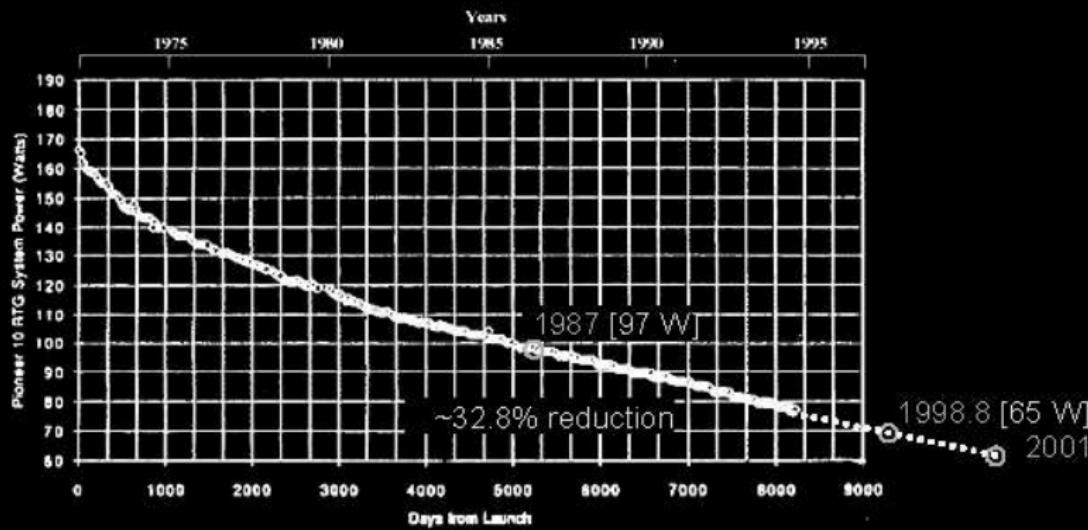
Next discussion: Errors ↔ possible conventional explanations

On-board error

Heat

Heat is an important source of error, but

- It is not strong enough (from geometry) to explain the anomaly
- Exponential decay of heat (and related errors) **not** seen



Drag through dust?

Interplanetary medium

- Interplanetary dust
 - Hot wind plasma (mainly protons and electrons) distributed within the Kuiper belt (from 30 AU to 100 AU)
 - *Modelled* density $\rho_{\text{IPD}} \leq 10^{-24} \text{ g/cm}^3$ ([Man and Kimura 2000](#))
- Interstellar dust
 - Fractions of total dust (characterized by greater impact velocity)
 - *Measured* by Ulysses $\rho_{\text{ISD}} \leq 3 \cdot 10^{-26} \text{ g/cm}^3$

Effect on Pioneers ([Nieto et al, MPJ 2005](#))

- Drag on spacecraft $a_s = -K_s \rho v_s^2 A_s / m_s$
- Only dust of a density of $\rho \sim 3 \cdot 10^5 \cdot (\rho_{\text{IPD}} + \rho_{\text{ISD}})$ can lead to a Pioneer acceleration
- Dust cannot be the origin of the Pioneer acceleration

Errors by external effects

Further external effects

- Gravity from the Galaxy
- Gravity from dark matter distributed in the halo around the Solar system
- Errors in the planetary ephemerides, Earth orientation parameters, precession, and nutation

Rejected as explanation

Signal errors

The transponders

The transponders cannot show any drift because they transform the incoming frequency through multiplication with a simple ratio 240/221.

$$\frac{\nu_{\text{sent}}}{\nu_{\text{received}}} = \frac{240}{221}$$

This is implemented into the hardware and **can not** drift.

Signal errors

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$$\frac{\nu_{\text{sent}}}{\nu_{\text{received}}} = \frac{240}{221}$$

This is implemented into the hardware and **can not** drift.

Influence of medium on signals

Influence gives **redshifted** modification. Anomalous acceleration corresponds to blueshift.



Error budget: external effects

Sources of external systematic error

Error budget constituents	Bias 10^{-10} m/s^2	Uncertainty 10^{-10} m/s^2
Solar radiation pressure		± 0.001
From the mass uncertainty	+0.03	± 0.01
Solar wind contribution		$\pm < 10^{-5}$
Effects of the solar corona		± 0.02
Electro-magnetic Lorentz forces		$\pm < 10^{-4}$
Influence of the Kuiper belt's gravity		± 0.03
Influence of the Earth orientation		± 0.001
DSN Antennae: mechanical/phase stability		$\pm < 0.001$
Phase stability and clocks		$\pm < 0.001$
DSN station location		$\pm < 10^{-5}$
Effects of troposphere and ionosphere		$\pm < 0.001$

Error budget: simulation

Computational systematics

Error budget constituents	Bias 10^{-10} m/s^2	Uncertainty 10^{-10} m/s^2
Numerical stability of least-squares estimation		± 0.02
Accuracy of consistency/model tests		± 0.13
Mismodeling of maneuvers		± 0.01
Mismodeling of the solar corona		± 0.02
Annual/diurnal terms		± 0.32



Error budget: on-board effects

Sources of internal systematic error

Error budget constituents	Bias 10^{-10} m/s^2	Uncertainty 10^{-10} m/s^2
Radio beam reaction force	+1.10	± 0.11
Thermal/propulsion effects from RTGs:		
RTG heat reflected off the craft	-0.55	± 0.55
Differential emissivity of the RTGs		± 0.85
Non-isotropic radiative cooling of s/c		± 0.16
Expelled He produced within the RTGs	+0.15	± 0.16
Propulsive mass expulsion: gas leakage		± 0.56
Variation between s/c determinations	+0.17	± 0.17

Conclusion

Observation

All data analyses show that there is an anomalous acceleration

$$a_{\text{Pioneer}} = (8.74 \pm 1.33) \cdot 10^{-10} \text{ m/s}^2$$

of the Pioneer 10 and 11 spacecrafts.

► General theoretical description



Conclusion

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of the Pioneer 10 and 11 spacecrafts.

- No cause for the effect has been found
- All errors are much smaller than the effect

► General theoretical description



Conclusion

Observation

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Question

What is the origin of the acceleration ?

► General theoretical description



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Helicity–rotation coupling

Anderson & Mashhoon, PLA 2003

- ω and \mathbf{k} are frequency and wave vector measured by inertial observer
frequency measured by moving v and rotating Ω observer

$$\text{helicity operator for the photon } \widehat{\mathbf{H}} = \pm \frac{1}{\omega} \mathbf{k}$$

$$\omega' = \frac{1}{\sqrt{1 - v^2}} \left(\omega - \widehat{\mathbf{H}} \cdot \boldsymbol{\Omega} - \mathbf{v} \cdot \mathbf{k} \right) = \frac{1}{\sqrt{1 - v^2}} \left(\omega - \left(\mathbf{v} \mp \frac{\boldsymbol{\Omega}}{\omega} \right) \cdot \mathbf{k} \right)$$

- Simulates

$$\mathbf{v} \quad \rightarrow \quad \mathbf{v}' = \mathbf{v} \mp \frac{c}{\omega} \boldsymbol{\Omega}$$

Effect on Pioneer

too small



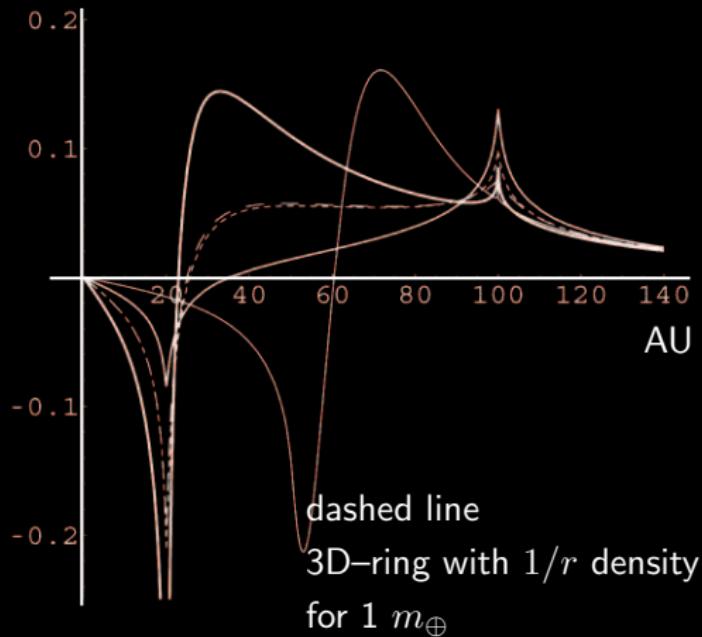
Masses in the Kuiper belt

Nieto, PRD 2005

Newton potential

$$U(r) = -G \int \frac{\mu(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3V'$$

10^{-8} m/s^2



Observation station errors

Drift of clocks

Quadratic drift of clocks on Earth may simulate the effect ([Anderson et al, PRD 2002](#), [Ranada, FP 2005](#))

$$t \rightarrow t + \frac{1}{2}a_P t^2, \quad a_P \sim 10^{-19} \text{ s}^{-1}$$

- Consistency with other satellites?
- Consistency with observation of pulsars and binary systems (define also clocks)?

Attempted explanations

Attempted explanations **within standard physics**

- An intriguing observation:

$$a_{\text{Pioneer}} \approx c H \approx a_{\text{MOND}}$$

- Influence of cosmology?
- Influence of dark matter?

Explanations using non-standard physics

- Running coupling constant (Jaekel and Reynaud, CQG 2004)
- Antisymmetric Gravity (Moffat 2004)
- Braneworld scenarios (Bertolami, CQG 2003)
- Projective geometry (Schmutzler, GRG 2000)
- ...

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The Questions

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- Does the coupling of the expansion to light lead to an effect?
- Does the cosmological expansion influence the physics in the Solar system?
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CL, Preuss & Dittus 2007

▶ Sun



Cosmology: Doppler ranging in the expanding universe

Conserved quantities

- Light

$$\nu_u(t)R(t) = \text{const.}$$

⇒ Hubble red shift

$$\nu_u(t) = \frac{R(t_0)}{R(t)}\nu_u(t_0) \approx (1 - H(t - t_0))\nu_u(t_0)$$

Cosmology: Doppler ranging in the expanding universe

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\Rightarrow Hubble red shift

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- Massive particles

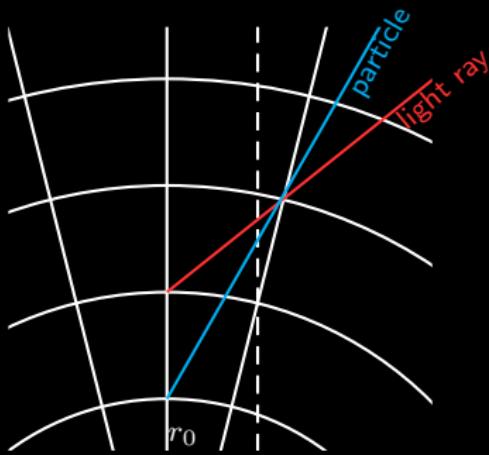
$$R^2(t) \frac{dr(s)}{ds} = \text{const.} \Leftrightarrow R(t) \frac{1}{\sqrt{1 - V^2(t)}} V(t) = \text{const.}$$

For small velocities $R(t)V(t) = \text{const.}$

\Rightarrow slowing down of velocity

$$V(t_2) = \frac{R(t_1)}{R(t_2)} V(t_1) = (1 - H(t_2 - t_1)) V(t_1)$$

Cosmology: Doppler ranging in the expanding universe



Kinematics: Distance and velocity

Distance D measured by time-of-flight of light rays

$$D = R(r_2 - r_1)$$

Change of measured distance ($r_1 = \text{const.}$)

$$\frac{d}{dt} D = \dot{R}(r_2 - r_1) + R\dot{r}_2 = HD + V_2$$

Trajectory at constant distance to observer has local velocity

$$0 = \dot{D} = HD + V_2 \quad \Rightarrow \quad V_2 = -HD.$$

Cosmology: Doppler ranging in the expanding universe

Distance and velocity of moving objects: Application to moving spacecrafts

Observer and observed spacecraft

- Pioneer spacecrafts move on geodesics: slowing down of the spacecraft
 - Cosmic red shift of frequency
 - Velocity of points of constant distance with respect to the cosmic substrate
- ⇒ final Doppler effect

$$\nu_2(t_2) = (1 - H(t_2 - t_1) - V_2^{\text{tot}}) \nu_0(t_1).$$

Here V_2^{tot} is the total velocity of the spacecraft with respect to cosmic substrate

$$V_2^{\text{tot}} = -H(t_2 - t_1) - V(t_1)H(t_2 - t_1)$$



Cosmology: Doppler ranging in the expanding universe

Result

$$\nu_2(t_2) = (1 + V(t_1)H(t_2 - t_1)) \nu_0$$



Cosmology: Doppler ranging in the expanding universe

Result

$$\nu_2(t_2) = (1 + V(t_1)H(t_2 - t_1)) \nu_0$$

Discussion

- Cosmological red shift and Doppler effect from the velocity with respect to coordinates given by constant distances cancel
- Object at constant distance cannot make a Doppler effect
- Only the satellite's slowing down is left over
- Results in chirp of Doppler signal related to an acceleration

$$a = HV = \frac{V}{c}cH$$

- Acceleration is V/c smaller than observed Pioneer acceleration $a_{\text{Pioneer}} \sim cH$.

Therefore, Doppler tracking in an expanding universe cannot account for the observed Pioneer Anomaly (agrees with Carrera & Giulini 2007)

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▶ Planet



Cosmology and Solar system physics

Small isolated mass in expanding universe

Ansatz

- Weak field ansatz

$$g_{\mu\nu} = b_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu} \ll b_{\mu\nu}.$$

- Linearized Einstein equations for $h_{\mu\nu}$

$$b^{\rho\sigma} D_\rho D_\sigma \bar{h}_{\mu\nu} + 2b^{\rho\sigma} R^\kappa{}_{\mu\rho\nu} \bar{h}_{\kappa\sigma} = 16\pi G T_{\mu\nu},$$

- Solution

$$h_{00} = \frac{2GM}{R} \frac{\cos(\sqrt{6}|\dot{R}|r)}{r} = \frac{2GM}{Rr} (1 - 3H^2(Rr)^2 \pm \dots)$$

- Lowest order: standard Newtonian potential with **measured** distance $R(t)r$
- Additional acceleration toward the gravitating body, second order in H
- Potential practically does not participate in the cosmic expansion

The Questions

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▶ Escape



Cosmology: Modification of planetary orbits

General question

Behavior of bound systems in expanding universe

- Electromagnetically bound systems (Audretsch & Schäfer 1975, Bonnor 2000)
- Gravitationally bound systems

Adiabatic invariants

- Action

$$\begin{aligned} S &= -mc^2 \int \sqrt{1 - 2\frac{U(\mathbf{x})}{c^2} - R^2(t)\frac{\dot{\mathbf{x}}^2}{c^2}} dt \\ &\approx \int \left(-mc^2 + mU(r) + \frac{m}{2}R^2(t)(\dot{r}^2 + r^2\dot{\varphi}^2) \right) dt \end{aligned}$$

- For weak time dependence of $R(t)$: two **adiabatic invariants**

$$I_\varphi = L, \quad I_r = -L + \frac{GmM}{R} \sqrt{\frac{m}{2|E|}}.$$

Cosmology: Modification of planetary orbits

Adiabatic invariants

- Result:

$$E = -\frac{m^3 G^2 M^2}{2(I_r + I_\varphi)}, \quad p = \frac{I_\varphi^2}{m^2 M R}, \quad e^2 = 1 - \left(\frac{I_\varphi}{I_r + I_\varphi} \right)^2$$

- E , e , and $R(t)p$ are nearly constant,
- Rp and e of the planetary orbits **practically do not participate in cosmic expansion**
- Stability of adiabatic invariants estimated by $\exp(-\tau/T)$, τ = characteristic time of change of external parameter (here: $\tau = 1/H$), T = characteristic time of the periodic motion (here: T = period of planets)
 $\Rightarrow \exp(-\tau/T) \sim e^{-10^{10}}$: any change of the adiabatic invariants is truly negligible
- Analysis applies to periodic orbits = bound orbits
- Secular increase of AU cannot be explained by that

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▶ Lambda



Cosmology: Modified PPN formalism

Escape orbits

- PPN inspired space-time metric

$$g_{00} = -1 + 2\alpha \frac{U}{c^2} - 2\beta \frac{U^2}{c^4}, \quad g_{ij} = \left(1 + 2\gamma \frac{U}{c^2}\right) R^2(t) \delta_{ij}.$$

- Equation of motion in terms of measured distances and times:

$$\begin{aligned} \frac{d^2 X^i}{dT^2} &= \frac{\partial U}{\partial X^i} \left(\alpha + \gamma \frac{1}{c^2} \left(\frac{dX}{dT} \right)^2 + (2\alpha^2 - \gamma\alpha - 2\beta) \frac{U}{c^2} \right) \\ &\quad - \left((\alpha + \gamma) \frac{1}{c^2} \frac{\partial U}{\partial X^j} \frac{dX^j}{dT} + \frac{\dot{R}}{R} \right) \frac{dX^i}{dT} \end{aligned}$$

- Cosmological expansion results in a decelerating drag term which is a factor v/c too small to account for the Pioneer effect
- Bound orbits behave differently than escape orbits

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Schwarzschild-(anti-)de Sitter space-time

- Post-Newtonian approximation:
Kagramanova, Kunz & CL, PLB 2006
Kerr, Hauck and Mashhoon, CQG 2003
Sereno & Jetzer, PRD 2006
- Analytic treatment:
Hackmann & CL, PRL 2008

The metric

- The metric

$$ds^2 = \alpha dt^2 - \alpha^{-1} dr^2 - r^2 (\sin^2 \theta d\phi^2) ,$$

with

$$\alpha = 1 - \frac{2M}{r} - \frac{1}{3}\Lambda r^2$$

and Λ is the cosmological constant and M the mass of the source.

