ULTRARELATIVISTIC BOOSTS OF BLACK RINGS

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We summarize the main results of recent studies of Aichelburg-Sexl ultrarelativistic limits of five dimensional vacuum and charged black rings.

1. Introduction

In 1959 Pirani argued that the geometry associated with a fast moving mass resembles a "plane" gravitational wave.[1] Later on, Aichelburg and Sexl (AS)[2] considered a limiting ("ultrarelativistic") boost of the Schwarzschild line element to determine the exact impulsive pp-wave[3] generated by a lightlike particle. In higher dimensions $D \geq 4$, the AS limit of static black holes[4] has been known for some time[5]

$$\mathrm{d}s^2 = 2\mathrm{d}u\mathrm{d}v + \mathrm{d}z_i\mathrm{d}z^i + H\delta(u)\mathrm{d}u^2 \qquad (i = 2, \dots, D-1), \tag{1}$$

$$\text{where} \quad H = -8\sqrt{2}p_M \ln\rho - \frac{3\pi\sqrt{2}}{2}\frac{p_Q^2}{\rho} \qquad (D = 4), \tag{2}$$

$$H = C_M \frac{p_M}{\rho^{D-4}} - C_Q \frac{p_Q^2}{\rho^{2(D-3)-1}} \qquad (D > 4), \tag{3}$$

$$\text{with} \quad C_M = \frac{16\pi\sqrt{2}}{(D-4)\Omega_{D-3}}, \quad C_Q = \frac{(2D-9)!!}{(D-3)!}\frac{2D-5}{(D-2)(D-3)}\frac{\pi\sqrt{2}}{2^{D-4}},$$

$\rho^2 = z_i z^i$, $\delta(u)$ is the Dirac delta, p_M/p_Q constants related to the mass/charge of the original spacetime[4] and Ω_{D-3} the area of a unit $(D-3)$-sphere. Such pp-waves have been employed in studies of classical formation of black holes in high energy collisions.[6–11] In the case of zero charge they can be straightforwardly generalized to include an external magnetic field.[12] The AS boost of rotating black holes[13] has been studied in.[14] Here we focus on $D = 5$ black rings (cf.[15–17] for more details).

2. Boost of black rings

Ultrarelativistic boosts of spacetimes rely on first identifying a notion of Lorentz boost (e.g., with respect to asymptotic infinity). Then one applies such a transformation to the metric and takes the singular limit when the boost parameter tends to the speed of light. Simultaneously, the mass is appropriately rescaled to zero.[2] The final metric depends on the boost direction. We shall use spatial "cartesian" coordinates (x_1, x_2, y_1, y_2) such that the 2-plane of the ring circle is (y_1, y_2), and study boosts along the axes x_1 and y_1, "orthogonal" and "parallel" to it.

Vacuum black ring In the *orthogonal* case, the ultrarelativistic boost of the vacuum black ring of[18] results[15,16] in the following pp-wave propagating along x_1

$$\mathrm{d}s^2 = 2\mathrm{d}u\mathrm{d}v + \mathrm{d}x_2^2 + \mathrm{d}y_1^2 + \mathrm{d}y_2^2 + H_{\perp}(x_2, y_1, y_2)\delta(u)\mathrm{d}u^2, \tag{4}$$

$$\begin{aligned} H_{\perp} &= \sqrt{2}\,\frac{3p_\lambda L^2 + (2p_\nu - p_\lambda)\xi^2}{\sqrt{(\xi+L)^2 + x_2^2}}\,K(k) + \sqrt{2}(2p_\nu - p_\lambda) \\ &\times \left[-\sqrt{(\xi+L)^2 + x_2^2}\,E(k) + \frac{\xi - L}{\xi + L}\frac{x_2^2}{\sqrt{(\xi+L)^2 + x_2^2}}\,\Pi(\rho, k) + \pi|x_2|\Theta(L-\xi)\right], \end{aligned} \tag{5}$$

$$\text{in which} \quad k = \sqrt{\frac{4\xi L}{(\xi+L)^2 + x_2^2}}, \qquad \rho = \frac{4\xi L}{(\xi+L)^2}, \qquad \xi = \sqrt{y_1^2 + y_2^2}, \tag{6}$$

and $\Theta(L-\xi)$ denotes the step function (cf. the appendix of[15] for definitions of the elliptic integrals K, E and Π). The null coordinates u and v are defined by

$$t = \frac{-u+v}{\sqrt{2}}, \qquad x_1 = \frac{u+v}{\sqrt{2}}, \tag{7}$$

and p_λ, p_ν and L are constants related to the mass, angular momentum and radius of the original ring. For black rings "in equilibrium" set $p_\lambda = 2p_\nu$ in eq. (5).

