BLACK STRING SOLUTIONS WITH NEGATIVE COSMOLOGICAL CONSTANT

ROBERT MANN

Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada and Department of Physics, University of Waterloo Waterloo

EUGEN RADU

Department of Mathematical Physics, National University of Ireland, Maynooth, Ireland

CRISTIAN STELEA

Department of Physics, University of Waterloo Waterloo

We present arguments for the existence of new black string solutions with negative cosmological constant. These higher-dimensional configurations have no dependence on the 'compact' extra dimension, and their conformal infinity is the product of time and $S^{d-3} \times R$ or $H^{d-3} \times R$. The configurations with an event horizon topology $S^{d-2} \times S^1$ have a nontrivial, globally regular limit with zero event horizon radius.

The physics of asymptotically Anti-de Sitter (AdS) black hole solutions is of particular interest due to the AdS/CFT conjecture. The thermodynamic properties of black objects in AdS offers the possibility of studying the nonperturbative aspects of certain conformal field theories living on the AdS boundary.

Here we present arguments for the existence of a class of configurations, which we interpret as the AdS counterparts of the $\Lambda = 0$ uniform black string solutions.[1] For such solutions the topological structure of the AdS boundary is the product of time and $S^{d-3} \times R$ or $H^{d-3} \times R$ and they correspond to black strings with the horizon topology $S^{d-3} \times S^1$ or $H^{d-3} \times R$ respectively (here the black string is wrapping the S^1 circle).

We consider the following parametrization of the d-dimensional line element (with $d \geq 5$)

$$ds^2 = a(r)dz^2 + \frac{dr^2}{f(r)} + r^2 d\Sigma^2_{k,d-3} - b(r)dt^2 \tag{1}$$

where the $(d-3)$–dimensional metric $d\Sigma^2_{k,d-3}$ is

$$d\Sigma^2_{k,d-3} = \begin{cases} d\Omega^2_{d-3} & \text{for } k = +1 \\ \sum_{i=1}^{d-3} dx_i^2 & \text{for } k = 0 \\ d\Xi^2_{d-3} & \text{for } k = -1\ , \end{cases} \tag{2}$$

where $d\Omega^2_{d-3}$ is the unit metric on S^{d-3}. By H^{d-3} we will understand the $(d-3)$–dimensional hyperbolic space, whose unit metric $d\Xi^2_{d-3}$ can be obtained by analytic continuation of that on S^{d-3}. The direction z is periodic with period L.

The event horizon is taken at constant $r = r_h$ where the metric functions $b(r)$ and $f(r)$ are vanishing while $a(r)$ takes positive values, $b(r) = b_1(r - r_h) + O(r -$

$r_h)^2$, $f(r) = f_1(r-r_h) + O(r-r_h)^2$, $b(r) = a_h + a_1(r-r_h) + O(r-r_h)^2$. For even d, the black string solutions admit at large r a power series expansion of the form:

$$\begin{aligned} a(r) &= \frac{r^2}{\ell^2} + \sum_{j=0}^{(d-4)/2} a_j (\frac{\ell}{r})^{2j} + c_z (\frac{\ell}{r})^{d-3} + O(1/r^{d-2}), \\ b(r) &= \frac{r^2}{\ell^2} + \sum_{j=0}^{(d-4)/2} a_j (\frac{\ell}{r})^{2j} + c_t (\frac{\ell}{r})^{d-3} + O(1/r^{d-2}), \\ f(r) &= \frac{r^2}{\ell^2} + \sum_{j=0}^{(d-4)/2} f_j (\frac{\ell}{r})^{2j} + (c_z + c_t)(\frac{\ell}{r})^{d-3} + O(1/r^{d-2}), \end{aligned} \tag{3}$$

where $a_j,\ f_j$ are constants depending on the index k and the spacetime dimension only and $\Lambda = -(d-1)(d-2)/(2\ell^2)$ is the cosmological constant. A similar expansion can be written for an odd number of spacetime dimensions, the main difference being the presence of logarithmic terms in the corresponding expressions. The mass M and the tension $\mathcal{T}$ of the black strings are found by using a boundary counterterm prescription,[2] being determined by the constants c_t and c_z in the asymptotic expansion at infinity