- $\Lambda < 0$ attraction, $\Lambda > 0$ repulsion

Schwarzschild-(anti-)de Sitter first order effects

Results

Observed effect	Estimate on Λ
gravitational redshift	$ \Lambda \leq 10^{-27} \text{ m}^{-2}$
perihelion shift	$ \Lambda \leq 3 \cdot 10^{-42} \text{ m}^{-2}$
light deflection	no effect
gravitational time delay	$ \Lambda \leq 6 \cdot 10^{-24} \text{ m}^{-2}$
geodetic precession	$ \Lambda \leq 10^{-27} \text{ m}^{-2}$
Pioneer anomaly	$\Lambda = -10^{-37} \text{ m}^{-2}$

Table: Estimates on Λ from Solar system observations.

- Cosmological constant has no influence on Solar system effects
- If some **other** Λ is assumed to account for the Pioneer anomaly, then it conflicts with Perihelion shift \Rightarrow A constant Λ **cannot** be responsible for Pioneer anomaly
- May not describe critical orbits (near separatrix) \rightarrow **needs exact calculations**

Geodesic eqn in Schwarzschild-(anti-)de Sitter space-time

Geodesic equation

$$0 = \frac{d^2x^\mu}{ds^2} + \{\overset{\mu}{\rho\sigma}\} \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds}, \quad g(u, u) = \epsilon$$

conservation of energy E and angular momentum L

effective equations

$$\left(\frac{dr}{d\varphi}\right)^2 = \frac{r^4}{L^2} \left(E^2 - \left(1 - \frac{r_S}{r} - \frac{1}{3}\Lambda r^2\right) \left(\epsilon + \frac{L^2}{r^2}\right) \right)$$

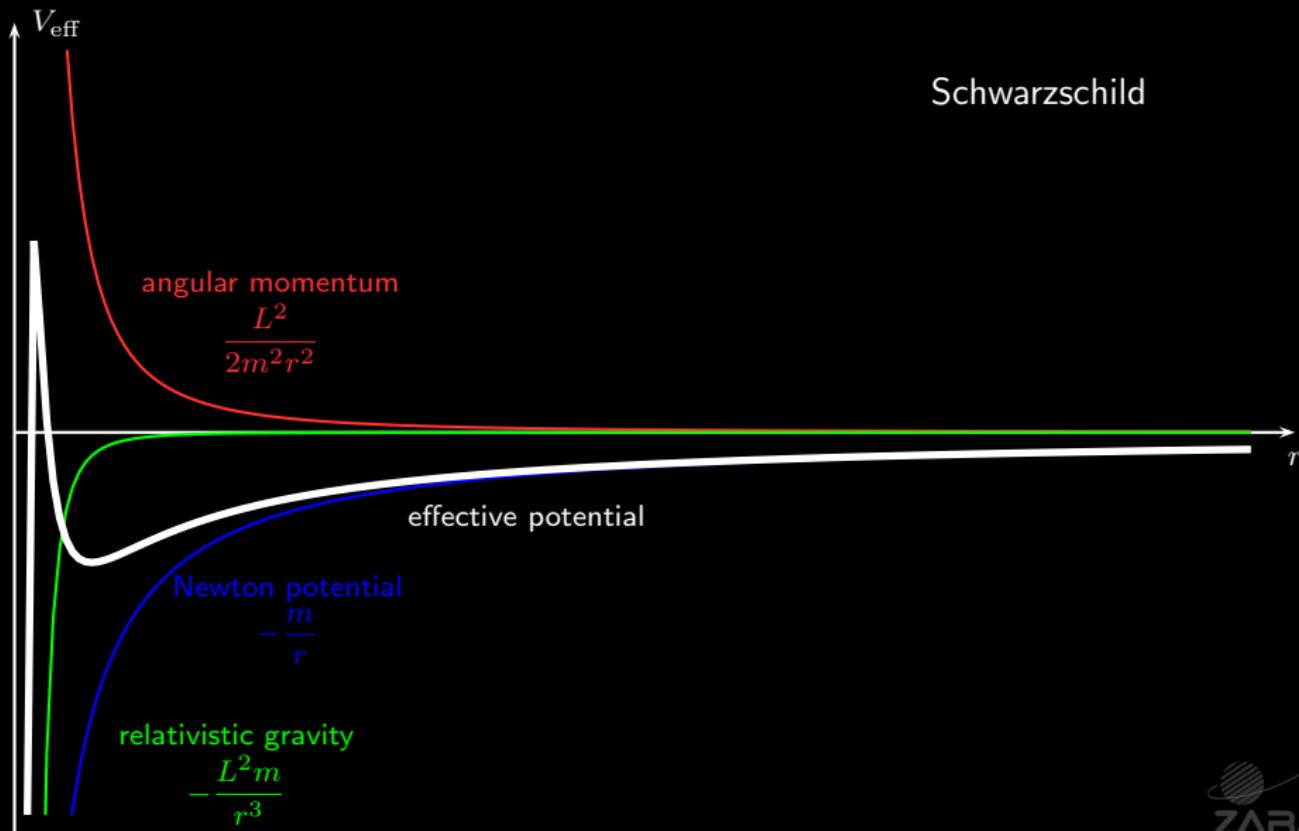
$$\left(\frac{dr}{ds}\right)^2 = E^2 - \left(1 - \frac{r_S}{r} - \frac{1}{3}\Lambda r^2\right) \left(\epsilon + \frac{L^2}{r^2}\right) = E^2 - V_{\text{eff}}(r)$$

$$\left(\frac{dr}{dt}\right)^2 = \frac{1}{E^2} \left(1 - \frac{r_S}{r} - \frac{1}{3}\Lambda r^2\right)^2 \left(E^2 - \left(1 - \frac{r_S}{r} - \frac{1}{3}\Lambda r^2\right) \left(\epsilon + \frac{L^2}{r^2}\right)\right)$$

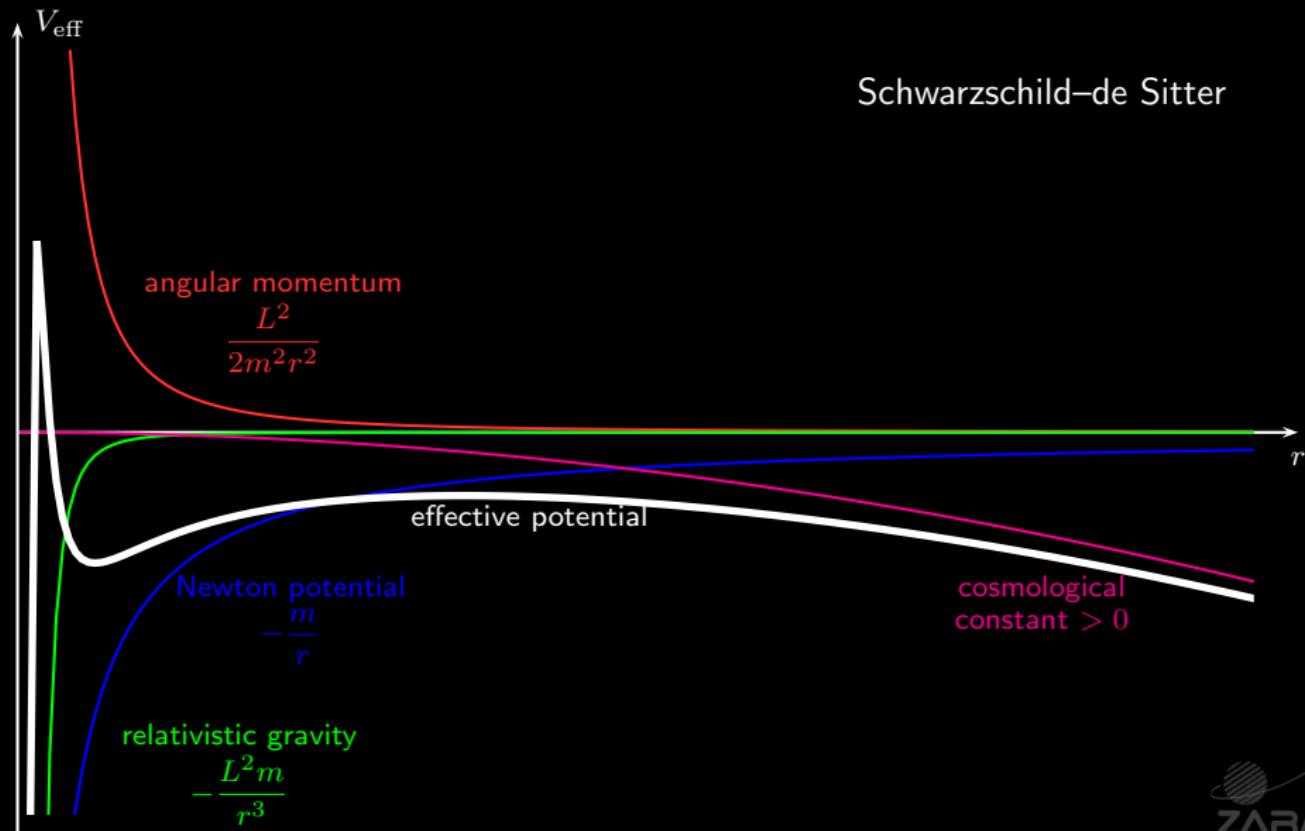
Enhancement of influence of Λ near critical orbits



