CAUSAL STRUCTURE AROUND SPINNING 5-DIMENSIONAL COSMIC STRINGS

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We present a numerical solution of a stationary 5-dimensional spinning cosmic string in the Einstein-Yang-Mills (EYM) model, where the extra bulk coordinate ψ is periodic. It turns out that when $g_{\psi\psi}$ approaches zero, i.e., a closed time-like curve (CTC) would appear, the solution becomes singular. We also investigated the geometrical structure of the static 5D cosmic string. Two opposite moving 5D strings could, in contrast with the 4D case, fulfil the Gott condition for CTC formation.

Keywords: Bulk space time; Einstein Yang-Mills; Cosmic strings; Closed timelike curves.

In recent years higher dimensional gravity is attracting much interest. The possibility that spacetime may have more than four dimensions is initiated by high energy physics and inspired by D-brane ideology in string theory. Our 4-dimensional spacetime (brane) is embedded in the 5-dimensional spacetime (bulk). It is assumed that all the standard model degrees of freedom reside on the brane, where as gravity can propagate into the bulk. The effect of string theory on classical gravitational physics is investigated by the low-energy effective action. In General Relativity(GR), gravitating non-Abelian gauge field, i.e., the Yang-Mills(YM) field, can be regarded as the most natural generalization of Einstein-Maxwell(EM) theory. In particular, particle-like, soliton-like and black hole solutions in the combined EYM models, shed new light on the complex features of compact object in these models.

Gravitating cosmic strings became of interest, when it was discovered that inflationairy cosmological models solved many shortcomings in the standard model. Inflation is triggered by a Higgs field (Φ) on the right hand side of the equations of Einstein. The Higgs field is also responsible for the formation of topological defects during the fabric of space time at the early stages of the universe. The last decades many physicians studied the consequences of topological defects in general relativity. One of the earliest investigation was the gravitational Aharonov-Bohm effect in a conical space time. The cosmic string can also be described as point particle in (2+1) dimensional space time. An interesting example of the richness of the (2+1) dimensional gravity, is the Gott space time.[1] An isolated pair of point particles, moving with respect to each other, may generate a surrounding region where close timelike curves (CTC) occur. It was not a surprise that one can prove that the Gott space time has unphysical features.[2]

It is quite natural to consider as a next step the non-Abelian EYM situation is context with cosmic string solutions and its unusual features like the formation of CTC's. There is some evidence that, by suitable choice of the gauge, CTC's will not emerge dynamically.[3,4] Let us now consider a 5-dimensional space time with action

$$\mathcal{S}=\frac{1}{16\pi}\int d^5x\sqrt{-g_5}\Big[\frac{1}{G_5}(R-\Lambda)+\kappa(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}-4R_{\alpha\beta}R^{\alpha\beta}+R^2)-\frac{1}{g^2}Tr\mathbf{F^2}\Big], \tag{1}$$

with G_5 the gravitational constant, Λ the cosmological constant, κ the Gauss-Bonnet coupling and g the gauge coupling. The coupled set of equations of the EYM-GB system will then become

$$\Lambda g_{\mu\nu}+G_{\mu\nu}+\kappa GB_{\mu\nu}=8\pi G_5T_{\mu\nu},\quad \mathcal{D}_\mu F^{\mu\nu a}=0, \tag{2}$$

with $G_{\mu\nu}$ the Einstein tensor, $GB_{\mu\nu}$ the Gauss-Bonnet tensor and $F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+g\epsilon^{abc}A^b_\mu A^c_\nu$.

Consider now the stationary axially symmetric 5-dimensional space time

$$ds^2=-F(r)(dt+\omega(r)d\psi)^2+dr^2+dz^2+A(r)^2r^2d\varphi^2+B(r)^2d\psi^2, \tag{3}$$

with the YM parameterization only in the brane ($A_r^{(a)}=A_z^{(a)}=A_\psi^{(a)}=0$):

$$A_t^{(a)}=\Big(\Phi(r)\cos\varphi,\Phi(r)\sin\varphi,0\Big),\quad A_\varphi^{(a)}=\Big(0,0,W(r)-1\Big). \tag{4}$$

