# KALUZA-KLEIN BLACK HOLE WITH GRAVITATIONAL CHARGE IN EINSTEIN-GAUSS-BONNET GRAVITY

HIDEKI MAEDA

*Graduate School of Science and Engineering, Waseda University, Tokyo 169-8555, Japan,*
*Department of Physics, Rikkyo University, Tokyo 171-8501, Japan,*
*and*
*Department of Physics, International Christian University, Tokyo 181-8585, Japan*
*hideki@gravity.phys.waseda.ac.jp*

NARESH DADHICH

*Inter-University Centre for Astronomy & Astrophysics, Post Bag 4, Pune 411 007, India*
*nkd@iucaa.ernet.in*

We obtain a new exact black-hole solution in Einstein-Gauss-Bonnet gravity with a cosmological constant which bears a specific relation to the Gauss-Bonnet coupling constant. The spacetime is a product of the usual 4-dimensional manifold with a $(n-4)$-dimensional space of constant negative curvature, i.e., its topology is locally $M^n \approx M^4 \times H^{n-4}$. The solution has two parameters and asymptotically approximates to the field of a charged black hole in anti-de Sitter spacetime. The most interesting and remarkable feature is that the Gauss-Bonnet term acts like a Maxwell source for large $r$ while at the other end it regularizes the metric and weakens the central singularity. It is a pure gravitational creation including Maxwell field in four-dimensional vacuum spacetime. This paper is based on the results in Ref. 1.

## 1. Model and basic equation

We write action of Einstein-Gauss-Bonnet gravity with a cosmological constant for $n \geq 5$,

$$S = \int d^n x \sqrt{-g}\biggl[\frac{1}{2\kappa_n^2}(R - 2\Lambda + \alpha L_{GB})\biggr], \tag{1}$$

where $\alpha$ is the Gauss-Bonnet (GB) coupling constant and all other symbols having their usual meaning. The GB Lagrangian is given by

$$L_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}, \tag{2}$$

where the Greek indices run $\mu = 0, 1, \cdots, n-1$. This form of action follows from low-energy limit of heterotic superstring theory.[2] In that case, $\alpha$ is identified with the inverse string tension and is positive definite, so we assume $\alpha \geq 0$ in this paper. It should be noted that $L_{GB}$ makes no contribution in the field equations for $n \leq 4$.

The gravitational equation following from the action (1) is given by

$$\mathcal{G}^\mu{}_\nu \equiv G^\mu{}_\nu + \alpha H^\mu{}_\nu + \Lambda\delta^\mu{}_\nu = 0, \tag{3}$$

where

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R, \tag{4}$$

$$H_{\mu\nu} \equiv 2\Bigl[RR_{\mu\nu} - 2R_{\mu\alpha}R^\alpha{}_\nu - 2R^{\alpha\beta}R_{\mu\alpha\nu\beta} + R_\mu{}^{\alpha\beta\gamma}R_{\nu\alpha\beta\gamma}\Bigr] - \frac{1}{2}g_{\mu\nu}L_{GB}. \tag{5}$$

We consider the $n$-dimensional spacetime locally homeomorphic to $M^4 \times K^{n-4}$ with the metric, $g_{\mu\nu} = \mathrm{diag}(g_{AB}, r_0^2\gamma_{ab})$, $A, B = 0, \cdots, 3$; $a, b = 4, \cdots, n-1$. Here $g_{AB}$ is an arbitrary Lorentz metric on $M^4$, $r_0$ is a constant and $\gamma_{ab}$ is the unit metric on the $(n-4)$-dimensional space of constant curvature $K^{n-4}$ with its curvature $\bar{k} = \pm 1, 0$. Then, we obtain the following no-go theorem on $M^4$:

**Theorem 1.1.** *If (i)* $r_0^2 = -2\bar{k}\alpha(n-4)(n-5)$ *and (ii)* $\alpha\Lambda = -(n^2-5n-2)/[8(n-4)(n-5)]$*, then* ${\mathcal{G}^A}_B = 0$ *for* $n \geq 6$ *and* $\bar{k}$ *and* $\Lambda$ *being non-zero.*

The conditions (i) and (ii) imply for $\alpha > 0$, $\bar{k} = -1$ and $\Lambda < 0$. Hereafter we set $\bar{k} = -1$, i.e., the local topology of the extra dimensions is $\mathcal{H}^{n-4}$, and obtain the vacuum solution satisfying the conditions (i) and (ii). The governing equation for $g_{AB}$ is then a single scalar equation on $M^4$, ${\mathcal{G}^a}_b = 0$, which is given by

$$\frac{1}{n-4}{}^{(4)}R + \frac{\alpha}{2}{}^{(4)}L_{GB} + \frac{2n-11}{\alpha(n-4)^2(n-5)} = 0, \tag{6}$$

where superscript (4) means the geometrical quantity on $M^4$.