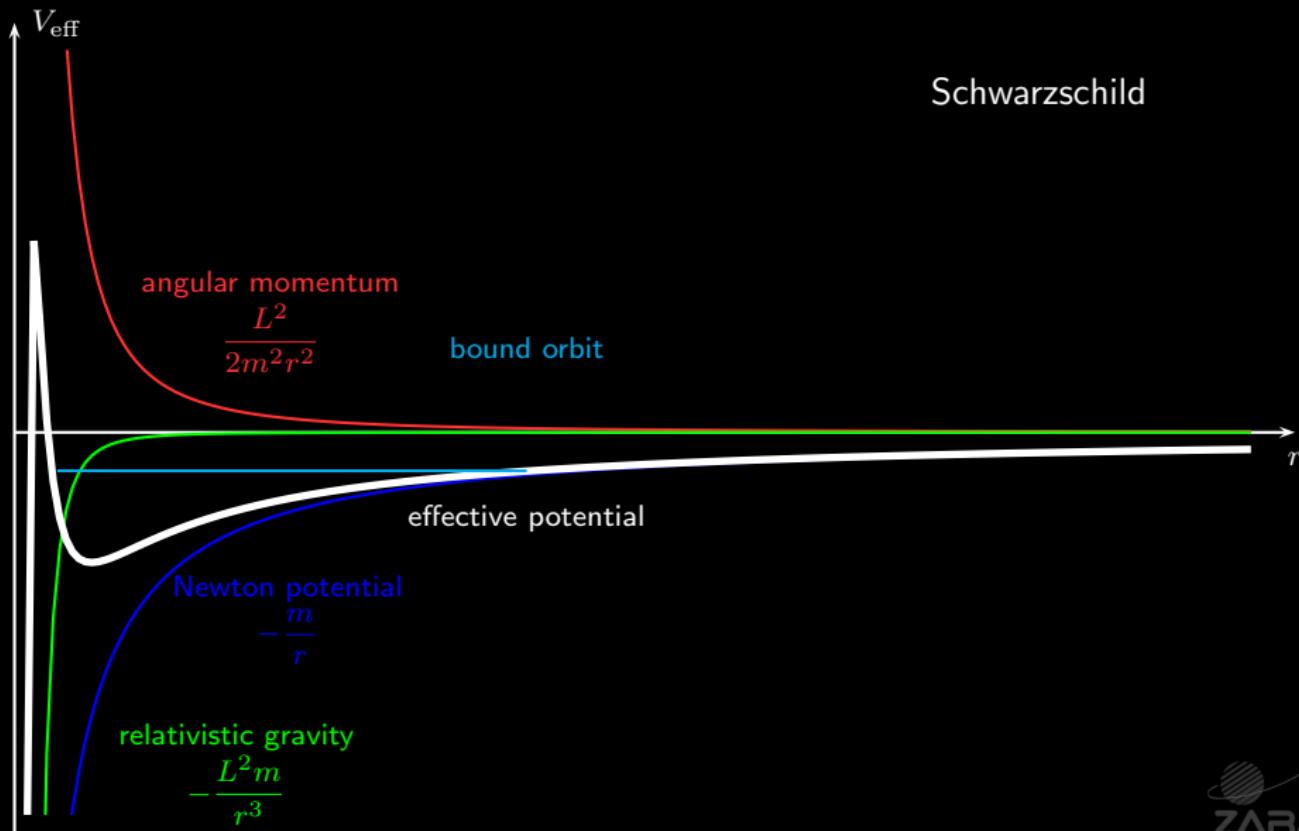
Geodesic equation: effective potential



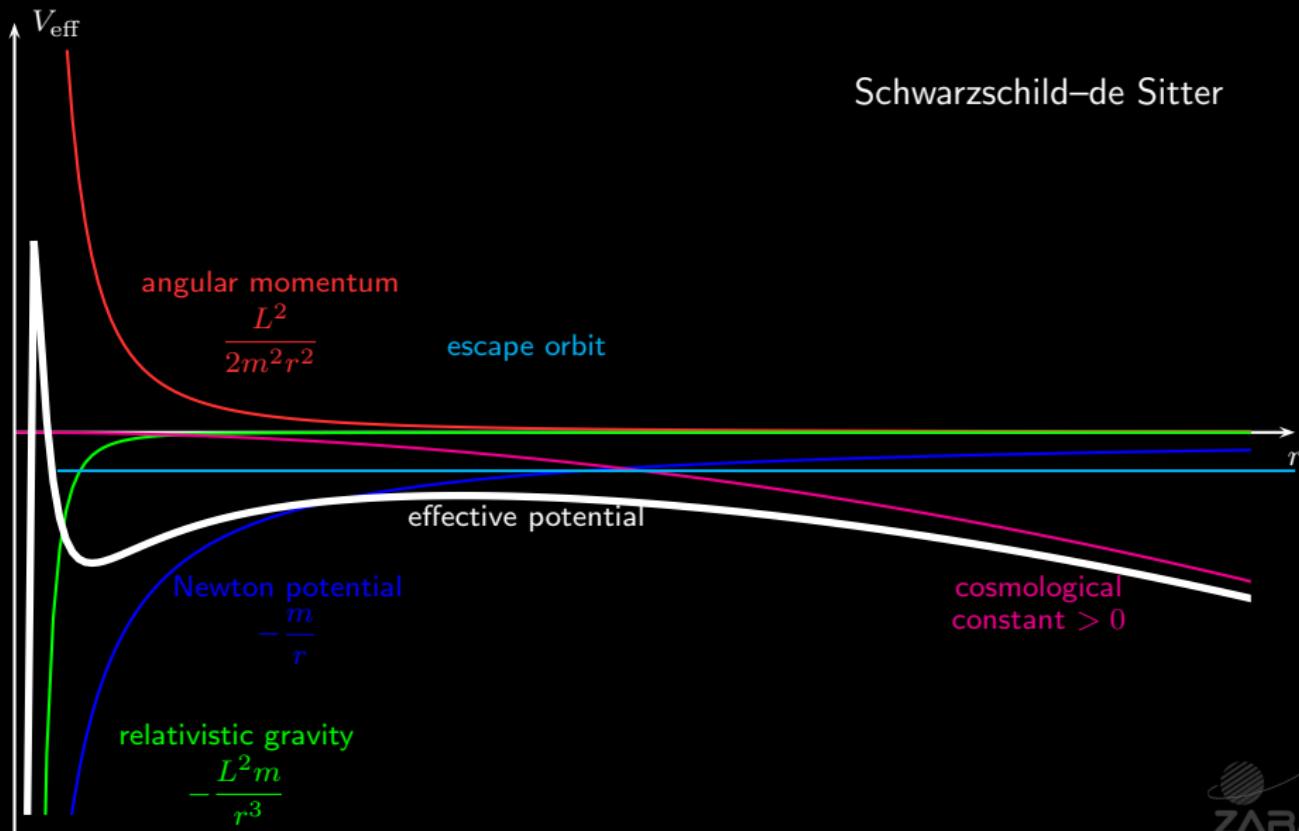
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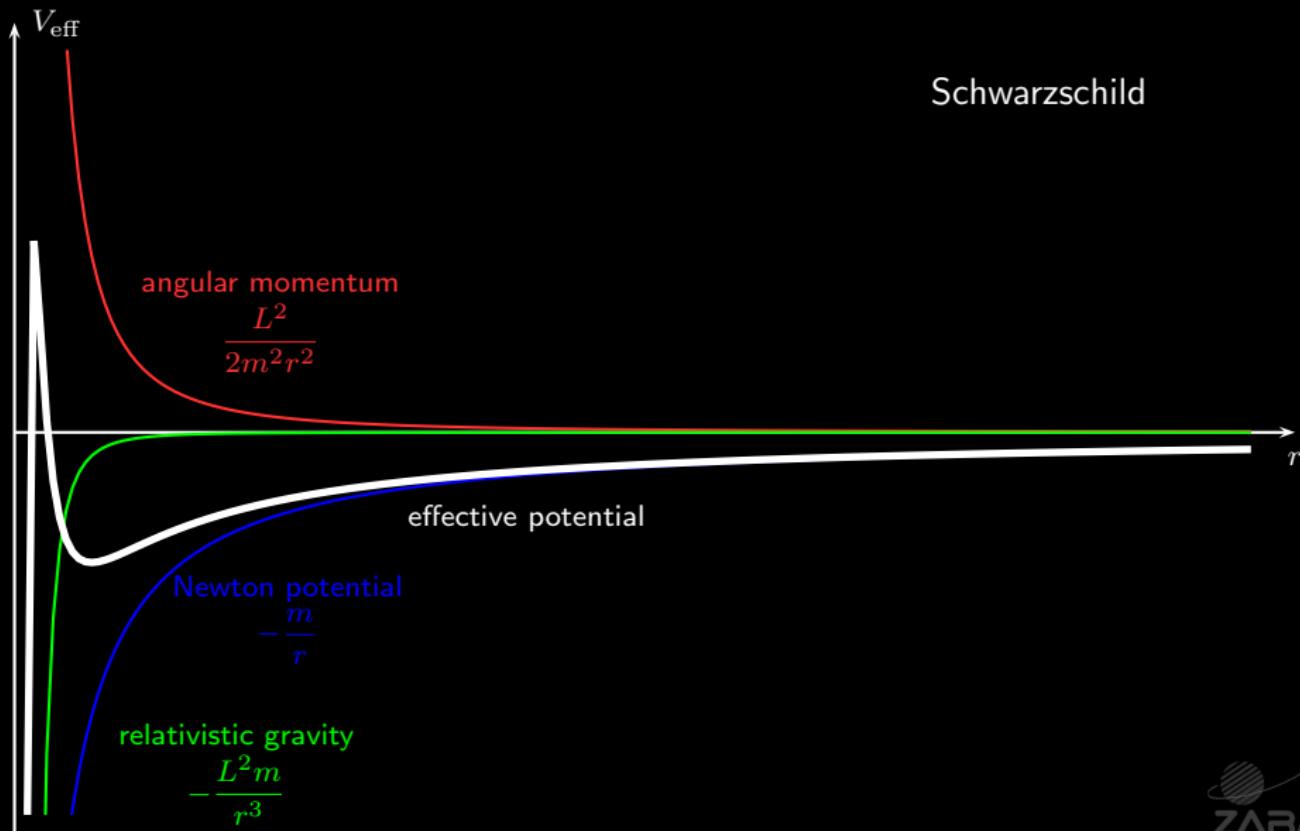
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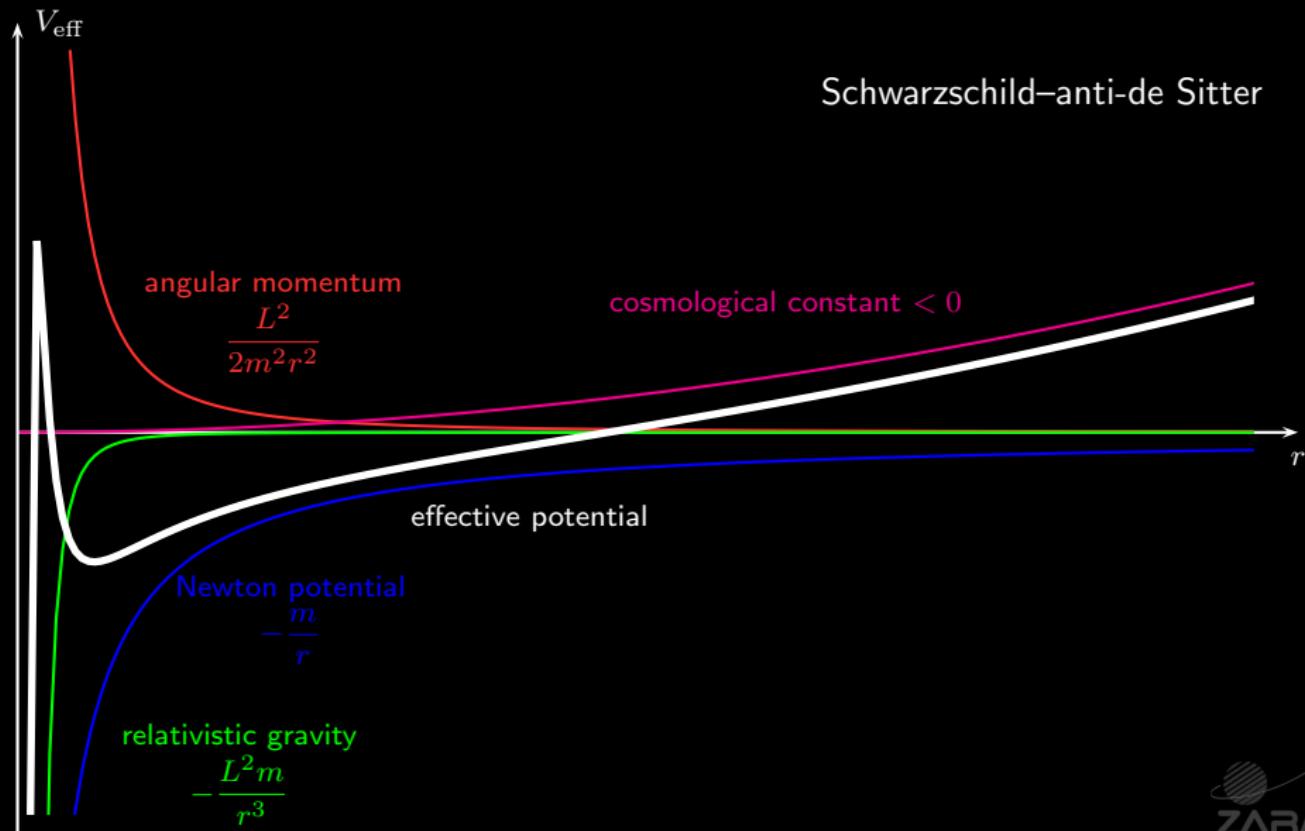
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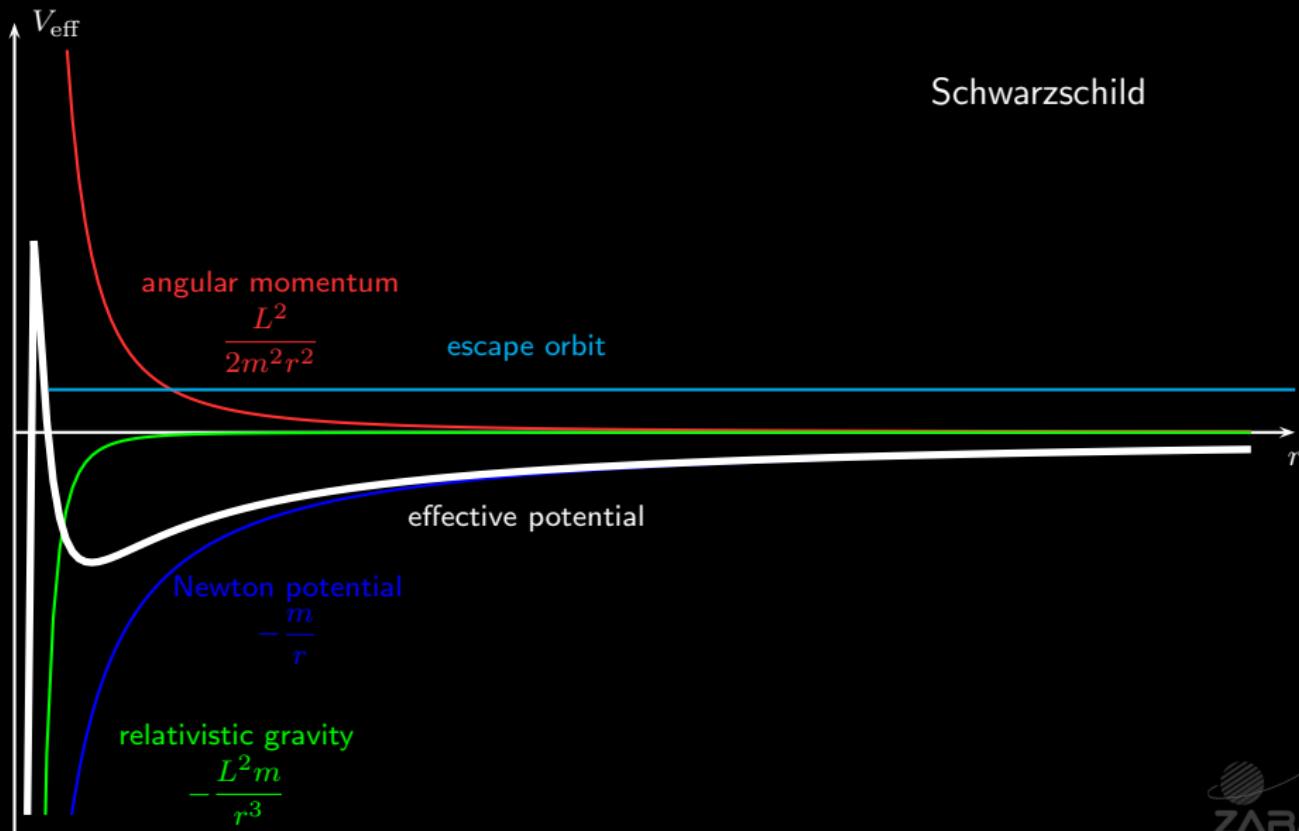
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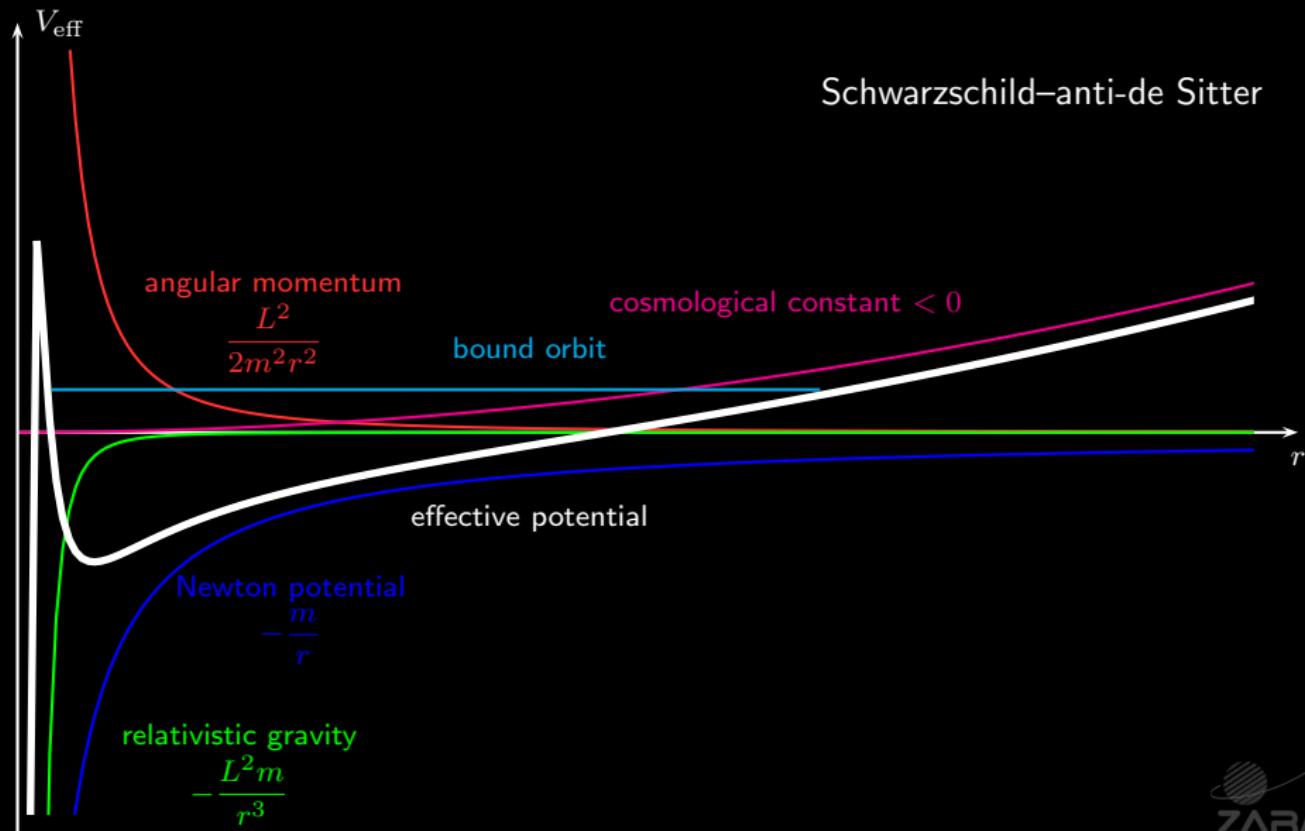
Geodesic equation: effective potential



Geodesic equation: effective potential



Geodesic equation: effective potential



Geodesic eqn in Schwarzschild-(anti-)de Sitter space-time

Effective equation of motion ($u = r_S/r$)

$$\left(u \frac{du}{d\varphi}\right)^2 = u^5 - u^4 + \epsilon \lambda u^3 + \left(\lambda(\mu - \epsilon) + \frac{1}{3}\rho\right) u^2 + \frac{\epsilon}{3} \lambda \rho =: P_5(u)$$

with

$$\lambda = \left(\frac{r_S}{L}\right)^2, \quad \mu = E^2, \quad \rho = \Lambda r_S^2$$

- Polynomial of 5th order → beyond elliptic integral: hyperelliptic integral
- P_5 possesses at most 4 real positive zeros
- $\epsilon = 0 \Rightarrow P_5(u) = u^2 P_3(u) \Rightarrow$ elliptic function \wp (Λ has no influence on light propagation)
- $\Lambda = 0 \Rightarrow P_5(u) = u^2 P_3(u) \Rightarrow$ elliptic function \wp (Hagihara, JJAG 1931)

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Geodesic eqn in Schwarzschild-(anti-)de Sitter space-time

Effective equation of motion ($u = r_S/r$)

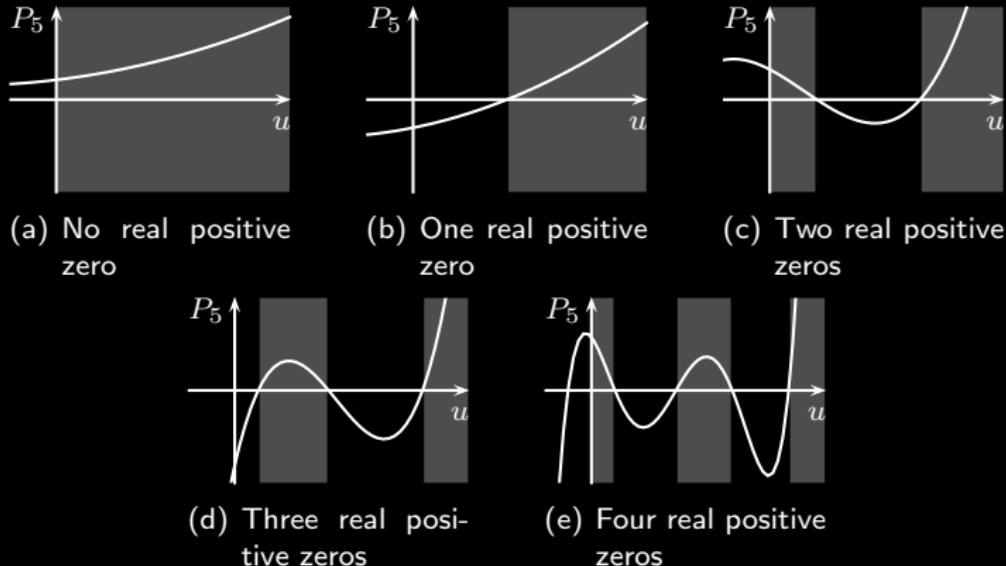
$$\left(u \frac{du}{d\varphi}\right)^2 = u^5 - u^4 + \epsilon \lambda u^3 + \left(\lambda(\mu - \epsilon) + \frac{1}{3}\rho\right) u^2 + \frac{\epsilon}{3} \lambda \rho =: P_5(u)$$

with

$$\lambda = \left(\frac{r_S}{L}\right)^2, \quad \mu = E^2, \quad \rho = \Lambda r_S^2$$

