Perturbatively non-uniform charged black strings: a new stable phase

UMPEI MIYAMOTO

Department of Physics, Waseda University, Okubo 3-4-1, Tokyo 169-8555, Japan
umpei at gravity.phys.waseda.ac.jp

HIDEAKI KUDOH

Department of Physics, UCSB, Santa Barbara, CA 93106, USA
and
Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan
kudoh at utap.phys.s.u-tokyo.ac.jp

The final fate of the Gregory-Laflamme instability is one of the most interesting problems in the black hole physics. We investigate the thermodynamical stability of gauge-charged non-uniform black strings with higher-order perturbations. We find that there exists a new stable state of non-uniform black strings, which can be the final fate of the dynamical instability in any dimensions.

1. Introduction

It is known that black objects with translational symmetries, such as black branes and black strings, suffer from the Gregory-Laflamme (GL) dynamical instability, breaking the translational symmetries.[1] To know the final fate of GL instability, it is useful to construct black objects, such as localized black holes, black strings and so on, in Kaluza-Klein spacetimes.[2] The whole phase structure in a vacuum spacetime has been clarified recently.[3] One of the most interesting features is the phase structure, therefore the final fate of GL instability, will depend on spacetime dimensions.[4] Since the gravity inevitably couples to other fundamental fields, such as gauge fields and dilaton, it is important to generalize these analyses to those of black objects with charges. As the first step, we construct non-uniform charged black strings perturbatively in various dimensions in this article [a].

2. Static perturbations of magnetic black strings

Let us consider $D = d+1$ dimensional spacetime with the gravity coupled to $(d-2)$-form field $\mathcal{F}_{d-2}$. The governing equations are

$$R_{\mu\nu} = \frac{1}{2(d-3)!}\mathcal{F}_{\mu}{}^{\mu_2\ldots\mu_{d-2}}\mathcal{F}_{\nu\mu_2\ldots\mu_{d-2}} - \frac{d-3}{2(d-1)!}g_{\mu\nu}\mathcal{F}^2, \quad \nabla_\mu \mathcal{F}^{\mu\mu_2\ldots\mu_{d-2}} = 0.$$

In addition, the form field satisfies the Bianchi identity, $d\mathcal{F}_{d-2} = 0$. We construct non-uniform black strings perturbatively with a metric ansatz given by

$$ds^2_{d+1} = -e^{2a(r,z)}f_+dt^2 + e^{2b(r,z)}f_-\left(\frac{dr^2}{f_+f_-^2} + dz^2\right) + e^{2c(r,z)}r^2 d\Omega^2_{d-2},$$

[a] This article is based on our paper 5, although a new result, which is related to the stability of non-uniform black strings in a grandcanonical ensemble, is added.

where $f_\pm(r) = 1 - (r_\pm/r)^{d-3}$ and $\mathcal{F}_{d-2} = Q_m \,\mathrm{Vol}_{\Omega_{d-2}}$. Here, $r = r_+$ and $r = r_-$ correspond to an outer and an inner horizons, respectively, and Q_m is proportional to a magnetic charge. Setting $a = b = c = 0$ gives the background uniform black strings. The physical property of the background solution to be noted is that the specific heat, which controls the thermodynamical stability, is negative for small charge ($0 \le Q < Q_{\rm GM}$) and positive for large charge ($Q_{\rm GM} < Q < M$) , where $Q_{\rm GM}$ is a certain critical charge. The Gubser-Mitra conjecture[6] asserts that the GL instability exists iff black objects are (locally) thermodynamically unstable. Indeed, we will see the GL instability does not exist for $Q \ge Q_{\rm GM}$ as a realization of the conjecture.

Now, we expand the metric functions $X(r,z)$ ($X = a, b, c$) around the uniform solution,

$$X(r,z) = \sum_{n=0}^{\infty} \epsilon^n X_n(r) \cos(nKz), \quad X_n(r) = \sum_{p=0}^{\infty} \epsilon^{2p} X_{n,p}(r), \quad K = \sum_{q=0}^{\infty} \epsilon^{2q} k_q,$$

where $X_{0,0}(r) = 0$ is imposed. Here K is the GL critical wavenumber and ϵ is an expansion parameter. Substituting these expansions into the Einstein equations, we obtain ODEs for $X_{n,p}(r)$, to be solved order by order.

3. Results and Discussion

Solving the first order perturbations, the GL critical mode for each given charge Q is obtained. The charge dependence of the critical wavenumber is shown in Fig. 1 (a). One can see that the critical wavenumber vanishes at $Q = Q_{\rm GM}$. In addition, we find that the vanishing of the wavenumber obeys a power law near the GM point, $k_0 \propto |Q - Q_{\rm GM}|^\beta$, irrespective of dimensions. The universal "critical exponent" β is nearly 1/2, which resembles a second-order phase transition. It would be interesting to investigate the universal behavior in this line.

