PERTURBATIVE STABILITY AND ABSORPTION CROSS SECTION IN STRING CORRECTED BLACK HOLES

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We consider a $d-$dimensional spherically symmetric dilatonic $\mathcal{R}^2$ string corrected black hole solution. We study its stability under tensor type gravitational perturbations and compute the absorption cross section for low frequency gravitational waves.

We are interested in studying the behavior of a string-corrected dilatonic black hole solution under perturbations in d spacetime dimensions,[1] setting any tensorial or fermionic fields to zero and taking as background metric

$$d\,s^2 = -f(r)\,d\,t^2 + f^{-1}(r)\,d\,r^2 + r^2\,d\,\Omega^2_{d-2} \tag{1}$$

with $d\Omega^2_{d-2} = \gamma_{ij}\,(\theta)\;d\theta^i\,d\theta^j$, $\gamma_{ij} = g_{ij}/r^2$ being the metric of a $(d-2)$-sphere S^{d-2}.

We take, in the effective action, only the leading $\mathcal{R}^2$ α' correction:[2]

$$\frac{1}{2\kappa^2}\int\sqrt{-g}\left[\mathcal{R} - \frac{4}{d-2}\left(\partial^\mu\phi\right)\partial_\mu\phi + \mathrm{e}^{\frac{4}{2-d}\phi}\frac{\lambda}{2}\mathcal{R}^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}\right]\mathrm{d}^dx, \tag{2}$$

with $\lambda = \frac{\alpha'}{2}, \frac{\alpha'}{4}, 0$ for bosonic, heterotic and superstrings, respectively.

The corrected field equation for the graviton is, to this order,

$$\mathcal{R}_{\mu\nu} + \lambda\mathrm{e}^{\frac{4}{2-d}\phi}\left(\mathcal{R}_{\mu\rho\sigma\tau}\mathcal{R}_\nu^{\ \rho\sigma\tau} - \frac{1}{2(d-2)}g_{\mu\nu}\mathcal{R}_{\rho\sigma\lambda\tau}\mathcal{R}^{\rho\sigma\lambda\tau}\right) = 0. \tag{3}$$

Here we only take tensorial perturbations to the metric (by using the dilaton field equation, we show that we can set $\delta\phi = 0$), given by $h_{\mu\nu} = \delta g_{\mu\nu}$:[3]

$$h_{ij} = 2r^2 H_T(r)\mathcal{T}_{ij}\left(\theta^i\right),\; h_{ir} = h_{it} = 0, h_{rr} = h_{tr} = h_{tt} = 0. \tag{4}$$

D_i is the S^{d-2} covariant derivative; $\mathcal{T}_{ij}$ are the eigentensors of the S^{d-2} laplacian, with eigenvalues $-k_T = 2 - \ell\left(\ell + d - 3\right), \ell = 2, 3, 4\ldots$, satisfying

$$\left(\gamma^{kl}D_kD_l + k_T\right)\mathcal{T}_{ij} = 0,\; D^i\mathcal{T}_{ij} = 0,\; g^{ij}\mathcal{T}_{ij} = 0. \tag{5}$$

Using the explicit form of the Riemann tensor for the metric (1) and its variations, computed from (4), and perturbing (3), we determine the equation for H_T, which we write in the form of a master equation (with r_* defined by $dr_*/dr = 1/f$)

$$\frac{\partial^2\Phi}{\partial r_*^2} - \frac{\partial^2\Phi}{\partial t^2} =: V_T\Phi, \tag{6}$$

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As explained in ref.4, we derive our master function and potential:

$$\Phi = \frac{H_T}{\sqrt{f}} \exp\left(\int \frac{\frac{f'}{f} + \frac{d-2}{r} + \frac{4}{r^3}(d-4)\lambda(1-f) - \frac{4}{r^2}\lambda f' - \frac{2}{rf}\lambda f'^2}{2 - \frac{4}{r}\lambda f'} dr \right),$$

$$\begin{aligned} V_{\mathsf{T}}[f(r)] = & \left(\frac{1}{1-2\lambda\frac{f'}{r}}\right)^2 \left(1 + \frac{4\lambda}{r^2}(1-f)\right) \left[\frac{d-4}{4r^2}\left(1+\frac{4\lambda}{r^2}(1-f)\right) + \frac{2\lambda f''-1}{2r^2}\right] \\ & + \frac{1}{1-2\lambda\frac{f'}{r}} \left[(k_T+2)\frac{f}{r^2} + 2(d-3)\frac{f(1-f)}{r^2} + \frac{d-8}{2}\frac{ff'}{r} - \frac{\lambda}{d-2} f\,(f'')^2 \right. \\ & \left. + 4\lambda\,(k_T+2)\frac{f(1-f)}{r^4} + 2(d-3)\lambda\frac{f(1-f)^2}{r^4} + 2(d-4)\lambda\frac{f(1-f)f'}{r^3} \right] \\ & + \frac{ff'}{r} + (d-4)\frac{f^2}{r^2}. \end{aligned} \tag{7}$$

To study the stability of a solution, we use the "S-deformation approach".[3] After having obtained the potential $V_T(f)$, we assume that its solutions are of the form $\Phi(r_*, t) = e^{i\omega t}\phi(r_*)$, such that $\partial\Phi/\partial t = i\omega\Phi$. The master equation is then written in the Schrödinger form $A\Phi = \omega^2\Phi$, and a solution to the field equation is then stable if the operator A is positive definite with respect to the following inner product:

$$\langle\phi, A\phi\rangle = \int_{-\infty}^{+\infty} \overline{\phi}(r_*) \left[-\frac{d^2}{dr_*^2} + V\right] \phi(r_*)\, dr_* = \int_{-\infty}^{+\infty} \left(|D\phi|^2 + \frac{Q}{f}|\phi|^2\right) dr_* \tag{8}$$