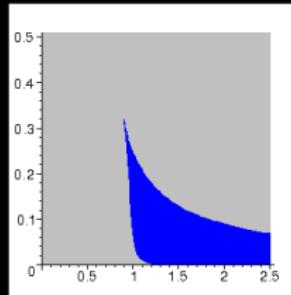
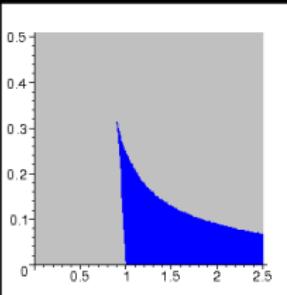
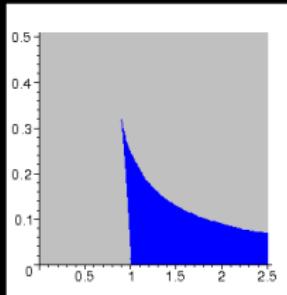
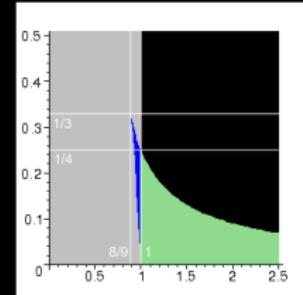
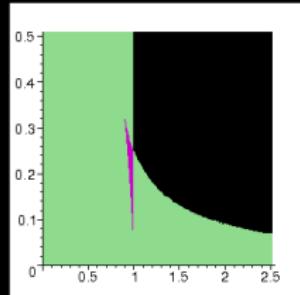
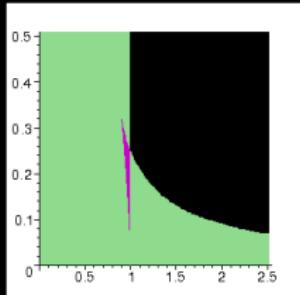
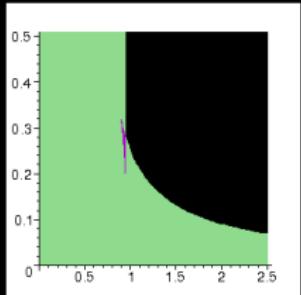
- Polynomial of 5th order → beyond elliptic integral: hyperelliptic integral
- P_5 possesses at most 4 real positive zeros
- $\epsilon = 0 \Rightarrow P_5(u) = u^2 P_3(u) \Rightarrow$ elliptic function \wp (Λ has no influence on light propagation)
- $\Lambda = 0 \Rightarrow P_5(u) = u^2 P_3(u) \Rightarrow$ elliptic function \wp (Hagihara, JJAG 1931)

Orbits in Schwarzschild-(anti-)de Sitter space-time



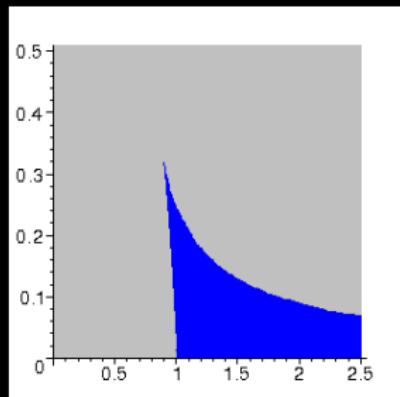
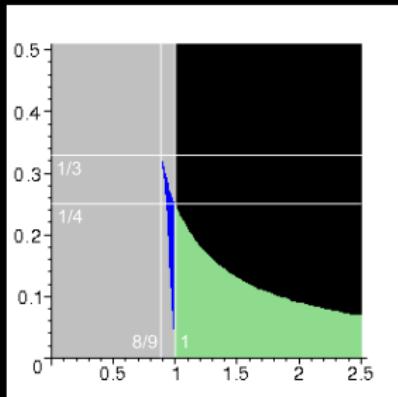
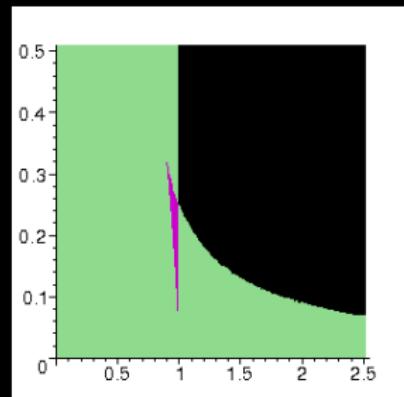
- Five possibilities of having real positive zeros of $P_5(u)$
- Zeros correspond to the zeros of $V_{\text{eff}} = E$
- Bound non-terminating, quasi-periodic orbits exist (planetary orbits) only for three or more positive zeros

Orbits in Schwarzschild-(anti-)de Sitter space-time

(f) $\Lambda = -10^{-5} \text{ km}^{-2}$ (g) $\Lambda = -10^{-10} \text{ km}^{-2}$ (h) $\Lambda = -10^{-45} \text{ km}^{-2}$ (i) $\Lambda = 0$ (j) $\Lambda = 10^{-45} \text{ km}^{-2}$ (k) $\Lambda = 10^{-10} \text{ km}^{-2}$ (l) $\Lambda = 10^{-5} \text{ km}^{-2}$

violet = 4, blue = 3, green = 2, gray = 1, black = 0 zeros of P_5

Orbits in Schwarzschild-(anti-)de Sitter space-time

(m) $\Lambda = -10^{-45} \text{ km}^{-2}$ (n) $\Lambda = 0$ (o) $\Lambda = 10^{-45} \text{ km}^{-2}$

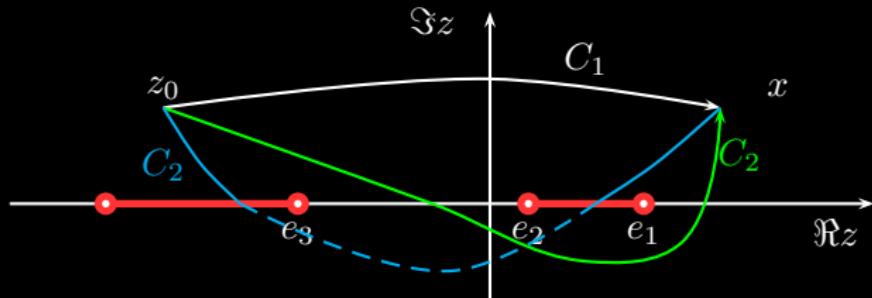
color	# zeros > 0	types of orbits
black	0	particle from infinity to singularity
gray	1	bound terminating orbits
green	2	bound terminating and escape orbits
blue	3	bound terminating and quasi-periodic bound orbits
violet	4	bound terminating, escape and quasi-periodic bound orbits

Analytic solution of geodesic eqn in $S(a)dS$ -space-time

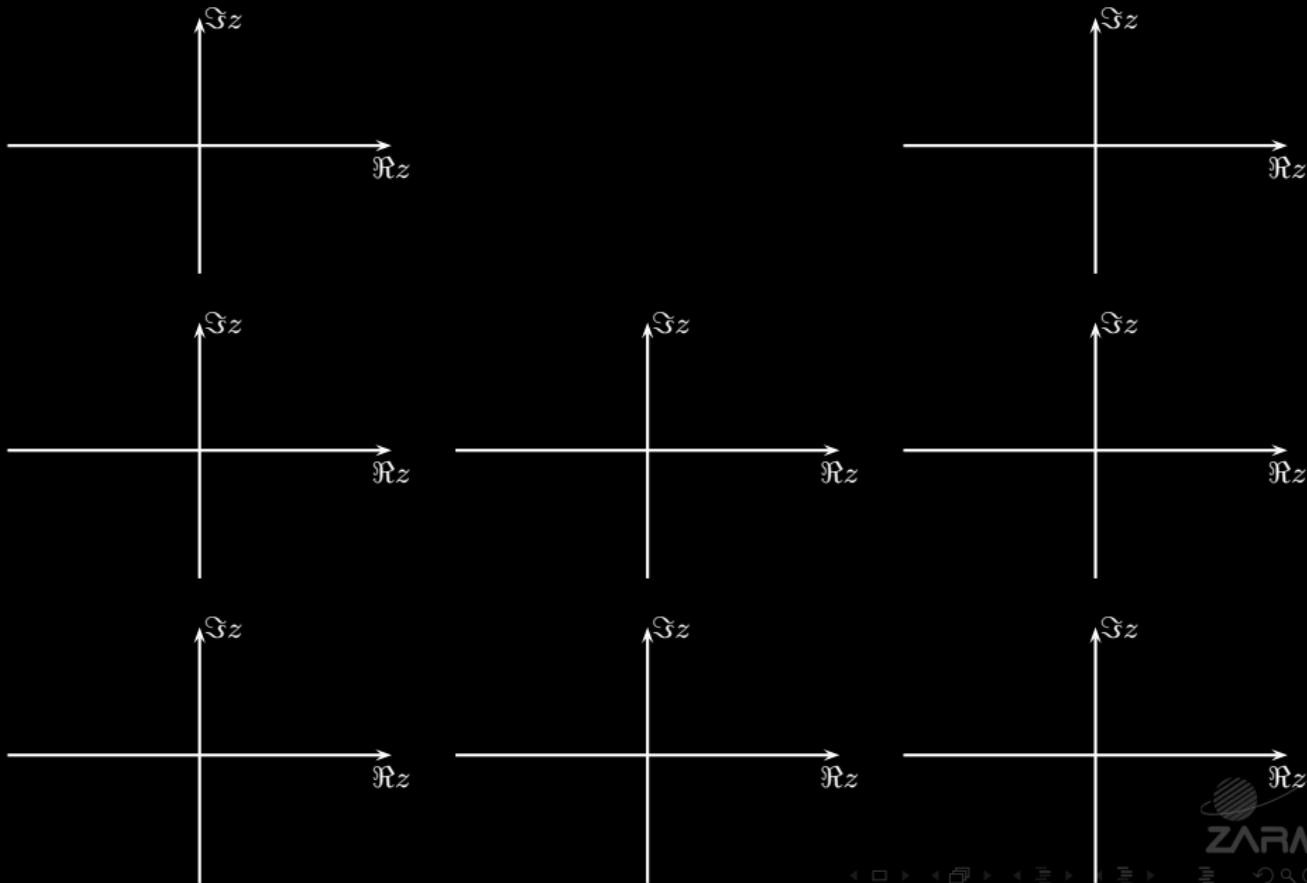
Separation of variables

$$\varphi - \varphi_0 = \int_{u_0}^u \frac{u' du'}{\sqrt{P_5(u')}}$$

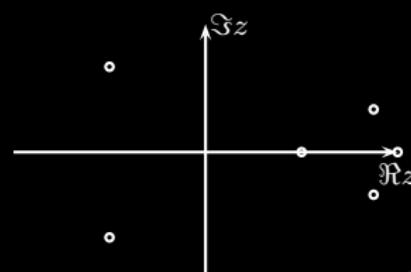
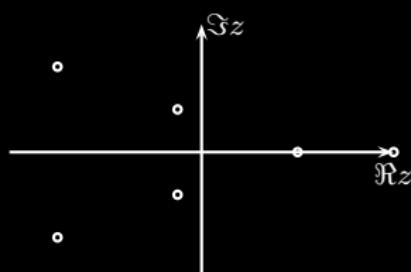
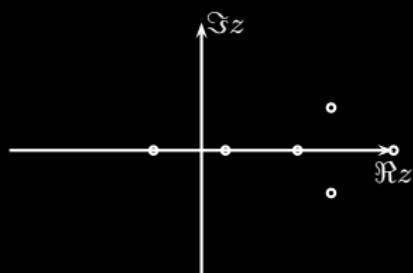
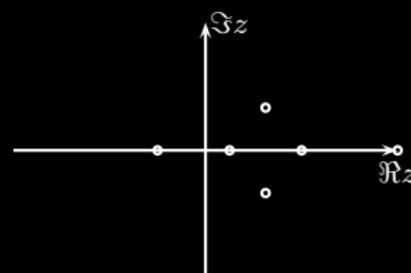
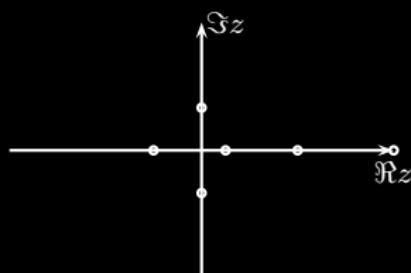
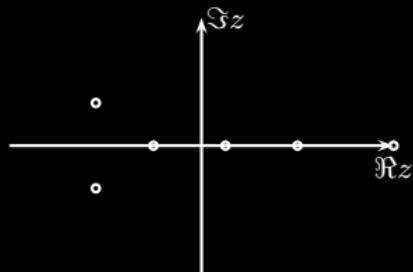
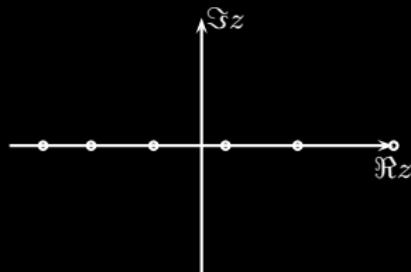
- Looked for: $u = u(\varphi) \leftrightarrow$ inversion problem
- Not well defined in complex plane
- Uniqueness of integration: u is function with 4 periods



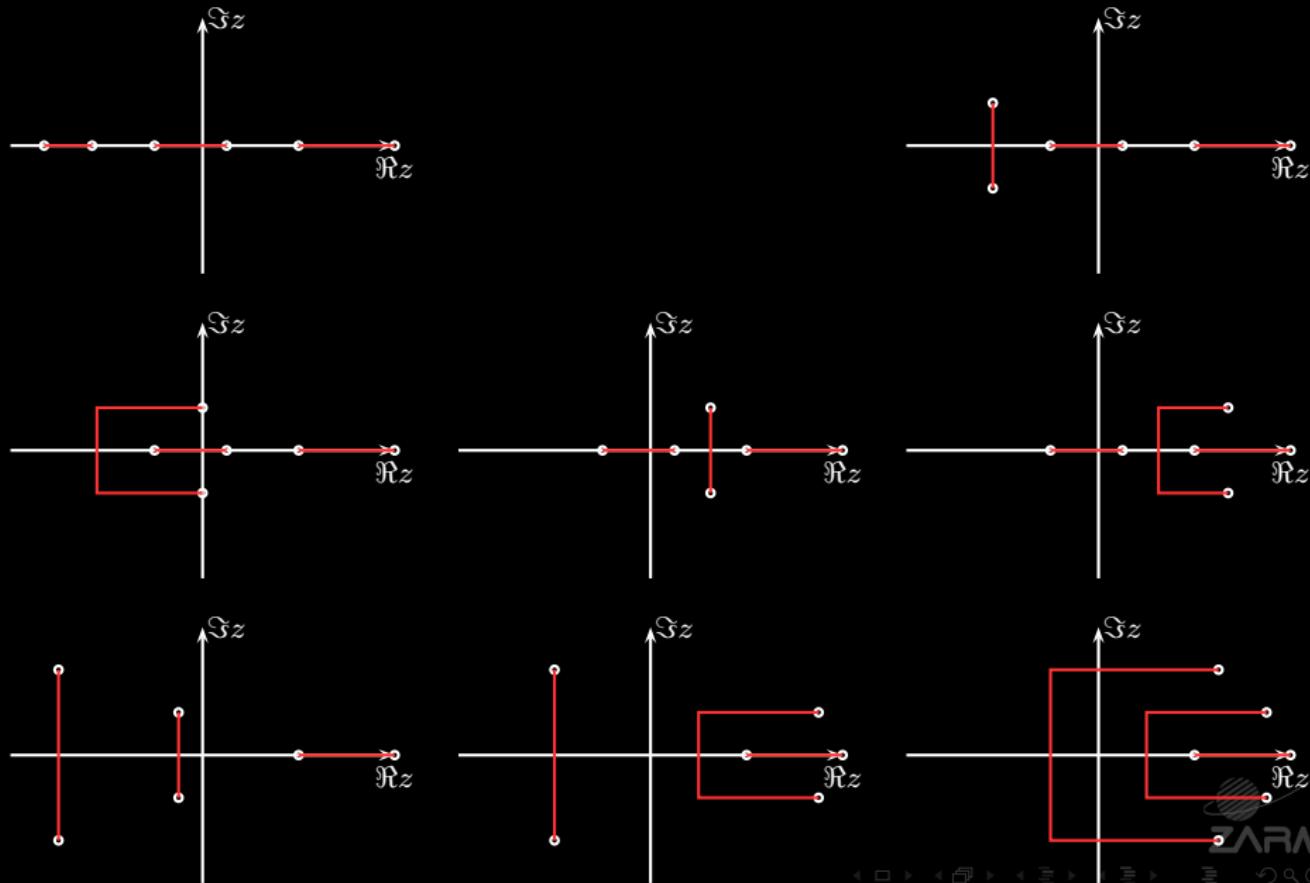
Analytic solution of geodesic eqn in $S(a)dS$ -space-time