Solving the higher-order perturbations (up to third order), we can compare the entropy S, Helmholtz free energy F and Gibbs free energy G between Uniform and Non-Uniform black strings in suitable ensembles. We denote the differences of these thermodynamical functions by

$$S_{\rm NU}/S_{\rm U} \simeq 1 + \sigma_2 \epsilon^4, \qquad F_{\rm NU}/F_{\rm U} \simeq 1 + \rho_2 \epsilon^4, \qquad G_{\rm NU}/G_{\rm U} \simeq 1 + \kappa_2 \epsilon^4.$$

The charge dependence of the coefficients, $(\sigma_2, \rho_2, \kappa_2)$, are shown in Figs. 1 (b)-(d) for $D = 6, 10, 14$. Here let us focus on the entropy σ_2. For $D \le 13$, σ_2, being negative initially at $Q = 0$, increases as the charge increases, and it becomes positive at some critical charge $Q = Q_{\rm I,cr}$. Increasing the charge furthermore, σ_2 falls off and becomes negative at a second critical charge $Q = Q_{\rm II,cr}$. For $D \ge 14$, since σ_2 is initially positive even at $Q = 0$,[4] the first critical charge is absent, and only the "second" critical charge exists. For $Q_{\rm I,cr} < Q < Q_{\rm II,cr}$, the non-uniform string have a larger entropy than the uniform one, and the transition would be higher order.

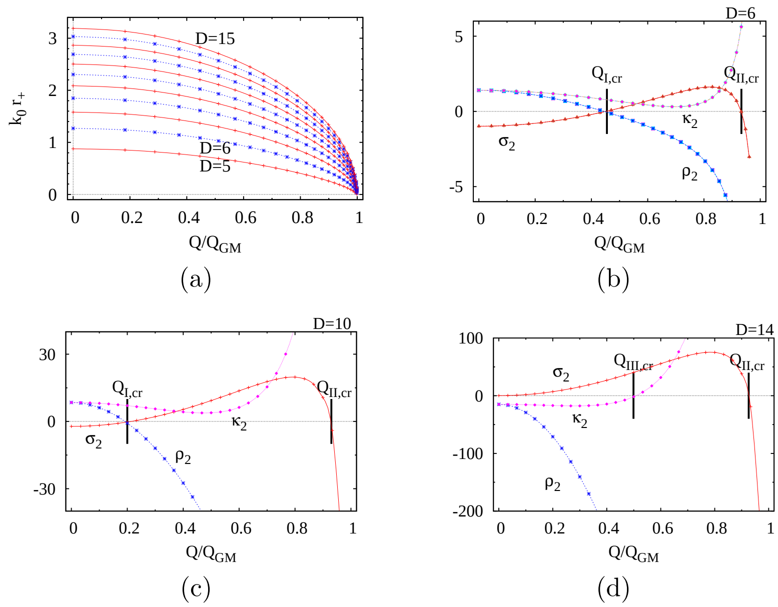

Fig. 1. (a): The charge dependance of the GL critical mode. (b)-(d): The difference of thermodynamical functions between uniform and non-uniform strings for $D = 6$, 10 and 14.

Next, let us focus on the Gibbs free energy. For $5 \leq D \leq 12$, κ_2 is always positive, showing the non-uniform blanch is unstable. While, for $D \geq 13$, κ_2 is negative around $Q = 0$ and monotonically increases to be positive for $Q > Q_{\rm III,cr}$.

The sign change of $(\sigma_2, \rho_2, \kappa_2)$ is an interesting phenomenon. We can say that the charge controls the stability of non-uniform strings as well as that of uniform strings. Fully non-linear construction of non-uniform strings, including dilaton, and the application to the gauge theory via the gauge/gravity dual will be interesting generalization of this work.

References

1. R. Gregory and R. Laflamme, *Phys. Rev. Lett.* **70**, 2837 (1993); G. T. Horowitz and K. Maeda, *Phys. Rev. Lett.* **87**, 131301 (2001); M. W. Choptuik et al., *Phys. Rev. D* **68**, 044001 (2003).
2. S. S. Gubser, *Class. Quant. Grav.* **19**, 4825 (2002); Troels Harmark. *Phys. Rev. D* **69**, 104015 (2004); D. Gorbonos and B. Kol, *J. High Energy Phys.* 06 (2004) 053.
3. T. Wiseman, *Class. Quant. Grav.* **20**, 1137 (2003); H. Kudoh and T. Wiseman, *Phys. Rev. Lett.* **94**, 161102 (2005).
4. E. Sorkin, *Phys. Rev. Lett.* **93**, 031601 (2004); H. Kudoh and U. Miyamoto, *Class. Quant. Grav.* **22**, 3853 (2005).
5. U. Miyamoto and H. Kudoh, *J. High Energy Phys.* 12 (2006) 048.
6. S. S. Gubser and I. Mitra, hep-th/0009126; *J. High Energy Phys.* 08 (2001) 018.