Short distances, black holes, and TeV gravity

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The Hawking effect can be rederived in terms of two-point functions and in such a way that it makes it possible to estimate, within the conventional semiclassical theory, the contribution of ultrashort distances at I^+ to the Planckian spectrum. Thermality is preserved for black holes with $\kappa l_P \ll 1$. However, deviations from the Planckian spectrum can be found for mini black holes in TeV gravity scenarios, even before reaching the Planck phase.

1. Introduction

In 1974 Hawking predicted the thermal emission of quanta by black holes[1] using semiclassical gravity . The deep connection of this result with thermodynamics and, in particular, with the generalized second law, strongly support its robustness and its interpretation as a low-energy effect, not affected by the particular underlying theory of quantum gravity[2] . However[3] , ultrahigh frequencies (or ultrashort distances) seem to play a crucial role in the derivation of the Hawking effect. Any emitted quanta, even those with very low frequency at future infinity, will suffer a divergent blueshift when propagated backwards in time and measured by a freely falling observer. The exponential redshift of the event horizon provides, to the external observer, a glimpse of the world at very short-distance scales, where semiclassical tools are not well justified. All derivations of Hawking radiation seem to invoke Planck-scale physics in a fundamental way, which makes it unclear the way to parameterize the contribution of transplanckian physics in black hole radiation.

We propose an alternative to the standard approach in terms of Bogolubov coefficients to derive the Hawking effect. In our approach, the correlation functions of the matter fields are used to compute the spectrum of the emitted particles. This provides an explicit way to evaluate the contribution of ultrashort distances (Planck-scale) to the spectrum of Hawking quanta within the semiclassical approach.

Let us assume, for the sake of simplicity, that ϕ is a massles, neutral, and minimally coupled scalar field. One can easily verify that the number operator can be obtained from the following projection

$$a^{out\dagger}_i a^{out}_j = \int_\Sigma d\Sigma_1^\mu d\Sigma_2^\nu [u_i^{out}(x_1)\overleftrightarrow{\partial}_\mu][u_j^{out*}(x_2)\overleftrightarrow{\partial}_\nu]\left\{\phi(x_1)\phi(x_2) - \langle out|\phi(x_1)\phi(x_2)|out\rangle\right\} \ , \tag{1}$$

where $u_i^{out}(x)$ is a normalized positive frequency mode with respect to the inertial time at future infinity, and Σ represents a suitable initial value hypersurface. Therefore, the number of particles in the i^{th} mode measured by the "out" observer in the "in" vacuum is given by $\langle in|N_i^{out}|in\rangle \equiv \langle in|N_{ii}^{out}|in\rangle$, where $N_{ij}^{out} \equiv \hbar^{-1} a^{out}{}_i^{\dagger} a_j^{out}$ can be easily worked out using the above expression. Let us now apply (1) to the formation process of a Schwarzschild black hole and restrict the "out" region to future null infinity (I^+). The "in" region is, as usual, defined by past null infinity (I^-). At I^+ we can consider the normalized radial plane-wave modes $u_{wlm}^{out}(t,r,\theta,\phi) = u_w(u) r^{-1} Y_{lm}(\theta,\phi)$, where $u_w(u) = \frac{e^{-iwu}}{\sqrt{4\pi w}}$ and u is the null retarded time. Using these modes in (1) one finds[4,5] (for simplicity we omit the factor $\delta_{l_1 l_2}\delta_{m_1 m_2}$)

$$\langle in|N_{i_1 i_2}^{out}|in\rangle = -\frac{|t_{lm}(w)|^2 \delta(w_1 - w_2)}{2\pi\sqrt{w_1 w_2}} \int_{-\infty}^{+\infty} dz e^{-i\frac{(w_1+w_2)}{2}z} \left[\frac{\kappa^2 e^{-\kappa z}}{(e^{-\kappa z}-1)^2} - \frac{1}{z^2}\right]$$

where $z = u_1 - u_2$ represents the "distance" between the points u_1 and u_2 and $t_{lm}(w)$ are the transmission coefficients. To get the Planckian spectrum, there remains to perform the integration in z

$$\langle in|N_w^{out}|in\rangle = \frac{-|t_{lm}(w)|^2}{2\pi w} \int_{-\infty}^{+\infty} dz e^{-iwz} \left[\frac{\kappa^2 e^{-\kappa z}}{(e^{-\kappa z}-1)^2} - \frac{1}{z^2}\right] = \frac{|t_{lm}(w)|^2}{e^{2\pi w \kappa^{-1}} - 1} \ .$$