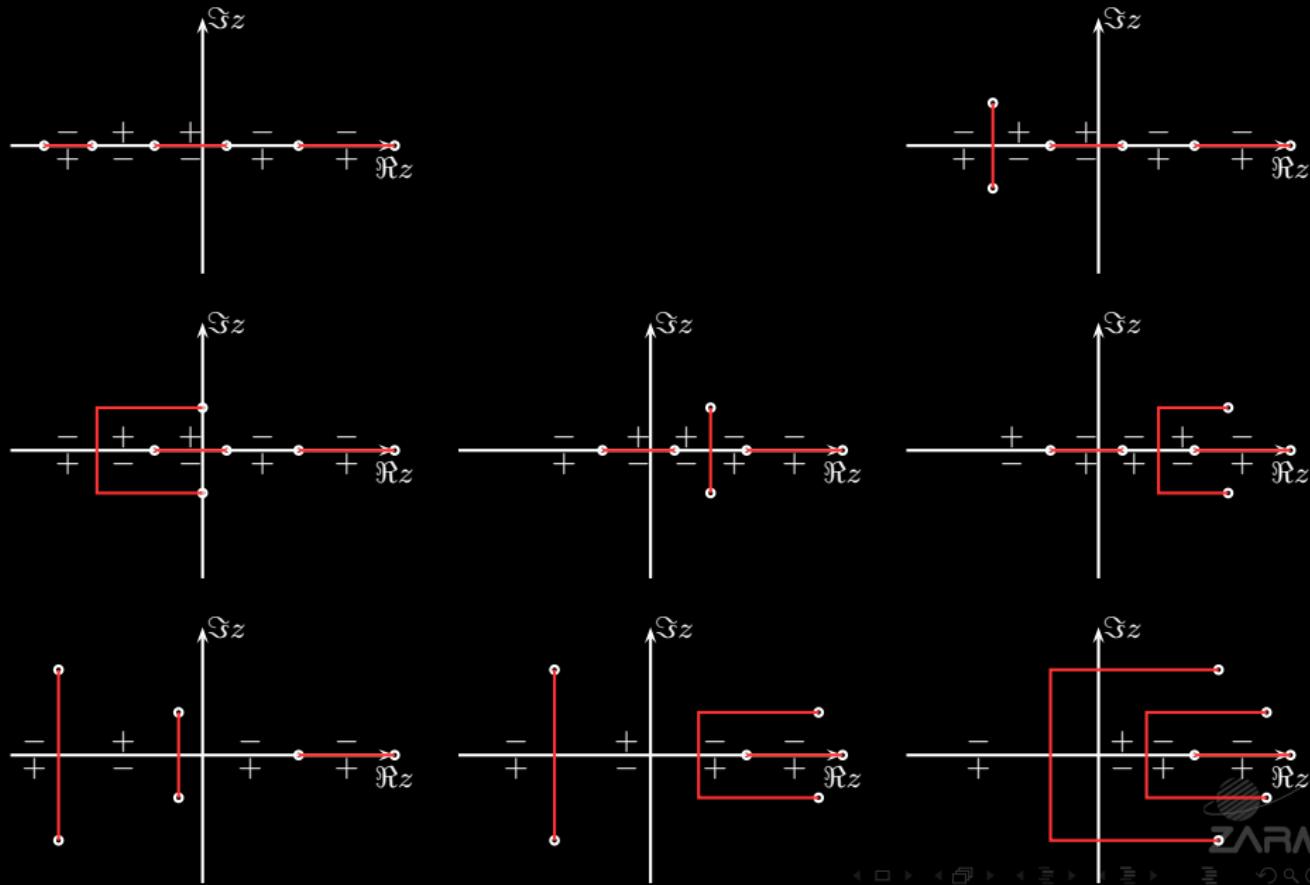
Analytic solution of geodesic eqn in $S(a)dS$ -space-time



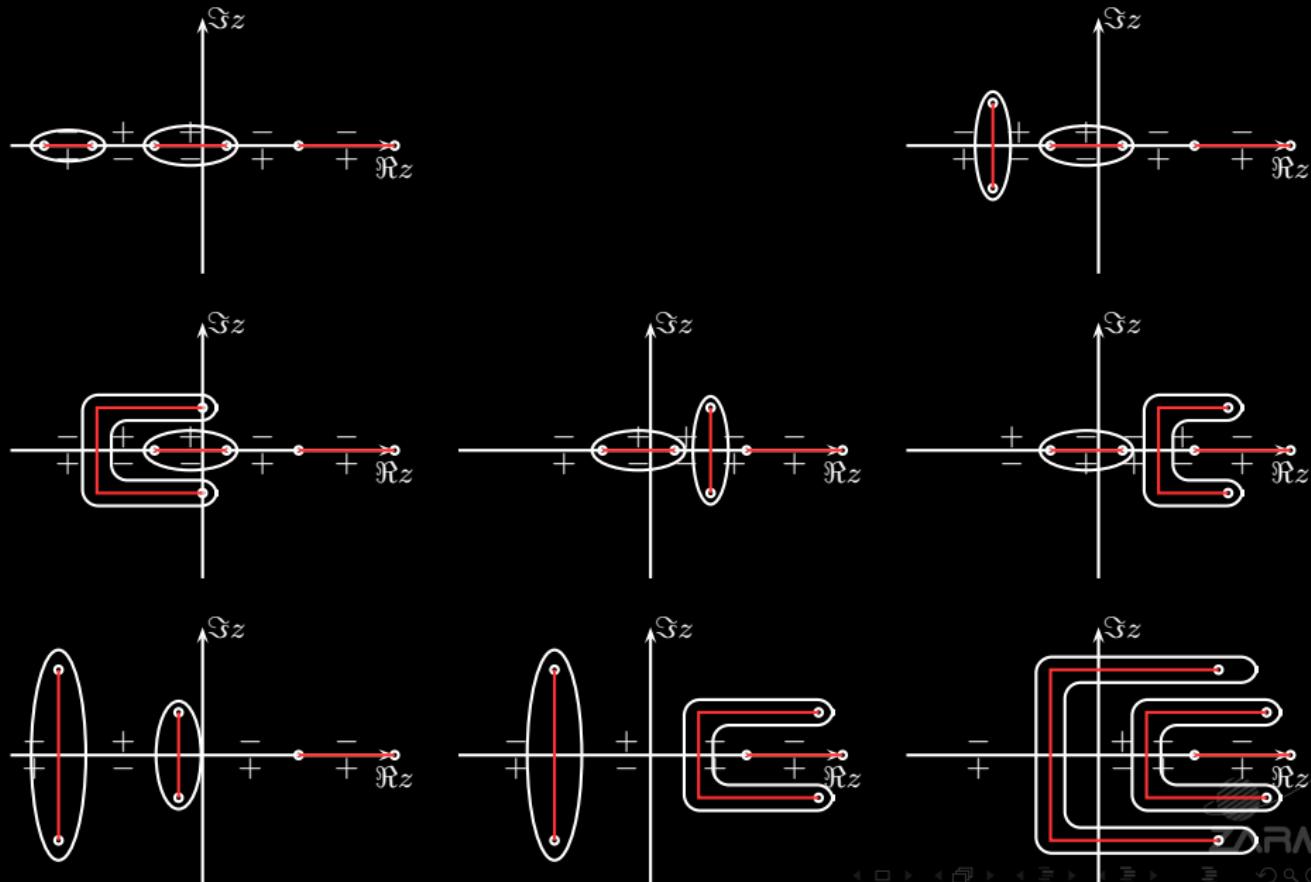
Analytic solution of geodesic eqn in $S(a)dS$ -space-time



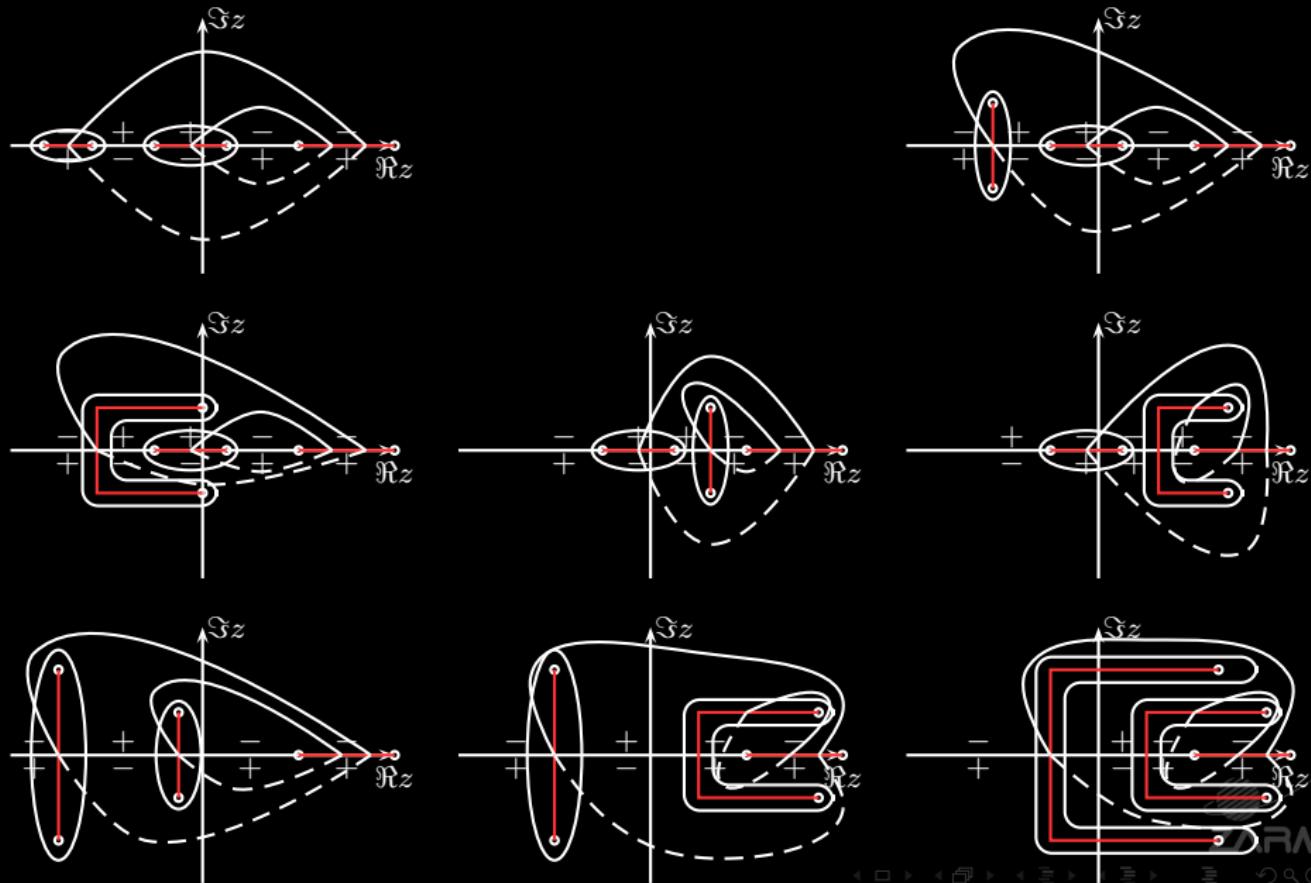
Analytic solution of geodesic eqn in $S(a)dS$ -space-time



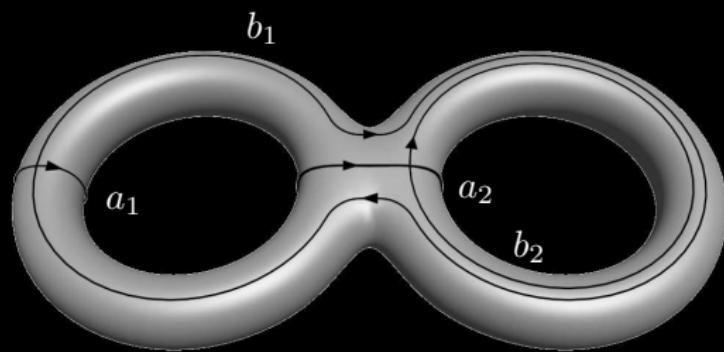
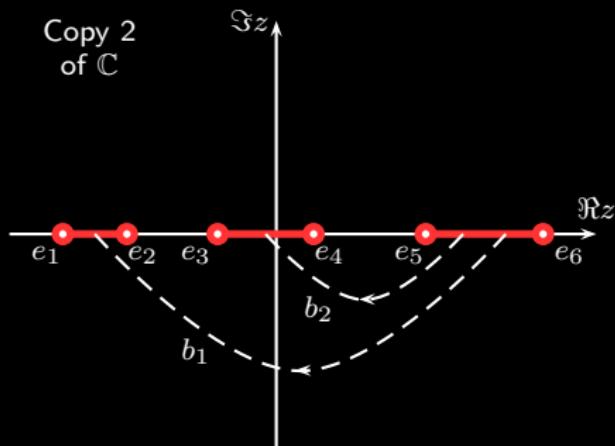
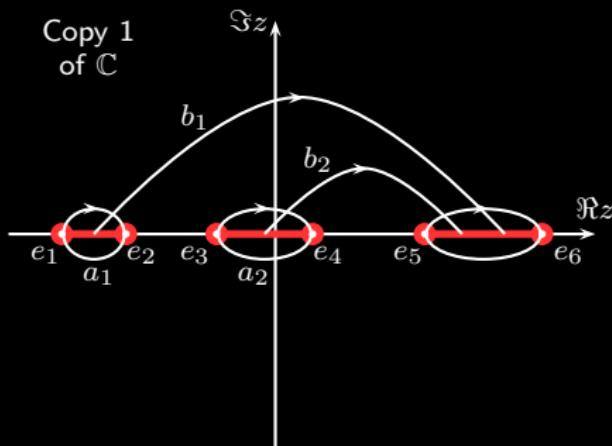
Analytic solution of geodesic eqn in $S(a)dS$ -space-time



Analytic solution of geodesic eqn in $S(a)dS$ -space-time



Analytic solution of geodesic eqn in $S(a)dS$ -space-time



$P_5 \leftrightarrow X = \text{pretzel}$



Analytic solution of geodesic eqn in $S(a)dS$ -space-time

Holomorphic and associated meromorphic differentials

$$\begin{aligned} dz_1 &:= \frac{dx}{\sqrt{P_5(x)}} , & dz_2 &:= \frac{xdx}{\sqrt{P_5(x)}} \\ dr_1 &:= \frac{3x^3 - 2x^2 + \lambda x}{4\sqrt{P_5(x)}} dx , & dr_2 &:= \frac{x^2 dx}{4\sqrt{P_5(x)}} \end{aligned}$$

Period matrices $(2\omega, 2\omega')$ and $(2\eta, 2\eta')$

$$\begin{aligned} 2\omega_{ij} &:= \oint_{a_j} dz_i , & 2\omega'_{ij} &:= \oint_{b_j} dz_i \\ 2\eta_{ij} &:= - \oint_{a_j} dr_i , & 2\eta'_{ij} &:= - \oint_{b_j} dr_i \end{aligned}$$

Normalized differentials and their period matrix

$$d\vec{z} \rightarrow d\vec{v} = (2\omega)^{-1} d\vec{z}, \quad (2\omega, 2\omega') \rightarrow (1_2, \tau) \quad \text{with} \quad \tau = \omega^{-1} \omega'$$



Analytic solution of geodesic eqn in $S(a)dS$ -space-time

Preliminaries – definitions

- **Theta function** $\vartheta : \mathbb{C}^2 \rightarrow$ (for construction of functions with 4 periods)

$$\vartheta(\vec{z}; \tau) := \sum_{\vec{m} \in \mathbb{Z}^2} e^{i\pi \vec{m}^t (\tau \vec{m} + 2\vec{z})}$$

- Periodicity: $\vartheta(\vec{z} + 1_2 \vec{n}; \tau) = \vartheta(\vec{z}; \tau)$
- Quasi-periodicity: $\vartheta(\vec{z} + \tau \vec{n}; \tau) = e^{-i\pi \vec{n}^t (\tau \vec{n} + 2\vec{z})} \vartheta(\vec{z}; \tau)$
- **Theta function with characteristics** $\vec{g}, \vec{h} \in \frac{1}{2}\mathbb{Z}^2$

$$\vartheta[\vec{g}, \vec{h}](\vec{z}; \tau) := e^{i\pi \vec{g}^t (\tau \vec{g} + 2\vec{z} + 2\vec{h})} \vartheta(\vec{z} + \tau \vec{g} + \vec{h}; \tau)$$

- **Sigma function** $\sigma(\vec{z}) = C e^{-\frac{1}{2} \vec{z}^t \eta \omega^{-1} \vec{z}} \vartheta \left[\begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1/2 \end{pmatrix} \right] ((2\omega)^{-1} \vec{z}; \tau)$
- **Generalized Weierstrass functions**

$$\wp_{ij}(\vec{z}) = -\frac{\partial}{\partial z_i} \frac{\partial}{\partial z_j} \log \sigma(\vec{z}) = \frac{\partial_i \sigma(\vec{z}) \partial_j \sigma(\vec{z}) - \sigma(\vec{z}) \partial_i \partial_j \sigma(\vec{z})}{\sigma^2(\vec{z})}$$

Analytic solution of geodesic eqn in $S(a)dS$ -space-time

Only formalism where $1/\sqrt{P_5}$ appears:

Jacobi's inversion problem

Determine \vec{u} for given $\vec{\varphi}$ from $\vec{\varphi} = \vec{A}_{u_0}(\vec{u})$ (Abel map), i.e.