(see ref. 4 for the details), with $D = \frac{d}{dr_*} - \frac{fH_T}{\Phi}\frac{d}{dr}\left(\frac{\Phi}{H_T}\right)$ and

$$\begin{aligned} \frac{Q}{f} = & \frac{1}{1-2\lambda\frac{f'}{r}}\frac{1}{r^2} \left[(k_T+2)\left(1+\frac{4\lambda}{r^2}(1-f)\right) + (2d-6)(1-f)\left(1+\frac{\lambda}{r^2}(1-f)\right) \right. \\ & \left. - 2rf' - \frac{\lambda}{d-2}(f'')^2 r^2 \right]. \end{aligned} \tag{9}$$

All that is necessary to guarantee the stability is to check the positivity of $\frac{Q}{f}$.

In ref. 4 we considered the $\mathcal{R}^2$-corrected black hole solution of the type of (1) studied in ref. 2, taking a coordinate system in which the horizon radius R_H is not changed. Assuming $R_H \gg \sqrt{\lambda}$, for this solution $f(r)$ is given by

$$f(r) = \left(1 - \left(\frac{R_H}{r}\right)^{d-3}\right)\left[1 - \lambda\frac{(d-3)(d-4)}{2}\frac{R_H^{d-5}}{r^{d-1}}\frac{r^{d-1}-R_H^{d-1}}{r^{d-3}-R_H^{d-3}}\right]. \tag{10}$$

We showed[4] that $\frac{Q}{f} > 0$; therefore this solution is stable under tensor perturbations.

In Einstein-Hilbert gravity, for any spherically symmetric black hole in arbitrary dimension the absorption cross section of minimally coupled massless scalar fields equals the area of the black hole horizon,[5] a result which suggests a universality of the low-frequency absorption cross sections of generic black holes. Since the equation describing gravitational perturbations to a black hole solution allows for a study of scattering in this spacetime geometry, we tried to extend this result to the higher-derivative corrected black hole (10), focusing only on the leading contribution to the scattering process: the s-wave, with $\ell = 0$. The low-frequency regime $R_H \omega \ll 1$ allows us to fully analytically solve the problem by using the technique of matching solutions near the event horizon to solutions at asymptotic infinity. In both these regions the potential $V_{\mathsf{T}}[f(r)]$ vanishes, and the master equation reduces to a free-field equation whose solutions are plane waves in the tortoise coordinate.

Near the event horizon, $r \simeq R_H$, since we are computing the absorption cross section, we shall consider the general solution for an incoming plane wave $H_T(r_*) = A_{\rm near} e^{i\omega r_*}$; after expanding $V_{\mathsf{T}}(r)$ and $r_*(r)$ this solution becomes

$$H_T(r) \simeq A_{\rm near} \left(1 + i \frac{R_H \omega}{d-3} \left(1 + \frac{(d-1)(d-4)}{2} \frac{\lambda}{R_H^2} \right) \log \left(\frac{r - R_H}{R_H} \right) \right). \quad (11)$$

Close to infinity, one must consider a superposition of incoming and outgoing waves which becomes, expressed in the original radial coordinate, $H_T(r) = (r\omega)^{(3-d)/2} \left[A_{\rm as} \, J_{(d-3)/2}(r\omega) + B_{\rm as} \, N_{(d-3)/2}(r\omega) \right]$; at low frequencies, $r\omega \ll 1$,

$$H_T(r) \simeq A_{\rm as} \frac{1}{2^{\frac{d-3}{2}} \Gamma\left(\frac{d-1}{2}\right)} + B_{\rm as} \frac{2^{\frac{d-3}{2}} \Gamma\left(\frac{d-3}{2}\right)}{\pi \, (r\omega)^{d-3}} + \mathcal{O}(r\omega). \quad (12)$$

In order to compute the absorption cross section, one needs the fluxes per unit area $J = \frac{1}{2i} \left(H_T^\dagger(r_*) \frac{dH_T}{dr_*} - H_T(r_*) \frac{dH_T^\dagger}{dr_*} \right)$. Near the horizon this quantity is given by $J_{\rm near} = \omega \left| A_{\rm near} \right|^2$; close to infinity we analogously have $J_{\rm as} = \frac{2}{\pi} r^{2-d} \omega^{3-d} \left| A_{\rm as} B_{\rm as} \right|$. In order to match the coefficients $A_{\rm near}$, $A_{\rm as}$ and $B_{\rm as}$, one needs to interpolate between the solutions near the event horizon and at asymptotic infinity. This requires solving the master equation in the intermediate region between the horizon and infinity. The full computation can be found in ref. 4, where it is shown that

$$\begin{aligned} A_{\rm as} &= 2^{\frac{d-3}{2}} \Gamma\left(\frac{d-1}{2}\right) A_{\rm near}, \\ B_{\rm as} &= -\frac{i\pi \, (R_H \omega)^{d-2}}{2^{\frac{d-3}{2}} (d-3) \Gamma\left(\frac{d-3}{2}\right)} \left(1 + \frac{(d-1)(d-4)}{2} \frac{\lambda}{R_H^2} \right) A_{\rm near}. \end{aligned} \quad (13)$$

With these results one obtains the scattering cross section

$$\sigma_T^{\ell=0} = \frac{\int r^{d-2} J_{\rm as} d\Omega_{d-2}}{J_{\rm near}} = A_H \left(1 + \frac{(d-1)(d-4)}{2} \frac{\lambda}{R_H^2} \right). \quad (14)$$

We conclude that the absorption cross section is increased due to the α' corrections, although it is still proportional to the area of the event horizon. The same happens to the black hole entropy, although its α' correction has a different value.[2,4]

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