Higher Dimensional Rotating Charged Black Holes

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We discuss a new analytic solution to the Einstein-Maxwell field equations that describes electrically charged black holes with a slow rotation and with a single angular momentum in all higher dimensions. We also compute the gyromagnetic ratio of these black holes.

1. Introduction

Black holes remain to be one of the most intriguing and puzzling object of study in higher dimensional spacetimes. It is widely believed that the remarkable features of black holes in four dimensions, such as the equilibrium and uniqueness properties as well as quantum properties of evaporation of mini-black holes might have played a profound role in understanding the nature of fundamental theories in higher dimensions. Therefore, of particular interest is the study of black hole solutions in higher-dimensional gravity theories as well as in string/M-theory.

The first higher dimensional solutions for static black holes with the spherical topology of the horizon have been discussed by Tangherlini.[1] These solutions generalize the spherically symmetric Schwarzschild and Reissner-Nordstrom solutions of four-dimensional general relativity. For the static black holes the uniqueness and the stability properties still survive[2] in higher dimensions, however the situation is different for rotating black holes. The exact solution for the rotating black holes was found by Myers and Perry.[3] The solution is not unique, unlike its four dimensional counterpart, the Kerr solution. There exists a rotating black ring solution[4] in five dimensions with the horizon topology of $S^2 \times S^1$ which may have the same mass and angular momentum as the Myers-Perry solution. However, the counterparts of the Myers-Perry black holes in higher dimensional Einstein-Maxwell theory have not been found yet. A numerical treatment of the problem for some special cases in five dimensions was given in Ref.[5] Here, as a first step towards the desired exact metric, we shall discuss the intermediate case of *slow rotation* and give a new analytical solution that describes electrically charged black holes with a single angular momentum in all higher dimensions.

2. Metric ansatz

The strategy of obtaining the familiar Kerr-Newman solution in general relativity is based on either using the metric ansatz in the Kerr-Schild form or applying the method of complex coordinate transformation to a non-rotating charged black hole. When employing in $N + 1$ dimensional spacetime both approaches lead us to the

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following metric ansatz

$$ds^2 = -\left(1 - \frac{m}{r^{N-4}\,\Sigma} + \frac{q^2}{r^{2(N-3)}\,\Sigma}\right) dt^2 - \frac{2a\left(mr^{N-2} - q^2\right)\sin^2\theta}{r^{2(N-3)}\,\Sigma}\, dt\, d\phi$$
$$+ \left(r^2 + a^2 + \frac{a^2\left(mr^{N-2} - q^2\right)\sin^2\theta}{r^{2(N-3)}\,\Sigma}\right)\sin^2\theta\, d\phi^2 + \frac{r^{\,N-2}\,\Sigma}{\Delta}\, dr^2$$
$$+\Sigma\, d\theta^2 + r^2\cos^2\theta d\Omega^2_{N-3}\ , \tag{1}$$

where the parameters m , a and q are related to the mass, angular momentum and electric charge of the black hole. The metric function

$$\Delta = r^{N-2}(r^2 + a^2) - m\, r^2 + q^2\, r^{4-N} \tag{2}$$

and $d\Omega^2_{N-3}$ is the metric on a unit $(N-3)$-sphere.

It is straightforward to verify that the source-free Maxwell equations in the background of the metric (1) admit the potential one-form field[6]

$$A = -\frac{Q}{(N-2)\, r^{N-4}\,\Sigma}\left(dt - a\sin^2\theta\, d\phi\right)\ . \tag{3}$$

where Q is the electric charge of the black hole. For $N = 3$, inspecting the simultaneous system of the Einstein-Maxwell equations with this potential form and with the metric (1) shows that it is satisfied for $q^2 = GQ^2$. This is the case of a Kerr-Newman black hole in four dimensions. However, for $N \geq 4$, the consistent solution to the system of equations is obtained only when we restrict ourselves to a slow rotation.

3. Results

The metric for slowly rotating and charged black holes with a single angular momentum in all higher dimensions has the form

$$ds^2 = -\left(1 - \frac{m}{r^{N-2}} + \frac{q^2}{r^{2(N-2)}}\right) dt^2 + \left(1 - \frac{m}{r^{N-2}} + \frac{q^2}{r^{2(N-2)}}\right)^{-1} dr^2$$
$$-\frac{2\,a\sin^2\theta}{r^{N-2}}\left(m - \frac{q^2}{r^{N-2}}\right) dt\, d\phi + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2 + \cos^2\theta d\Omega^2_{N-3}\right) \tag{4}$$

where the parameter q is given by

$$q = \pm Q\left[\frac{8\pi G}{(N-2)\,(N-1)\, A_{N-1}}\right]^{1/2} \tag{5}$$

and A_{N-1} is the area of a unit $(N-1)$-sphere. The associated electromagnetic field is described by the potential one-form

$$A = -\frac{Q}{(N-2)\, r^{N-2}}\left(dt - a\sin^2\theta\, d\phi\right)\ . \tag{6}$$

The metric (4) generalizes the higher dimensional Schwarzschild-Tangherlini[1] solution to include an arbitrarily small angular momentum of the black holes. For $N=4$ we have the metric in five dimensions.[7]

It is clear that a rotating charged black hole must also have a magnetic dipole moment. In our case, it is determined from the far distant behaviour of the magnetic field in the metric (4). We find that the associated magnetic $(N-2)$-form field has the following orthonormal components[6]

$$B_{\hat{r}\,\hat{\chi}_1\hat{\chi}_2\ldots\hat{\chi}_{N-3}}=\frac{2\,Qa}{N-2}\,\frac{\cos\theta}{r^N}\,,\qquad B_{\hat{\theta}\,\hat{\chi}_1\hat{\chi}_2\ldots\hat{\chi}_{N-3}}=\frac{Qa\sin\theta}{r^N} \tag{7}$$

which give the value of the magnetic dipole moment $\mu=Qa$. From the asymptotic behavior of the metric (4) we find the specific angular momentum $j=am$ of the black hole. The latter allows us to rewrite the magnetic dipole moment in terms of total mass and total angular momentum of the black hole as follows

$$\mu=\frac{Q\,j}{m}=(N-1)\,\frac{Q\,J}{2\,M}\,. \tag{8}$$

This expression shows that a slowly rotating charged black hole in $N+1$ dimensions must have the gyromagnetic ratio

$$g=N-1\,. \tag{9}$$

The value of the gyromagnetic ratio agrees with that obtained earlier[8] for the Myers-Perry black hole with a test (small) electric charge in five dimensions. It also agrees with the numerical analysis of paper.[9]

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