$$\begin{aligned}\varphi_1 &= \int_{u_0}^{u_1} \frac{du}{\sqrt{P_5(u)}} + \int_{u_0}^{u_2} \frac{du}{\sqrt{P_5(u)}} \\ \varphi_2 &= \int_{u_0}^{u_1} \frac{udu}{\sqrt{P_5(u)}} + \int_{u_0}^{u_2} \frac{udu}{\sqrt{P_5(u)}}\end{aligned}$$

Solution of inversion problem

Solution \vec{u} of inversion problem given by

$$\begin{aligned}u_1 + u_2 &= 4\wp_{22}(\vec{\varphi}) \\ u_1 u_2 &= -4\wp_{12}(\vec{\varphi})\end{aligned}$$

Our problem: two positions \vec{u} , two angles $\vec{\varphi}$ \rightarrow requires reduction

Analytic solution of geodesic eqn in $S(a)dS$ -space-time

Only formalism where $1/\sqrt{P_5}$ appears:

Jacobi's inversion problem

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Solution of inversion problem

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Our problem: two positions \vec{u} , two angles $\vec{\varphi}$ \rightarrow requires reduction

Analytic solution of geodesic eqn in $S(a)dS$ -space-time

Rewrite inversion problem as (based on Enolskii, Pronine & Richter, JNS 2005)

$$\vec{\phi} = \vec{A}_\infty(\vec{u}) \quad \text{with} \quad \vec{\phi} = \vec{\varphi} - \int_{u_0}^{\infty} dz$$

Extraction of one component of \vec{u} (namely u_2 in Jacobi inv. prob.) through a limit

$$u_1 = \lim_{u_2 \rightarrow \infty} \frac{u_1 u_2}{u_1 + u_2} = \frac{\sigma(\vec{\phi}_\infty) \partial_1 \partial_2 \sigma(\vec{\phi}_\infty) - \partial_1 \sigma(\vec{\phi}_\infty) \partial_2 \sigma(\vec{\phi}_\infty)}{(\partial_2 \sigma)^2(\vec{\phi}_\infty) - \sigma(\vec{\phi}_\infty) \partial_2 \partial_2 \sigma(\vec{\phi}_\infty)}$$

with

$$\vec{\phi}_\infty = \lim_{u_2 \rightarrow \infty} \vec{\phi} = \vec{A}_\infty(\vec{u}_\infty), \quad \vec{u}_\infty = \begin{pmatrix} u_1 \\ \infty \end{pmatrix}$$

One u but still two φ : Lemma (Mumford 1983 – Riemann vanishing theorem):
 $(2\omega)^{-1} \vec{A}_\infty(\vec{u}_\infty) \in \text{Theta-divisor } \Theta_{K_\infty} = \{\vec{z} \mid \vartheta(\vec{z} + \vec{K}_\infty) = 0\}$ with the vector of
 Riemann constants $\vec{K}_\infty = \tau \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1/2 \end{pmatrix}$
 $\Rightarrow \sigma(\vec{\phi}_\infty) = 0$, gives a relation between $\phi_{\infty,1}$ and $\phi_{\infty,2}$.

Analytic solution of geodesic eqn in $S(a)dS$ -space-time

Define

$$\vec{\varphi}_\Theta := \begin{pmatrix} x \\ \varphi - \varphi'_0 \end{pmatrix} \quad \text{with} \quad \varphi'_0 = \varphi_0 + \int_{u_0}^{\infty} dz_2$$

and choose x so that $(2\omega)^{-1}\vec{\varphi}_\Theta \in \Theta_{\vec{K}_\infty}$.

Complete analytic solution of equation of motion in Schwarzschild-de Sitter space-time

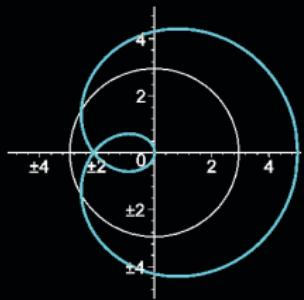
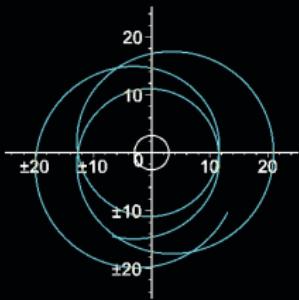
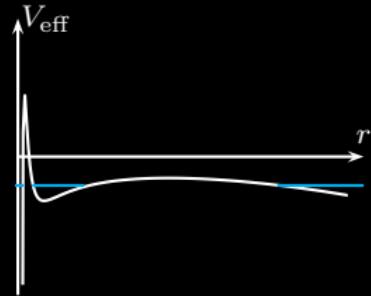
$$u(\varphi) = u_1 = -\frac{\partial_1 \sigma(\vec{\varphi}_\Theta)}{\partial_2 \sigma(\vec{\varphi}_\Theta)} \quad \Rightarrow \quad r(\varphi) = \frac{r_S}{u(\varphi)} = -r_S \frac{\partial_2 \sigma(\vec{\varphi}_\Theta)}{\partial_1 \sigma(\vec{\varphi}_\Theta)}$$

where $(2\omega)^{-1}\vec{\varphi}_\Theta \in \Theta_{\vec{K}}$

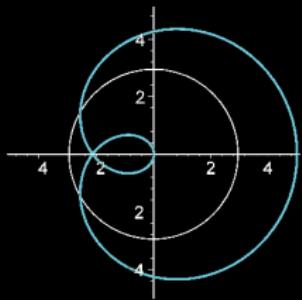
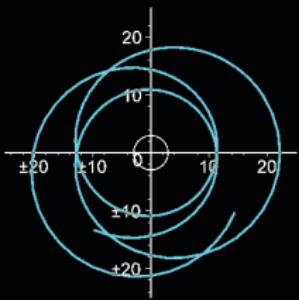
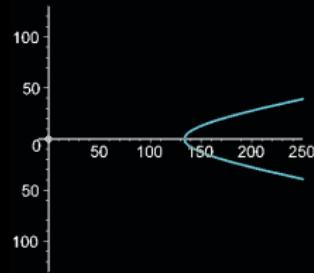
New result, **Hackmann & CL**, PRL 2008



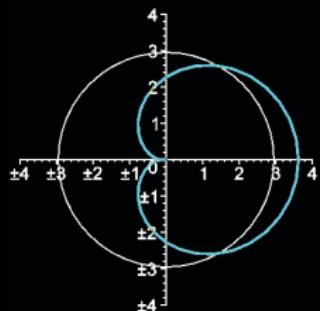
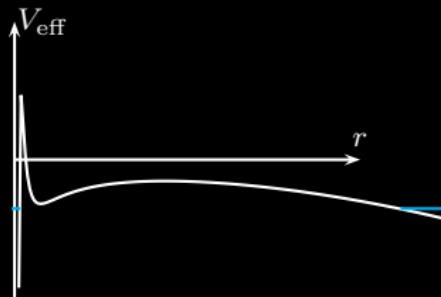
Orbits

(p) $\Lambda = 0, r_0 = 5.010\text{km}$ (q) $\Lambda = 0, r_0 = 20.951\text{km}$ 

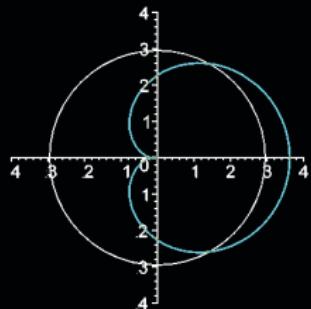
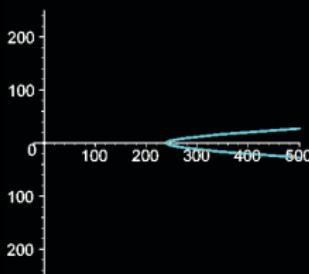
(r) effective potential

(s) $\Lambda = 10^{-5}\text{km}^{-2}, r_0 = 5.013\text{km}$ (t) $\Lambda = 10^{-5}\text{km}^{-2}, r_0 = 22.185\text{km}$ (u) $\Lambda = 10^{-5}\text{km}^{-2}, r_0 = 133.60\text{km}$

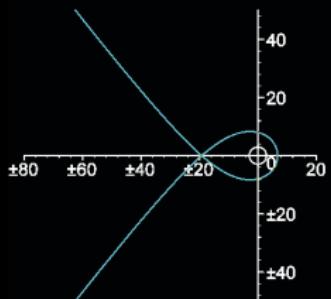
Orbits

(v) $\Lambda = 0$, $r_0 = 3.625\text{km}$ 

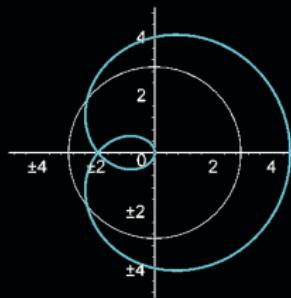
(w) effective potential

(x) $\Lambda = 10^{-5}\text{km}^{-2}$, $r_0 = 3.625\text{km}$ (y) $\Lambda = 10^{-5}\text{km}^{-2}$, $r_0 = 237.61\text{km}$

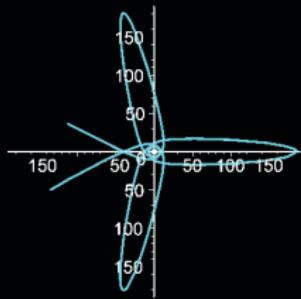
Orbits



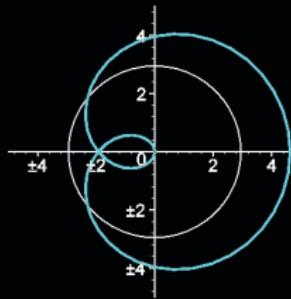
(z) $\Lambda = 0, r_0 = 6.776\text{km}$



() $\Lambda = 0, r_0 = 4.642\text{km}$



() $\Lambda = -10^{-5}\text{km}^{-2}, r_0 = 185.37\text{km}$ () $\Lambda = -10^{-5}\text{km}^{-2}, r_0 = 4.639\text{km}$



Application to Pioneer

Influence of Λ on Pioneer satellites (orbital parameters from Nieto & Anderson 2005)

$$\begin{array}{lll} \text{Pioneer 10:} & \mu = 1.000\,000\,001\,43, & \lambda = 2.855\,572\,373\,82 \cdot 10^{-9} \\ \text{Pioneer 11:} & \mu = 1.000\,000\,001\,22, & \lambda = 1.340\,740\,574\,59 \cdot 10^{-9} \end{array}$$

- For given angle φ : $r_\Lambda(\varphi) - r_{\Lambda=0}(\varphi) \approx 10^{-3}$ m
- For given distance $r = 65$ AU: $r\varphi_\Lambda(r) - r\varphi_{\Lambda=0}(r) \approx 10^{-5}$ m

\Rightarrow Form of the Pioneer orbits practically does not change.

Cosmological constant cannot be origin of Pioneer anomaly.
In principle we need also $\varphi = \varphi(t)$ and Doppler tracking.

Post-Schwarzschild of perihelion shift

Perihelion shift in Schwarzschild-de Sitter (for bound orbit, Kraniotis & Whitehouse 2003)

$$\delta\varphi_{\text{perihelion}} = 2\pi - \omega_{22} = 2\pi - \oint \frac{x dx}{\sqrt{P_5(x)}}$$

Approximation to first order in Λ

$$\frac{x}{\sqrt{p_5(x)}} = \frac{1}{\sqrt{P_3(x)}} - \frac{2}{3} m^2 \frac{x^2 + \lambda}{x^2 P_3(x) \sqrt{P_3(x)}} \Lambda + \mathcal{O}(\Lambda^2)$$

with $P_3(x) = x^3 - x^2 + \lambda x + \lambda(\mu - 1)$ Schwarzschild polynomial

This has to be integrated: gymnastics in elliptic integration

Post-Schwarzschild of perihelion shift

structure of result

$$\begin{aligned}
 \oint_{a_2} \frac{x dx}{\sqrt{P_5(x)}} &= \oint_{a_2} \frac{1}{\sqrt{P_3(x)}} - \frac{2}{3} m^2 \oint_{a_2} \frac{x^2 + \lambda}{x^2 P_3(x) \sqrt{P_3(x)}} \Lambda + \mathcal{O}(\Lambda^2) \\
 &= \omega_1 + \textcolor{blue}{\Lambda} \frac{m^2}{96} \left(\sum_{j=1}^3 \frac{\eta_1 + \omega_1 z_j}{(\wp''(\rho_j))^2} \left(1 + \frac{\lambda}{(4z_j + \frac{1}{3})^2} \right) \right. \\
 &\quad \left. + \lambda \left(\frac{2\eta_1 - \frac{1}{6}\omega_1}{16(\wp'(u_0))^2} + \frac{6}{16} \frac{\wp''(u_0)}{(\wp'(u_0))^5} (\zeta(u_0) - \eta_1 u_0) \right) \right) \\
 &\quad + \mathcal{O}(\textcolor{blue}{\Lambda}^2)
 \end{aligned}$$

- Needs introduction of r_{\min} and r_{\max} or a and e for interpretation
- Needs relativistic approximation for interpretation



Further applications

Formalism can be further applied to

- Geodesic equation in Plebanski–Demianski space-times
- Geodesic equation in Schwarzschild space-times in 5, 6, 7, 9, and 11 dimensions
- Geodesic equation in Schwarzschild–de Sitter space-times in 5, 6, 7, 9, 11 dimensions
- Geodesic equation in Reissner-Nordström space-times in 5 and 7 dimensions
- Geodesic equation in Reissner-Nordström–de Sitter space-times in 5 and 7 dimensions
- crystal with anharmonic potential of 6th order
(Hackmann, Kagramanova, Kunz & C.L. in preparation)

No scheme known for polynomials of order 7 or higher



The Questions

Questions

- Does the coupling of the expansion to light lead to an effect?
- Does the cosmological expansion influence the physics in the Solar system?
 - Does the gravitational field of the Sun feel the expansion?
 - Do planetary orbits (bound orbits) feel the expansion?
 - Do escape orbits behave differently than bound orbits?
- Does a cosmological constant influence the physics in the Solar system?
- Additional Yukawa coupling?
- MOdified Newtonian Dynamics



Outline

1 The situation of standard physics

- The status
- Problems?

2 Anomalies related to the Solar system

- Ranging and tracking
- The anomalies

3 The Pioneer anomaly

- The mission
- Modeling and errors

4 Attempts at explanation

5 Cosmology and local physics

- Physics in the expanding universe
- Physics in the Schwarzschild–(anti-)de Sitter space–time
- **Gravity on large scales**
- MOND

6 Summary and outlook

- Summary
- Further studies
- A new Pioneer Exploration mission



Gravity on large scales

Models

- Yukawa (rotation curves of galaxies, Sanders, AA 1986)

$$U(r) = U_{\text{Newton}}(r) \left(1 + \alpha e^{-r/\lambda} \right)$$

- Running coupling constant model (Jaekel & Reynaud, CQG 2004)

$$U(r) = U_{\text{Newton}}(r) + ar$$

- Parameter a fixed by Pioneer effect
- Pioneer effect should be $\sim v^2$
- Compatible with all other Solar system effects (Standish 2008)
- Higher-dim. braneworld models (Dvali, Gabadadze & Porrati, PLA 2002)

$$U(r) = U_{\text{Newton}}(r) \pm 2 \sqrt{U_{\text{Newton}}(r_0) \frac{r}{r_0}}, \quad r_0 \sim 5 \text{ Gpc}$$

- Perihelion shift: $\sim 5 \mu\text{as}/\text{y}$
- Secular increase of AU: increase of $\sim 5 \text{ m/cy}$

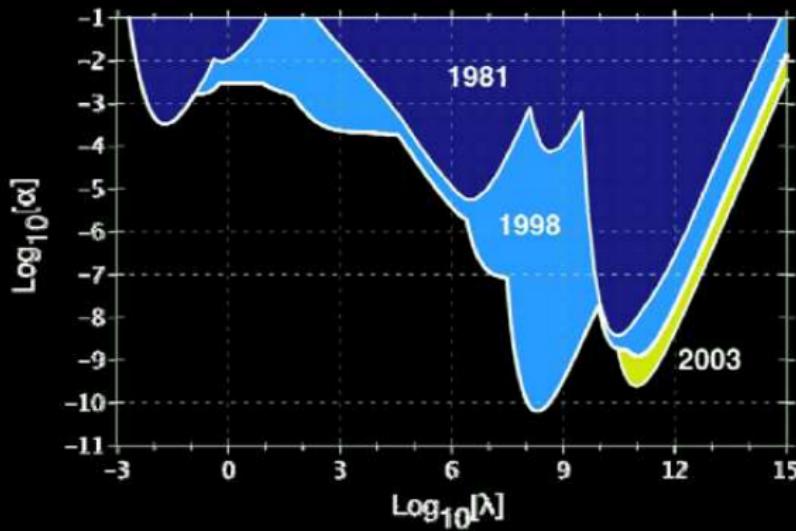
Yukawa modification: A viable model?