The interesting aspect of the above expression is that it allows us to explicitly evaluate the contribution of distances to the thermal spectrum. To be more explicit, the integral

$$I(w,\kappa,\epsilon) = -\frac{1}{2\pi w} \int_{-\epsilon}^{+\epsilon} dz e^{-iwz} \left[\frac{\kappa^2 e^{-\kappa z}}{(e^{-\kappa z}-1)^2} - \frac{1}{z^2}\right]$$

can be regarded as the contribution coming from distances $z \in [-\epsilon, \epsilon]$ to the full spectrum. This integral can be solved analytically. For details and the case of a massless spin $s = 1/2$ field see[4,5] . Obviously, in the limit $\epsilon \to \infty$, we recover the Planckian result $I(w,\kappa,\infty) = (e^{2\pi w\kappa^{-1}} - 1)^{-1}$. For a rotating black hole the result is similar with the usual replacement of w by $\tilde{w} \equiv w - m\Omega_H$ (m is the axial angular momentum quantum number of the emitted particle and Ω_H the angular velocity of the horizon).

On the other hand, if we take ϵ of order of the Planck length $l_P = 1.6 \times 10^{-33} cm$, we obtain that the contribution to the thermal spectrum at the typical emission frequency, $w_{typical} \sim \kappa/2\pi \equiv T_H$, due to transplanckian scales is of order κl_P. This contribution is negligible for macroscopic black holes with typical size much bigger than the microscopic Planck length. In fact, for three solar masses black holes the contribution to the total spectrum, $(e^{2\pi w\kappa^{-1}} - 1)^{-1}$, at $w_{typical}$ is of order $10^{-38}\%$. We need to look at high frequencies, $w/w_{typical} \approx 96$, to get contributions of the same order as the total spectrum itself. This is why Hawking thermal radiation is very robust, as it has been confirmed in analysis based on acoustic black holes[6] . Our results, in addition, indicate that when the product κl_P is of order unity, the contribution of short distances to the Planckian spectrum is not negligible. The integral $I(w, \kappa, \epsilon)$ gives values similar to $(e^{2\pi w \kappa^{-1}} - 1)^{-1}$ when $w/w_{typical}$ is not very high. This happens in the case of black holes predicted by TeV gravity scenarios[7,8] . For detailed and recent results see[9] . Assuming a drastic change of the strength of gravity at short distances due to n extra dimensions (a Planck mass M_{TeV} of 1 TeV) and for a $(4+n)$-dimensional Kerr black hole with surface gravity $\kappa \sim 0.6 - 1\ TeV^{-1}$ (this means $M \sim 5 - 10$ TeV when a=0), we obtain that, at $\tilde{w} = \kappa/2\pi = T_H$, around the 20% of the spectrum comes from distances shorter than the new Planck length $l_{TeV} \sim 10^{-17} cm$, for $n = 2 - 6$ and for spin zero particles. Moreover, at frequencies $\tilde{w} \approx 3T_H$ the contribution of ultra-short distances is of order of the total spectrum itself. For massless $s = 1/2$-particles the results can be obtained from the formulaes of[5] . In this case the contribution from ultrashort distances is smaller than for spin zero and it is around the 0.2% of the spectrum at $\tilde{w} = \kappa/2\pi = T_H$. In addition we find that, for $\kappa = 0.9 - 1$, and $n = 6$ we need to go to frequencies $\tilde{w} \approx 5.5T_H$ and $\tilde{w} \approx 5.6T_H$, respectively, to find short-distance contributions of order of the fermionic thermal spectrum $(e^{2\pi \tilde{w} \kappa^{-1}} + 1)^{-1}$. For $\kappa = 0.6 - 0.8$, and $n = 2$ we obtain $\tilde{w} \approx 6.2T_H$ and $\tilde{w} \approx 6.9T_H$, respectively. Therefore, in TeV gravity scenarios the spectrum of Hawking quanta is sensitive to transplanckian physics and significant deviations from the thermal spectrum can emerge in the "semiclassical" phase of the evaporation.

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