The equations are easily solved numerically. From a combination of the YM equations, we obtain also for the angular momentum component ω a first order expression

$$\omega'=\frac{B\omega(2FB'-BF')}{F(F\omega^2+B^2)}, \tag{5}$$

which can be used in the numerical code. We also have a constraint equation from the Einstein equations

$$g_{\psi\psi}=\frac{FB^2(W')^2-\frac{\Lambda FB^2A^2r^2}{8\pi G}}{(\Phi^2W^2+A^2r^2(\Phi')^2)}. \tag{6}$$

So a negative cosmological constant will keep $g_{\psi\psi}$ positive, which is desirable. Further, from $\nabla_\mu T^{\mu\nu}=0$, we obtain in combination with the YM equations the same equation as Eq.(5), which proves the consistency of the system. Without Λ, the solution becomes singular when $g_{\psi\psi}$ approaches zero. So it seems that CTC formation is prevented by singular behaviour. In figure 1 we plotted a typical solution for negative Λ (no GB contribution) . We see that $g_{\psi\psi}$ remains positive and no singular behaviour is observed. So again no CTC will form. An interesting question is what will happen with the reduced (3+1) dimensional model when the dz^2 term in the metric is omitted, on the analogy of the Gott pair in (2+1) dimensional space time.

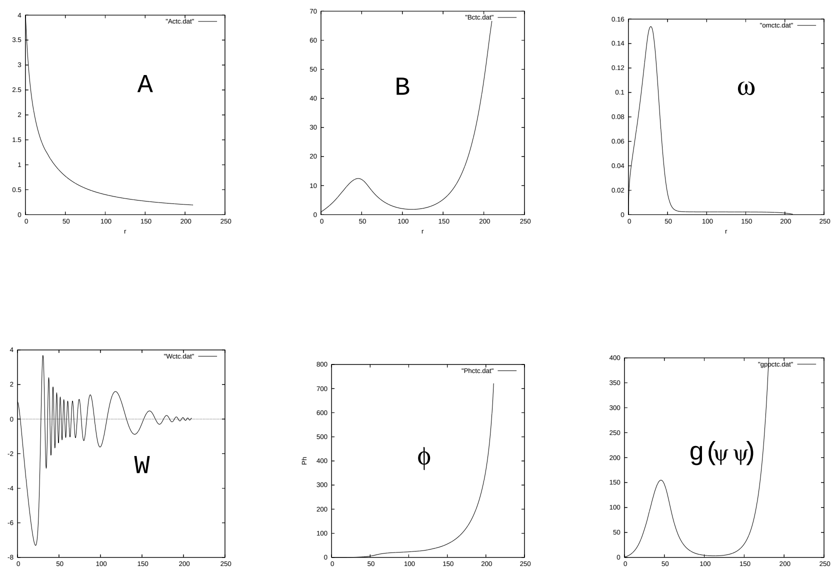

Fig.1 Typical solution for negative Λ

The effectively (3+1) dimensional point particles can be Lorentz-boosted in opposite directions and one can close space by two rotations about the two angle variables. The matching condition then yields

$$(\cos^2 2\beta - 1)(1 - 2\cosh^2 \xi) < \frac{1}{2}\sqrt{3}. \tag{7}$$

with $\beta = 4\pi G m = \pi(1 - 8\pi G\alpha)$, m the mass, α the angle deficit and $\tanh(\xi) = v$ the velocity of the point particles. Further we took equal angle deficits in the φ and ψ variables. The Gott pair will produce a CTC when[1] $\cosh\xi \sin\beta > 1$ If we choose the minimal possible value $\sin\beta = \frac{1}{\cosh\xi}$, we obtain

$$1 < \cosh\xi < \frac{\sqrt{110 + 22\sqrt{3}}}{11} \approx 1.11 \tag{8}$$

So in the static geometrical approach, there could be, in principle, a CTC. We conjecture that in the more realistic situation, where a matter field is present, this approach is not valid anymore.

References

1. J. R. Gott, *Phys. Rev. Lett.* **66**, 1126 (1990).
2. S. Deser, R. Jackiw and G. 't Hooft, *Phys. Rev. Lett.* **68**, 267 (1992).
3. R. J. Slagter, in *Proceedings of Eigth Marcel Grossmann Meeting*, edited by T. Piran and R. Ruffini (World Scientific, Singapore, 2006) pp. 602-604.
4. R. J. Slagter, *Phys. Rev. D* **54**, 4873 (1996), **gr-qc**/0609003.