The ansatz

- The ansatz

$$V(r) = G \frac{m}{r} \left(1 + \alpha e^{-r/\lambda} \right)$$

- Ansatz can explain many galactic rotation curves (**Sanders, AA 1984**)

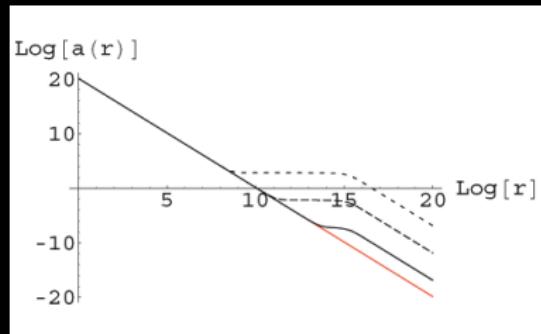
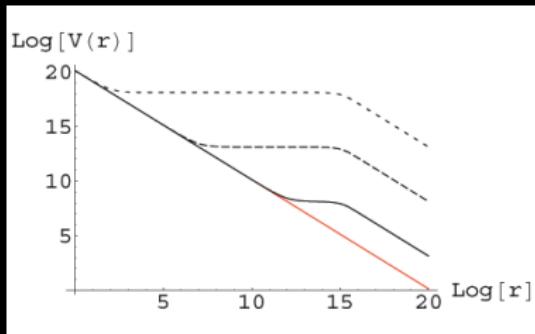


Yukawa modification: A viable model?

Acceleration

- Taylor $V(r) = (1 + \alpha) G \frac{M_\odot}{r} - \alpha G \frac{M_\odot}{\lambda} + \alpha G \frac{M_\odot}{2\lambda} \frac{r}{\lambda} - \alpha G \frac{M_\odot}{6\lambda} \frac{r^2}{\lambda^2} \pm \dots$
- acceleration $a(r) = \frac{\partial V}{\partial r} = -(1 + \alpha) G \frac{M_\odot}{r^2} + \alpha G \frac{M_\odot}{2\lambda^2} - \alpha G \frac{M_\odot}{3\lambda^2} \frac{r}{\lambda} \pm \dots$
- Identification $G_0 := (1 + \alpha) G = \text{observed gravitational constant for } r \rightarrow 0$
- acceleration

$$a(r) = -G_0 \frac{M_\odot}{r^2} + \frac{\alpha}{1 + \alpha} G_0 \frac{M_\odot}{2\lambda^2} - \frac{\alpha}{1 + \alpha} G_0 \frac{M_\odot}{3\lambda^2} \frac{r}{\lambda} \pm \dots$$



Yukawa modification: A viable model?

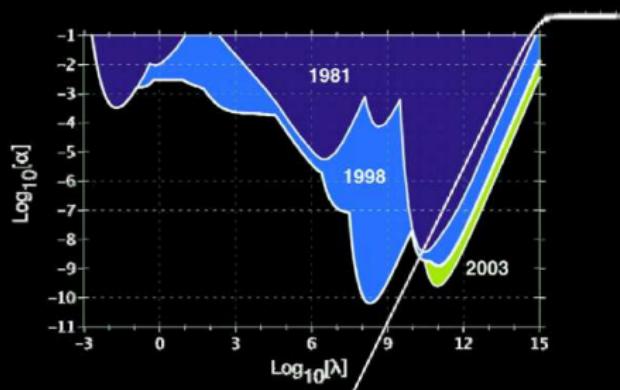
Compatibility

- Identification

$$a_{\text{Pioneer}} = \frac{\alpha}{1 + \alpha} G_0 \frac{M_\odot}{2\lambda^2} \quad \Rightarrow \quad \alpha = \frac{2\lambda^2 a_{\text{Pioneer}}}{G_0 M_\odot - 2\lambda^2 a_{\text{Pioneer}}}$$

$$\lambda \geq \sqrt{GM_\odot/(2a_{\text{Pioneer}})} \quad \Rightarrow \quad \lambda \geq 2.8 \cdot 10^{14} \text{ m}$$

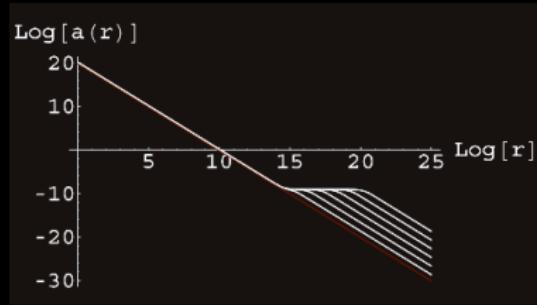
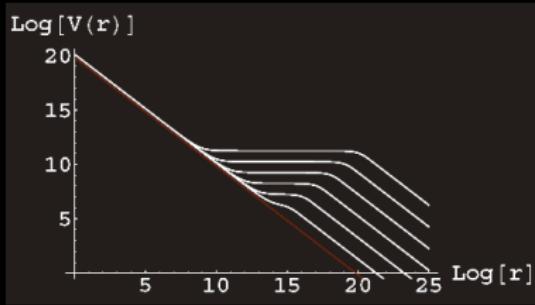
- $\log \lambda > 16$ with $\log |\alpha| \approx 0$ is compatible with existing data



Yukawa modification: A viable model?

Compatibility

- acceleration $a(r) = -G_0 \frac{M_\odot}{r^2} + a_{\text{Pioneer}} - \frac{2}{3} a_{\text{Pioneer}} \frac{r}{\lambda} \pm \dots$
- Next order term smaller by $\frac{2}{3} \frac{r}{\lambda} \leq 0.06$ — might account for the small decrease of the observed acceleration
- Result: rather strong $\alpha \approx -1$ Yukawa coupling with very long range; yields an acceleration plateau in some range between 1 and 100 AU.



V and a for parameters compatible with Pioneer acceleration. **red** = standard Newtonian case.
black = parameters $(\lambda, \alpha + 1) = (10^{15}, 7.07497 \cdot 10^{-2}), (10^{16}, 7.60784 \cdot 10^{-4}), (10^{17}, 7.61358 \cdot 10^{-6}), (10^{18}, 7.61364 \cdot 10^{-8}), (10^{19}, 7.61364 \cdot 10^{-10}), (10^{20}, 7.61369 \cdot 10^{-12})$

Yukawa modification: A viable model?

Compatibility with rotation curve fit

- Pioneer:
 $\log \lambda > 16, \quad \alpha + 1 \leq 10^{-5}$
- Galactic rotation curve:
 $\log \lambda > 16, \quad \alpha + 1 \leq 10^{-1}$
- Local strength: Modification by "Yukawa in Yukawa"

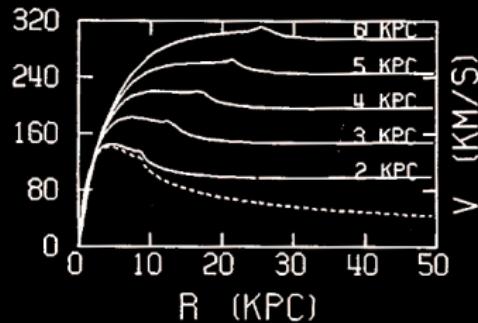


Figure 2: The dashed curve is the pure inverse square gravity rotation curve for a galaxy consisting of a truncated exponential disk ($\sigma = \sigma_0 \exp(-r/a)$ where $r < r_0 = 4.3 a$) with $a = 2$ kpc. The solid curves are rotation curves resulting from similar mass distributions but with a modified gravitational potential (e.g. 2) integrated over the mass distribution. Here $\alpha = -0.92$ and $r_0 = 40$ kpc. The central surface density (σ_0) is assumed constant and the disk length scale (a) is indicated for each curve.

The Questions

Questions

- Does the coupling of the expansion to light lead to an effect?
- Does the cosmological expansion influence the physics in the Solar system?
 - Does the gravitational field of the Sun feel the expansion?
 - Do planetary orbits (bound orbits) feel the expansion?
 - Do escape orbits behave differently than bound orbits?
- Does a cosmological constant influence the physics in the Solar system?
- Additional Yukawa coupling?
- MOdified Newtonian Dynamics

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MODified Newtonian Dynamics

MOND ansatz

Modification of Newton's second law = modification of relation between applied force and resulting acceleration ([Milgrom ApJ 1983, ...](#))

$$m\ddot{\mathbf{x}} = \mathbf{F} \quad \longrightarrow \quad m\ddot{\mathbf{x}}\mu(|\ddot{\mathbf{x}}|/a_0) = \mathbf{F}$$

with function

$$\mu(x) = \begin{cases} 1 & \text{for } |\ddot{\mathbf{x}}| \gg a_0 \\ x & \text{for } |\ddot{\mathbf{x}}| \ll a_0 \end{cases}$$

For Newtonian gravity

Newtonian force $\mathbf{F} = m\nabla U$

- large accelerations / large forces $\ddot{\mathbf{x}} = \nabla U$
- small accelerations / small forces $\ddot{\mathbf{x}}|\ddot{\mathbf{x}}| = a_0 \nabla U \rightarrow |\ddot{\mathbf{x}}| = \sqrt{a_0 |\nabla U|}$

MOND = modified inertia

Implications of MOND

Galactic rotation curves

- circular orbit, small distance, large accelerations:

$$a_{\text{centrifugal}} = \frac{v^2}{r} = \frac{GM}{r^2} \Rightarrow v = \sqrt{GM} r^{-1/2}$$

- circular orbit, large distance, small accelerations:

$$a_{\text{centrifugal}} = \frac{v^2}{r} = \sqrt{a_0 |\nabla U|} = \frac{\sqrt{a_0 GM}}{r} \Rightarrow v^4 = GM a_0$$

reproduces many galactic rotation curves for $a_0 \sim 10^{-9} \text{ m/s}^2$ – may also reproduce dynamics of galactic clusters

Pioneer anomaly

In strong Newtonian regime $a_0 \ll |\nabla U|$

- parametrization $\mu(x) = 1 - \xi x^{-1}$
- additional acceleration $a = \frac{GM}{r^2} + \xi a_0$

MOND reproduces Pioneer anomaly

Testing MOND

Torsion balance ([Gundlach et al, PRL 2007](#))

- transcribes acceleration into torque (for hollow cylinder with radius R):

$$\tau = \frac{I}{R} a \mu(a/a_0)$$
- takes $r\ddot{\theta} = a$ as acceleration
- no deviation from Newton's second law down to $a \sim 10^{-14}$ m/s²

Earlier experiment by [Abramovici and Vager, PRD 1986](#) (to $a \sim 10^{-9}$ m/s²)

These are no tests of MOND: is it not allowed to separate components

Free fall experiment ([Ignatiev, PRL 2007](#))

- Free fall with respect to galaxy:
- MOND-situation possible on Earth once a year for 0.1 s within 1 l volume

General question

- Gravity is what we explore by the free motion of particles and light rays
- General test of Newton's second law ... $0 = f(x, \dot{x}, \ddot{x}, \ddot{x}, \dots, x^{(n)})$

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Summary

Summary: Observation

- The Pioneer acceleration, a constant acceleration

$$a_{\text{Pioneer}} = (8.74 \pm 1.33) \cdot 10^{-10} \text{ m/s}^2$$

of the spacecrafts toward the sun seems to be a real physical effect

- No conventional explanation has been found
- Largest scaled experiment mankind ever carried out
- Potential indication for new physics?

Summary and outlook

Summary: theory

- Pioneer scenario treated with
 - Effects on radio signal
 - Effects on gravitational field of Sun
 - Effects on particle trajectory
- Despite $a_{\text{Pioneer}} \sim cH \sim a_{\text{MOND}}$, Pioneer anomaly cannot be explained by
 - Cosmic expansion
 - Cosmological constant

Outlook: Future work

- More on conventional physics ...
- New physics
 - Dilaton scenarios
 - Dark energy / Quintessence
 - Dark matter
 - Higher dimensional braneworld models

Baseline: PA probably related to some other unexplained effect

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Summary and outlook

Outlook: Future work on data analysis

- Data recovering almost completed: Doppler and telemetry data
- New analysis of **complete** data set

	previously used data set	
	time span AU	distance
Pioneer 10	3.1.1987 – 22.7.1998	40 – 70.5
Pioneer 11	5.1.1987 – 1.10.1990	22.4 – 31.7

	data set to be analyzed	
	time span AU	distance
Pioneer 10	8.9.1973 – 27.4.2002	4.6 – 80.2
Pioneer 11	10.4.1974 – 11.10.1994	1.0 – 41.7

- With different codes
- Independent modeling
- Direction of acceleration (toward Sun, Earth, along spin axis, along velocity)

Summary and outlook

Available data

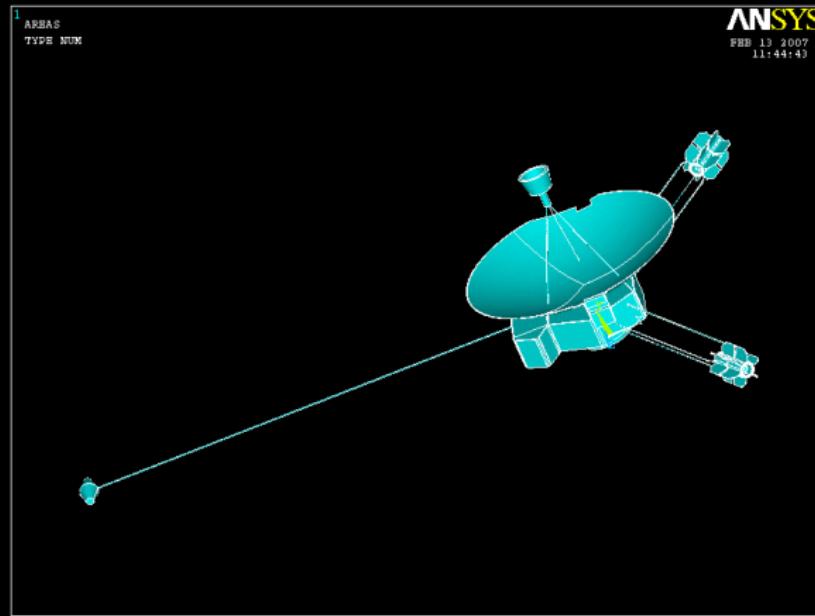
Parameters	Subsystem	Telemetry words
TEMPERATURES		
RTG fin root temps	Thermal	C201, C202, C203, C204
RTG hot junction temps	Thermal	C220, C219, C218, C217
TWT temperatures	Communications	C205, C206, C207, C228, C223, C221
Receiver temperatures	Communications	C222, C227
Platform temperatures	Thermal	C301, C302, C304, C318, C319, C320
PSA temperatures	Thermal	C225, C226
Thruster cluster temps	Propulsion	C309, C326, C310, C311, C312, C328, C325
SRA/SSA temperatures	ACS	C303, C317
Battery temperature	Power	C115
Propellant temperature	Propulsion	C327
N2 tank temperature	Propulsion	C130
Science instr temps	Science	E101, E102, E109, E110, E117, E118, E125, E128, E201, E209, E213, E221
VOLTAGES		
Calibration voltages	Data handling	C101, C102, C103
RTG voltages	Power	C110, C125, C131, C113
Battery/Bus voltages	Power	C106, C107, C117, C118, C119
TWT voltages	Communications	C224, C230
Science instr voltages	Science	E119, E129, E210, E211, E217, E220
CURRENTS		
RTG currents	Power	C127, C105, C114, C123
Battery/Bus currents	Power	C109, C126, C129
Shunt current	Power	C122, C209
TWT currents	Communications	C208, C211, C215, C216
Science instr currents	Science	E111, E112, E113
PRESSURE		
Propellant pressure	Propulsion	C210
OTHER ANALOG		
TWT power readings	Communications	C231, C214
Receiver readings	Communications	C111, C212, C232, C121, C229, C213

