NEW NONUNIFORM BLACK STRING SOLUTIONS *

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We present nonuniform vacuum black strings in five and six spacetime dimensions. We find qualitative agreement of the physical properties of nonuniform black strings in five and six dimensions. Our results offer further evidence that the black hole and the black string branches merge at a topology changing transition. The basic features of Einstein-Maxwell-dilaton black strings can be derived from those of the vacuum black strings after performing a Harrison transformation.

Black strings, present for $D \geq 5$ spacetime dimensions, have horizon topology $S^{D-3} \times S^1$. Uniform black strings possess a translational symmetry along the extracoordinate direction. As shown by Gregory and Laflamme,[1] these solutions are unstable below a critical value of the mass, where at the marginally stable uniform string a branch of nonuniform black strings arises.[2–5]

In addition to black string solutions there are also caged black hole solutions with an event horizon of topology S^{D-2}. The numerical results presented in[6] suggest that, for $D = 6$, the black hole and the nonuniform string branches merge at a topology changing transition.[7] By numerically constructing nonuniform black strings in $D = 5$ dimensions, we here give evidence, that this is also true in $D = 5$.[8]

We consider the Einstein action in a $D-$dimensional spacetime, where the black string solutions approach asymptotically the $D-1$ dimensional Minkowski-space times a circle $\mathcal{M}^{D-1} \times S^1$. The nonuniform black string solutions are found within the metric ansatz

$$ds^2 = -e^{2A(r,z)} f(r) dt^2 + e^{2B(r,z)} \left(\frac{dr^2}{f(r)} + dz^2 \right) + e^{2C(r,z)} r^2 d\Omega^2_{D-3}, \tag{1}$$

where $f = 1-(r_0 r)^{D-4}$. The asymptotic form of the relevant metric components[9,10]

$$g_{tt} \simeq -1 + \frac{c_t}{r^{D-4}}, \quad g_{zz} \simeq 1 + \frac{c_z}{r^{D-4}}, \tag{2}$$

yield mass and tension of the string solutions

$$M = \frac{\Omega_{D-3} L}{16\pi G}((D-3)c_t - c_z), \quad \mathcal{T} = \frac{\Omega_{D-3}}{16\pi G}(c_t - (D-3)c_z), \tag{3}$$

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where Ω_{D-3} is the area of the unit S^{D-3} sphere. The relative tension[11] $n = \mathcal{T}L/M$ is bounded, $0 \leq n \leq D-3$, where uniform string solutions have $n_0 = 1/(D-3)$.

A measure of the deformation of the solutions is given by the nonuniformity parameter λ[2]

$$\lambda = \frac{1}{2}\left(\frac{\mathcal{R}_{\max}}{\mathcal{R}_{\min}} - 1\right), \tag{4}$$

where $\mathcal{R}_{\max}$ and $\mathcal{R}_{\min}$ represent the maximum radius of a $(D-3)$-sphere on the horizon and the minimum radius, being the radius of the 'waist'. Thus for uniform black strings $\lambda = 0$, while the conjectured horizon topology changing transition should be approached for $\lambda \to \infty$.[4,12]

In Figure 1 we show the spatial embedding of the horizon into 3-dimensional space for the $D = 5$ nonuniform black string solutions. In these embeddings the proper radius of the horizon is plotted against the proper length along the compact direction, yielding a geometrical view of the nonuniformity of the solutions.

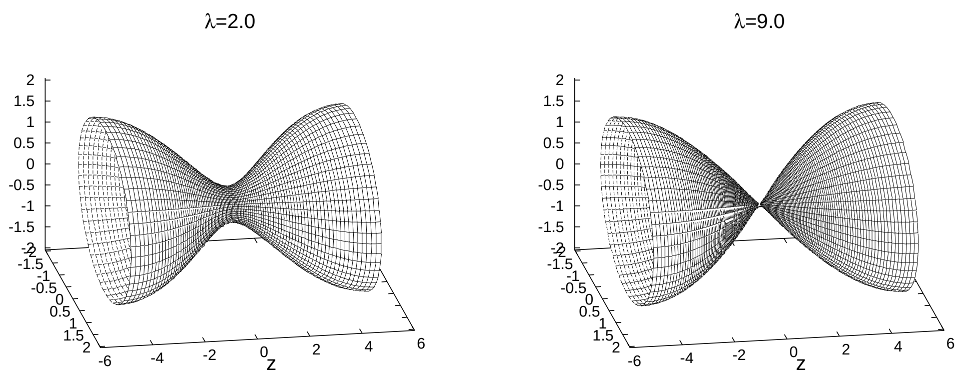

Fig. 1. The spatial embedding of the horizon of $D = 5$ nonuniform black string solutions with horizon coordinate $r_0 = 1$ and asymptotic length of the compact direction $L = L^{\rm crit} = 7.1713$, is shown for two values of the nonuniformity parameter, $\lambda = 2,\ 9$.

We exhibit in Figure 2 the mass of $D = 5$ and $D = 6$ nonuniform strings and black holes. Although we see a backbending of the nonuniform string branch in both $D = 5$ and $D = 6$ dimensions, not observed previously, because the nonuniform string branch had not been continued to sufficiently high deformation, all our data are consistent with the assumption, that the nonuniform string branch and the black hole branch merge at such a topology changing transition. In fact, extrapolation of the black hole branch towards this transition point appears to match well the (extrapolated) endpoint of the (backbending) part of the nonuniform string branch.

For the phase diagram this would mean that we would have a region $0 < n < n_{\rm b}$ with one branch of black hole solutions, then a region $n_{\rm b} < n < n_*$ with one

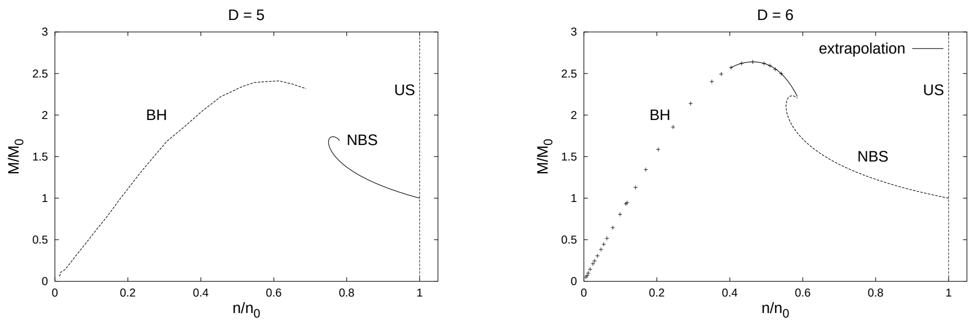

Fig. 2. The mass M of the $D = 5$ (a) and $D = 6$ (b) nonuniform string and black hole branches are shown versus the relative string tension n (in units of the uniform string quantities M_0 and n_0). The black hole data are from.[6] The $D = 6$ black hole branch is extrapolated towards the anticipated critical value n_*.

branch of black hole solutions and two branches of nonuniform string solutions, the ordinary one and the backbending one, and finally a region $n_* < n < n_0$ with only one branch of nonuniform string solutions. Thus the topology changing transition would be associated with n_*, and $n_\mathrm{b} < n < n_*$ would represent a middle region where three phases would coexist, one black hole and two nonuniform strings. This is strongly reminiscent of the phase structure of the rotating black ring–rotating black hole system in $D = 5$.[13]

Black string solutions of the Einstein-Maxwell-dilaton theory can be obtained via a Harrison transformation.[14] The basic features of these solutions can be derived from those of the vacuum black string configurations.[8]

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