$$M = M_0 + M_c^{(k,d)}\ , \quad M_0 = \frac{\ell^{d-4}}{16\pi G}\big[c_z - (d-2)c_t\big] L V_{k,d-3}\ , \tag{4}$$

$$\mathcal{T} = \mathcal{T}_0 + \mathcal{T}_c^{(k,d)}\ , \quad \mathcal{T}_0 = \frac{\ell^{d-4}}{16\pi G}\big[(d-2)c_z - c_t\big] V_{k,d-3}\ , \tag{5}$$

where $V_{k,d-3}$ is the total area of the angular sector. Here $M_c^{(k,d)}$ and $\mathcal{T}_c^{(k,d)}$ are Casimir-like terms which appear for an odd spacetime dimension only.[2]

These solutions have a nonzero temperature and an entropy

$$T_H = \frac{1}{4\pi}\sqrt{\frac{b_1}{r_h \ell^2}\big[(d-1)r_h^2 + k(d-4)\ell^2\big]}, \quad S = \frac{1}{4G} r_h^{d-3} V_{k,d-3} L \sqrt{a_h}, \tag{6}$$

and satisfy the Smarr relation

$$M + \mathcal{T} L = T_H S\ , \tag{7}$$

We have found black string numerical solutions with AdS asymptotics in all dimensions between five and twelve. We conjecture that they exist for any d and, in the case $k = 1$, for any value of the event horizon r_h. For all the solutions we studied, the metric functions $a(r)$, $b(r)$ and $f(r)$ interpolate monotonically between the corresponding values at $r = r_h$ and the asymptotic values at infinity, without presenting any local extrema. The metric functions $a(r)$, $b(r)$ and $f(r)$ of a typical solution are presented in Figure 1.

The condition for a regular event horizon implies in the $k = -1$ case the existence of a minimal value of r_h for a given Λ. The $k = 0$ solution appears to be unique, with only one free parameter, corresponding to the known planar topological black hole, $a = r^2$, $f = 1/b = -2M/r^{d-3} + r^2/\ell^2$.

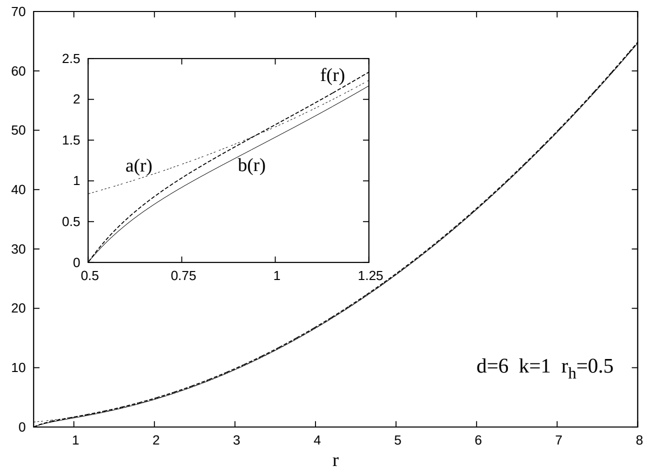

Fig. 1. The profiles of the metric functions $a(r)$, $b(r)$ and $f(r)$ are shown for a typical $k = 1$ black string solution with $d = 6$, $k = 1$ and $r_h = 0.5$.

As with the spherically symmetric Schwarzschild-AdS solutions, the temperature of the $k = 1$ black string solutions is bounded from below. The free energy of these solutions is positive for small r_h and negative for large r_h. The phase transition found in[3] occurs here as well. The $k = 1$ solutions have a nontrivial zero event horizon radius limit corresponding to AdS vortices. As $r_h \to 0$ we find *e.g.* $c_t(d = 6) \simeq -0.0801$, $c_t(d = 7) \simeq -0.0439$, $c_t(d = 8) \simeq 0.0403$, $c_t(d = 9) \simeq 0.0229$, while $c_t(d = 10) \simeq -0.0246$.

More details on these solutions are presented in.[4] Five dimensional AdS black string with an event horizon topology $S^2 \times S^1$ are also discussed in.[5]

Acknowledgments

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