Summary and outlook

Available data

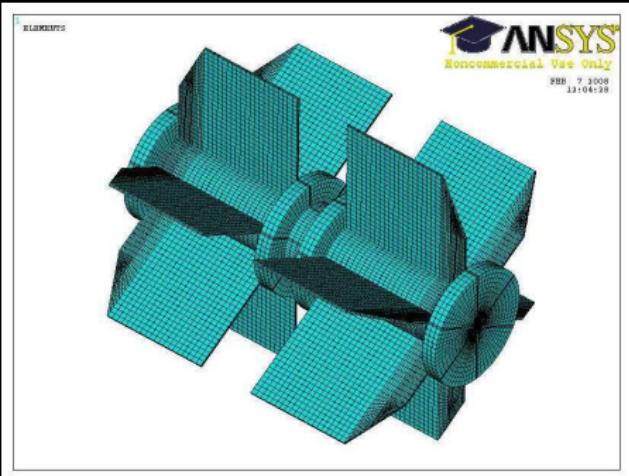
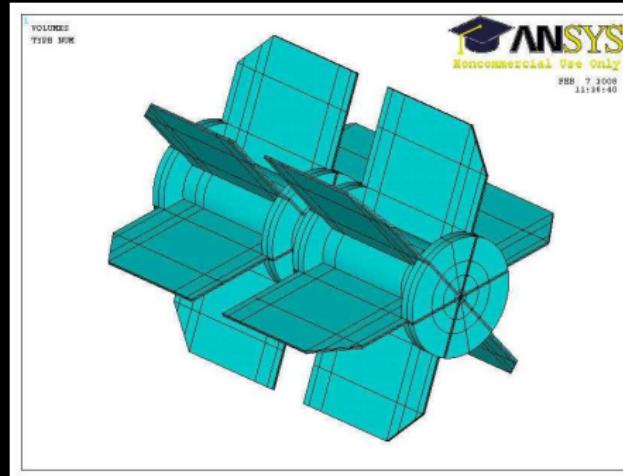
Parameters	Subsystem	Telemetry words
OTHER ANALOG		
TWT power readings	Communications	C231, C214
Receiver readings	Communications	C111, C212, C232, C121, C229, C213
BINARY/BIT FIELDS		
Conscan	Communications	C313, C314, C315, C316
Stored commands	Electrical	C305, C306, C307
Thruster pulse counts	Propulsion	C329, C321, C322, C330
Status bits	Data handling	C104
	Power	C128
	Electrical	C120, C132, C324, C332
	Communications	C308
Power switches	Electrical	C108, C124
Roll attitude	Data handling	C112, C116
Precession	ACS	C403, C411, C412, C415, C416, C422, C423, C424, C425, C428, C429, C430
Spin/roll	Data handling	C405, C406, C407, C408, C417
Delta V	ACS	C413, C414, C426
ACS status	Propulsion	C409
Star sensor	ACS	C410, C427, C431, C432
	ACS	C404, C419, C420, C421
SCIENCE INSTRUMENTS		
Status/housekeeping	Science	E108, E124, E130, E202, E224, E131, E132, E208
JPL/HVM readings	Science	E103, E104, E105, E106, E107, E203, E204, E205
UC/CPI readings	Science	E114, E115, E116, E206, E212, E214, E215, E216
GE/AMD readings	Science	E122, E123, E222, E223
GSFC/CRT readings	Science	E126, E127
LaRC/MD readings	Science	E207

Outlook: thermal model



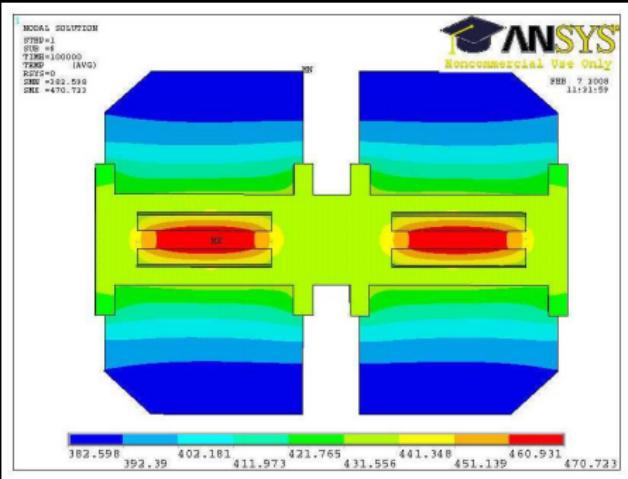
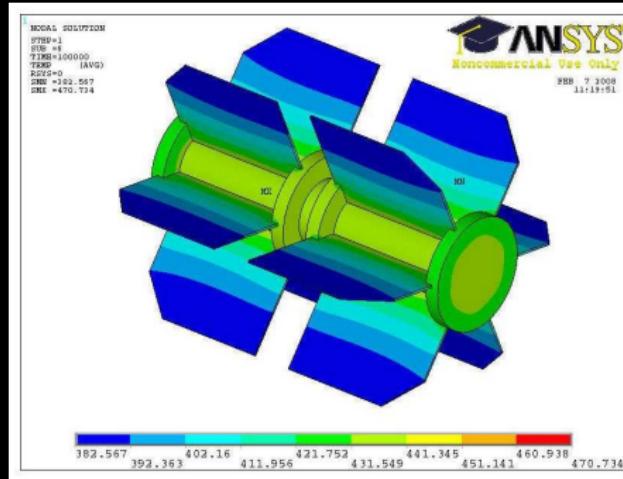
B. Rievers et al 2008

Outlook: thermal model



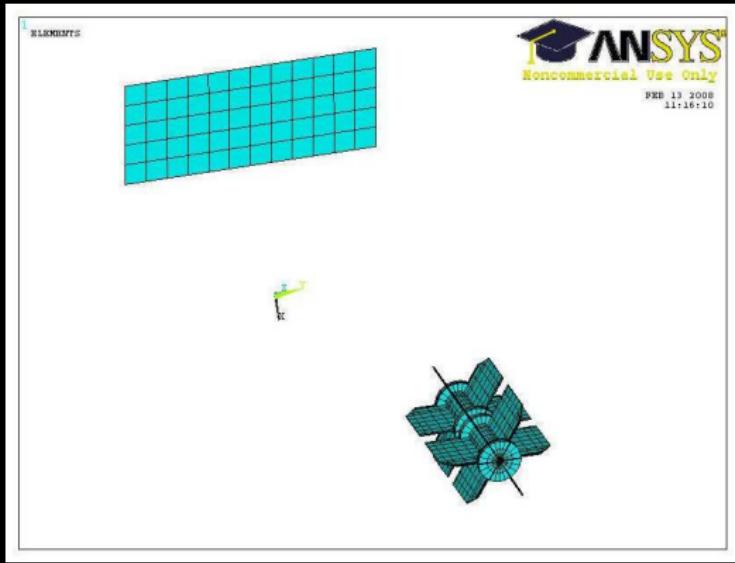
B. Rievers et al 2008

Outlook: thermal model



B. Rievers et al 2008

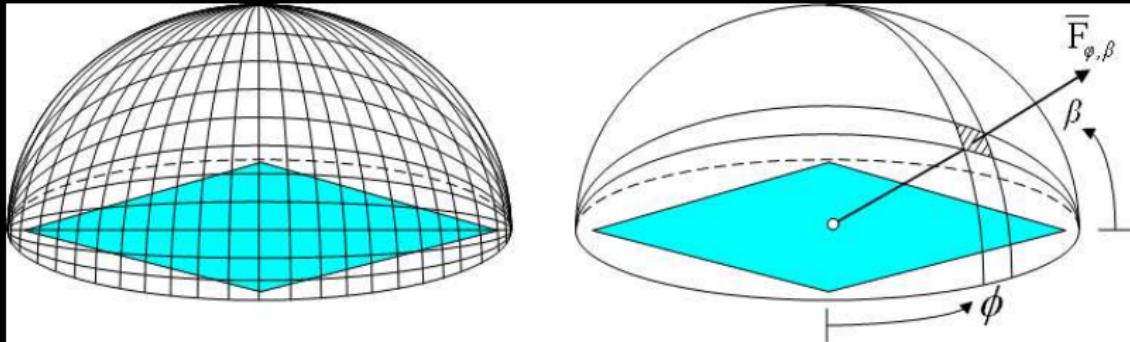
Outlook: thermal model



B. Rievers et al 2008

Outlook: thermal model

Computation of solid angle force $F_{\theta\varphi} \implies$ force generated by heat radiating surfaces dA

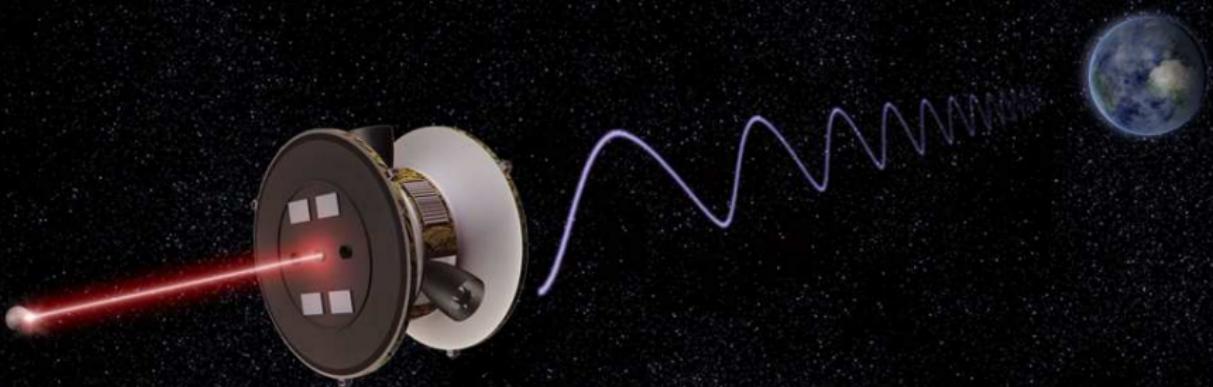


B. Rievers et al 2008: This model gives 12.5% of the Pioneer anomaly
More information about material properties and cross check with housekeeping data and orbital data

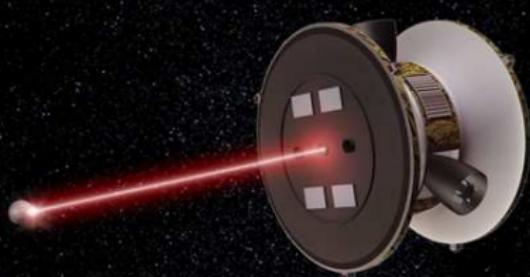
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Outlook: New mission



Outlook: New mission



- Active spacecraft, passive test mass
- Accurate tracking of test mass
- 2-step-tracking
 - Radio: Earth — spacecraft
 - Laser: spacecraft — test mass
- Flexible formation: varying distance
- Test mass is at environmentally quiet distance from spacecraft ~ 200 m
- Occasional manoeuvres to maintain formation

New Pioneer Explorer should be equipped with stable clock

For Pioneer anomaly:

$$\frac{\Delta_{\text{Pioneer}} \nu}{\nu} = \frac{1}{c^2} \int_{\text{Jupiter}}^{90 \text{ AU}} a_{\text{Pioneer}} dx = 10^{-13}$$

ESA Cosmic Vision

ESA Cosmic Vision call

4 Pioneer proposals

4 of 13 Fundamental Physics proposals!

- Class M missions
 - Deep Space Gravity Explorer (DSGE) – Bremen
 - ODYSSEY – Paris
 - Zacuto – Lisboa
- Class L mission
 - Search for Anomalous Gravitation using Atomic Sensors (SAGAS) – Paris

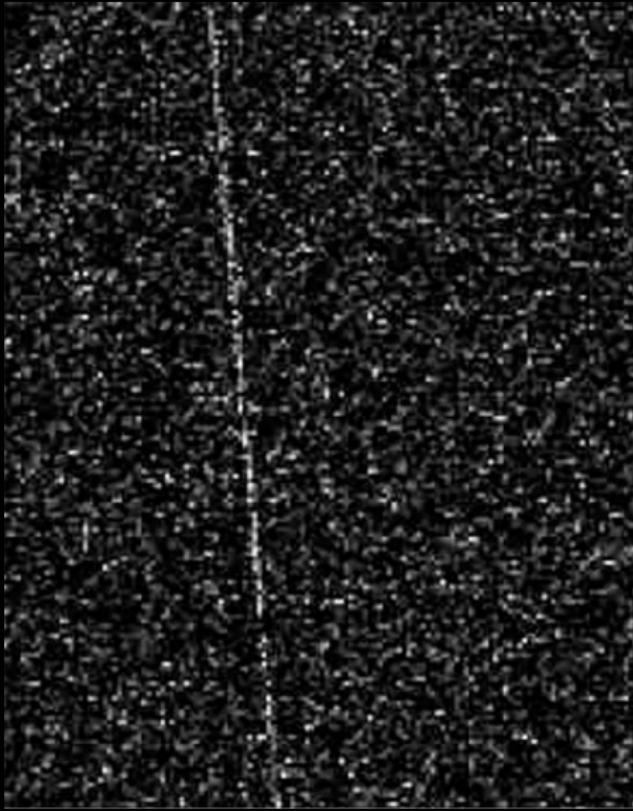
Pioneer coordination meeting Paris, April 20

Merger to two missions

- Class M Mission: ODYSSEY (more conventional, macroscopic test mass)
- Class L Mission: SAGAS (with atomic interferometry and clocks as gravity sensors)

Problem RTGs

The end



Position of Pioneer 10 on 29.1.2004

distance from Sun	87.89 AU
position, SE_lat SE_lon	(3.0° , 77.4°)
speed relative to Sun	12.24 km/s
distance from Earth	13.38 Gkm
Round-trip light time	$\approx 24\text{h } 32\text{min}$

The latest successful precession manoeuvre to point the s/c to Earth was accomplished on 11 Feb 2000 (distance from the Sun of 75 AU)

Picture of Pioneer 10 in ~ 80 AU as seen from Arecibo in 2000



Gravity modifications

Gravity: Physics of space and time = physics of clocks and particle trajectories

General ansatz for metric

Clocks:

$$g_{\mu\nu} = \begin{pmatrix} -1 + 2U & g_{0i} \\ g_{i0} & \delta_{ij} (1 - U - V) \end{pmatrix} \quad ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

V non-Newtonian potential

General modification of particle motion

Universality of Free Fall

$$0 = v^\nu \partial_\mu v^\nu + H^\mu(x, v) = v^\nu \partial_\mu v^\nu + \left\{ \begin{smallmatrix} \mu \\ \rho\sigma \end{smallmatrix} \right\} v^\mu v^\sigma + \underbrace{\gamma^\mu(x, v)}_{\text{non-metric}}$$



Particle trajectories

Nonrelativistic approximation

Expansion for small velocities

$$\begin{aligned} \frac{d^2x^i}{dt^2} &= - \left(\left\{ {}^i_{\mu\nu} \right\} - \left\{ {}^0_{\mu\nu} \right\} \frac{dx^i}{dt} \right) \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} + \frac{1}{\left(\frac{dt}{ds} \right)^2} \left(\gamma^i(v, x) - \frac{dx^i}{dt} \gamma^0(v, x) \right) \\ &\approx \underbrace{\partial_i U}_{\text{Newton}} + \underbrace{(\partial_i h_j - \partial_j h_i) \dot{x}^j}_{\text{Lense-Thirring}} + \dot{x}^2 \partial_i V + \dot{x}^i \dot{V} + \Upsilon^i + \Upsilon^i_j \dot{x}^j + \Upsilon^i_{jk} \dot{x}^j \dot{x}^k + \dots \end{aligned}$$

Pioneer acceleration?

We trivially obtain a constant acceleration for

$$\begin{aligned} V(x) &= \alpha_N r \quad \text{or} \quad \alpha'_N t \\ \Upsilon^i(x) &= \Upsilon^i \\ \Upsilon^i_j(x) &= \Upsilon^i_j \\ \Upsilon^i_{jk}(x) &= \Upsilon^i_{jk} \end{aligned}$$

Particle trajectories

Ansatz for gravity

If V and all Υ are assumed to be of **gravitational origin**: Ansatz (spherical symmetric mass M , asymptotic flatness)

$$V = A_0 \frac{GM}{r}$$

$$\Upsilon^i = A_{11} \frac{GM}{r^2} \frac{r^i}{r}$$

$$\Upsilon_j^i = A_{21} \frac{GM}{r^2} \frac{r^i r^j}{r^2} + A_{22} \frac{GM}{r^2} \delta_j^i$$

$$\Upsilon_{jk}^i = A_{31} \frac{GM}{r^2} \frac{r^i r^j r^k}{r^3} + A_{32} \frac{GM}{r^2} \frac{r^i}{r} \delta_{jk} + A_{33} \frac{GM}{r^2} \frac{r^j}{r} \delta_k^i.$$



Particle trajectories

Accelerations

the above terms lead to accelerations

$$\ddot{x}^i = A_0 \dot{x}^2 \frac{\partial_i U}{c^2} \sim \text{relativistic correction to Newton}$$

$$\ddot{x}^i = A_{11} \frac{GM}{r^2} \frac{r^i}{r} \sim \text{Newton}$$

$$\ddot{x}^i = \underbrace{(A_{21} + A_{22}) \frac{GM}{r^2} \frac{\dot{r}_\parallel^i}{c}}_{0 \text{ at perigee}} + \underbrace{A_{22} \frac{GM}{r^2} \frac{\dot{r}_\perp^i}{c}}_{\substack{\text{maximum at perigee} \\ \text{applies to flybys}}} \sim 10^{-9}, 10^{-4} \frac{\text{m}}{\text{s}^2}$$

$$\ddot{x}^i = \underbrace{A_{31} \frac{GM}{r^2 c^2} \frac{r^i (\mathbf{r} \cdot \dot{\mathbf{r}})^2}{r^3} + A_{32} \frac{GM}{c^2 r^2} \frac{r^i}{r} \dot{r}^2}_{\text{too small to account for } a_{\text{Pioneer}} \text{ or flyby-}\Delta v} + A_{33} \frac{GM}{c^2 r^2} \frac{\dot{r}^i}{r} (\mathbf{r} \cdot \dot{\mathbf{r}}),$$

Gravity modifications

Remarks I: On our model

- Universality of Free Fall still valid
- No energy conservation
- Gravity cannot be transformed away (Einstein's elevator not valid)
- May apply to flyby anomaly if restricted to escape orbits
- Unstable bound orbits
- r -dependence of all terms: Does not give constant Pioneer anomalous acceleration

Remarks II: General

- In general, we observe bound orbits only
- There are only a very few escape orbits (satellites, some comets)
- **Observational basis of escape orbits very poor**

[◀ Back to Pioneer anomaly](#)