## 2. Kaluza-Klein black-hole solution

We seek a static solution with the metric on $M^4 \approx M^2 \times K^2$ reading as:

$$g_{AB}dx^A dx^B = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Sigma_{2(k)}^2, \tag{7}$$

where $d\Sigma_{2(k)}^2$ is the unit metric on $K^2$ and $k = \pm 1, 0$. Then, Eq. (6) yields the general solution for the function $f(r)$:

$$f(r) = k + \frac{r^2}{2(n-4)\alpha}\left[1 \mp \sqrt{1 - \frac{2n-11}{3(n-5)} + \frac{4(n-4)^2\alpha^{3/2}\mu}{r^3} - \frac{4(n-4)^2\alpha^2 q}{r^4}}\right], \tag{8}$$

where $\mu$ and $q$ are arbitrary dimensionless constants. The solution does not have the general relativistic limit $\alpha \to 0$. There are two branches of the solution indicated by sign in front of the square root in Eq. (8), which we call the minus- and plus-branches.

There exists a central curvature singularity at $r = 0$ as well as the branch singularity at $r = r_{\mathrm{b}} > 0$ where the term inside the square root in Eq. (8) is zero. This solution can represent a black hole depending on the parameters. The $n$-dimensional black hole with $(n-4)$-dimensional compact extra dimensions is called the Kaluza-Klein black hole. The warp-factor of the submanifold $r_0^2$ is proportional to GB parameter $\alpha$ which is supposed to be very small. Thus, compactifying $H^{n-4}$ by appropriate identifications, we obtain the Kaluza-Klein black-hole spacetime with small and compact extra dimensions.

The function $f(r)$ is expanded for $r \to \infty$ as

$$f(r) \approx k \mp \frac{\alpha^{1/2}\mu\sqrt{3(n-4)(n-5)}}{r} \pm \frac{\alpha q\sqrt{3(n-4)(n-5)}}{r^2} + \frac{r^2}{2(n-4)\alpha}\left(1 \mp \sqrt{\frac{n-4}{3(n-5)}}\right). \tag{9}$$

This is the same as the Reissner-Nordström-anti-de Sitter (AdS) spacetime for $k = 1$ in spite of the absence of the Maxwell field. This suggests that $\mu$ is the mass of the central object and $q$ is the charge-like new parameter. Further, the solution (8) agrees with the solution in the Einstein-GB-Maxwell-$\Lambda$ system having the topology of $M^n \approx M^2 \times K^{n-2}$ although it does not admit $n = 4$.[3] The new "gravitational charge" $q$ is generated by our choice of the topology of spacetime, splitting it into a product of the usual 4-spacetime and a space of constant curvature. Thus, the solution (8) manifests gravitational creation of the Maxwell field, i.e., "matter without matter".

It is noted that $f(0) = k \mp \sqrt{-q}$, which produces a solid angle deficit and it represents a spacetime of global monopole. This means that at $r = 0$ curvatures will diverge only as $1/r^2$ and so would be density which on integration over volume will go as $r$ and would therefore vanish. This indicates that singularity is weak as curvatures do not diverge strongly enough.

## 3. Summary

We have thus found a new Kaluza-Klein vacuum black hole solution (8) of Einstein-Gauss-Bonnet gravity with topology of product of the usual 4-spacetime with a negative constant curvature space. In this solution we have brought the GB effects down on four dimensional black hole. This solution manifests gravitational creation of the Maxwell field and asymptotically resembles a charged black hole in AdS background. What really happens is that GB term regularizes the metric and weakens the singularity while the presence of extra dimensional hyperboloid space generates the Kaluza-Klein modes giving rise to the Weyl charge. This class exact solutions is further investigated in Ref. 4.

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