In the case of a *parallel* boost the final pp-wave, now propagating along y_1, is

$$\mathrm{d}s^2 = 2\mathrm{d}u\mathrm{d}v + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}y_2^2 + H_{||}(x_1, x_2, y_2)\delta(u)\mathrm{d}u^2, \tag{8}$$

$$\begin{aligned} H_{||} &= \left[2(2p_\lambda - p_\nu)L^2 + (2p_\nu - p_\lambda)a^2\left(1 + \frac{L^2+\eta^2}{a^2 - y_2^2}\right)\right. \\ &\left.+ 2\sqrt{p_\lambda(p_\lambda - p_\nu)}Ly_2\left(1 - \frac{L^2+\eta^2}{a^2 - y_2^2}\right)\right]\frac{\sqrt{2}}{a}K(k') - 2\sqrt{2}(2p_\nu - p_\lambda)aE(k') \\ &+ \frac{\sqrt{2}}{2}\left[(2p_\nu - p_\lambda)y_2 - 2\sqrt{p_\lambda(p_\lambda - p_\nu)}L\right]\left[-\frac{\eta^2 + L^2}{ay_2}\frac{a^2 + y_2^2}{a^2 - y_2^2}\,\Pi(\rho', k') + \pi\,\mathrm{sgn}(y_2)\right], \end{aligned} \tag{9}$$

$$\begin{aligned} \text{where} \quad k' &= \frac{\left(a^2 - \eta^2 - y_2^2 + L^2\right)^{1/2}}{\sqrt{2}a}, \qquad \rho' = -\frac{(a^2 - y_2^2)^2}{4a^2 y_2^2}, \\ a &= \left[(\eta^2 + y_2^2 - L^2)^2 + 4\eta^2 L^2\right]^{1/4}, \qquad \eta = \sqrt{x_1^2 + x_2^2}, \end{aligned} \tag{10}$$

and the null coordinates are now defined by

$$t = \frac{-u+v}{\sqrt{2}}, \qquad y_1 = \frac{u+v}{\sqrt{2}}. \tag{11}$$

Static charged black ring Static charged black rings were found in[19] (up to a misprint in $F_{\mu\nu}$) in the Einstein-Maxwell theory, see also.[20–22] After an *orthogonal* boost of such solutions, one obtains[17] a *pp*-wave (4) with

$$H^c_\perp = \sqrt{2}p_\lambda \Bigg[\left(3L^2 \frac{1+e^2}{1-e^2} + \xi^2 + x_2^2 \frac{\xi+L}{\xi-L} \right) \frac{K(k)}{\sqrt{(\xi+L)^2+x_2^2}} - \sqrt{(\xi+L)^2+x_2^2}\, E(k) - \frac{\xi+L}{\xi-L} \frac{(\xi-L)^2+x_2^2}{\sqrt{(\xi+L)^2+x_2^2}} \Pi(\rho_0,k) + \frac{\pi}{2}|x_2| \Bigg], \quad (12)$$

k and ξ as in eq. (6) and $\rho_0 = -(\xi-L)^2/x_2^2$. The parameter e is related[17] to the electric charge of the original static spacetime.

For a *parallel* boost, one finds a metric (8) with

$$H^c_{||} = \sqrt{2}p_\lambda \Bigg[\left(2L^2 \frac{1+2e^2}{1-e^2} + a^2 + a^2 \frac{L^2+\eta^2}{a^2-y_2^2} \right) \frac{1}{a} K(k') - 2aE(k') - \frac{\eta^2+L^2}{2a} \frac{a^2+y_2^2}{a^2-y_2^2} \Pi(\rho',k') + \frac{\pi}{2}|y_2| \Bigg], \quad (13)$$

with k', ρ', a and η as in eq. (10).

See[16] for the AS limit of the supersymmetric black ring of.[23]

References

1. F. A. E. Pirani, *Proc. R. Soc. A* **252**, 96 (1959).
2. P. C. Aichelburg and R. U. Sexl, *Gen. Rel. Grav.* **2**, 303 (1971).
3. R. Penrose, *Int. J. Theor. Phys.* **1**, 61 (1968).
4. F. R. Tangherlini, *Nuovo Cimento* **27**, 636 (1963).
5. C. O. Loustó and N. Sánchez, *Int. J. Mod. Phys. A* **5**, 915 (1990).
6. D. M. Eardley and S. B. Giddings, *Phys. Rev. D* **66**, 044011 (2002).
7. E. Kohlprath and G. Veneziano, *JHEP* **06**, 057 (2002).
8. H. Yoshino and Y. Nambu, *Phys. Rev. D* **66**, 065004 (2002).
9. S. B. Giddings and V. S. Rychkov, *Phys. Rev. D* **70**, 104026 (2004).
10. H. Yoshino and V. S. Rychkov, *Phys. Rev. D* **71**, 104028 (2005).
11. H. Yoshino and R. B. Mann, *Phys. Rev. D* **74**, 044003 (2006).
12. M. Ortaggio, *JHEP* **05**, 048 (2005).
13. R. C. Myers and M. J. Perry, *Ann. Phys. (N.Y.)* **172**, 304 (1986).
14. H. Yoshino, *Phys. Rev. D* **71**, 044032 (2005).
15. M. Ortaggio, P. Krtouš and J. Podolský, *Phys. Rev. D* **71**, 124031 (2005).
16. M. Ortaggio, J. Podolský and P. Krtouš, *JHEP* **12**, 001 (2005).
17. M. Ortaggio, *J. Phys. Conf. Ser.* **33**, 386 (2006), gr-qc/0601093.
18. R. Emparan and H. S. Reall, *Phys. Rev. Lett.* **88**, 101101 (2002).
19. D. Ida and Y. Uchida, *Phys. Rev. D* **68**, 104014 (2003).
20. H. K. Kunduri and J. Lucietti, *Phys. Lett. B* **609**, 143 (2005).
21. S. S. Yazadjiev (2005), hep-th/0507097.
22. M. Ortaggio and V. Pravda, *JHEP* **12**, 054 (2006).
23. H. Elvang, R. Emparan, D. Mateos and H. S. Reall, *Phys. Rev. Lett.* **93**, 211302 (2004).