CHARGED ROTATING BLACK HOLES IN HIGHER DIMENSIONS*

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We consider charged rotating black holes in higher odd dimensions in theories such as Einstein-Maxwell, Einstein-Maxwell-dilaton, and Einstein-Maxwell-Chern-Simons. Retricted to stationary axisymmetric black holes with equal-magnitude angular momenta and a horizon of spherical topology, we present an Ansatz for the metric and the matter fields where the angular dependence is explicitly given and all the unknowns are functions of the radial coordinate only. As a consequence, the field equations reduce to ordinary differential equations. These black holes resemble, in general, their uncharged counterparts, the Myers-Perry black holes. However, when the Chern-Simons term is present new surprising effects appear, such as counterrotation, negative horizon mass, or violation of uniqueness.

Higher dimensional black holes received much interest in recent years, in particular in the context of string theory, and with the advent of brane-world theories, raising the possibility of direct observation in future high energy colliders.[1]

Static charged asymptotically flat black hole solutions of Einstein-Maxwell (EM) theory exist for all spacetime dimensions $D \geq 4$.[2,3] The generalization of the Kerr metric to higher dimensions was obtained by Myers and Perry,[3] while the higher dimensional generalization of the Kerr-Newman metric is still not known analytically.

In contrast to pure EM theory, exact solutions of higher dimensional charged rotating black holes are known in theories with more symmetries or fields. For instance, if a dilaton field is included, with the Kaluza-Klein value for the coupling constant, one may produce black hole solutions to the corresponding Einstein-Maxwell-dilaton (EMD) field equations by using the Myers and Perry solutions as seeds.[4]

Another example would be the presence of a Chern-Simons (CS) term. Surprisingly, for $D = 5$ the addition of that term allows to solve the Einstein-Maxwell-Chern-Simons (EMCS) field equations exactly in the special case when the CS coupling constant is set to the supergravity value,[5] and analytic solutions describing charged, rotating black holes are known.[6–8]

For arbitrary values of the coupling constants of these theories (EM, EMD, EMCS) no analytic solution is known. The numerical approach is the only possibility then to study these black hole configurations.[9,10]

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We here concentrate on charged stationary axisymmetric black holes with equal-magnitude angular momenta and a horizon of spherical topology in odd dimensional EM, EMD, and EMCS theories. Under these assumptions it is possible to give a general Ansatz for the metric and the matter fields, involving only unknown functions of the radial coordinate.

Denoting the dimension of the spacetime by $D(= 2N + 1)$, the metric for these equal-magnitude angular momenta black holes reads

$$\begin{aligned} ds^2 = & -f dt^2 + \frac{m}{f}\left[dr^2 + r^2 \sum_{i=1}^{N-1}\left(\prod_{j=0}^{i-1}\cos^2\theta_j\right) d\theta_i^2\right] \\ & + \frac{n}{f} r^2 \sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k\left(\varepsilon_k d\varphi_k - \frac{\omega}{r}dt\right)^2 \\ & + \frac{m-n}{f} r^2 \left\{\sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k d\varphi_k^2 - \left[\sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k\varepsilon_k d\varphi_k\right]^2\right\} , \end{aligned} \tag{1}$$

where $\theta_0 \equiv 0$, $\theta_i \in [0, \pi/2]$ for $i = 1, \ldots, N-1$, $\theta_N \equiv \pi/2$, $\varphi_k \in [0, 2\pi]$ for $k = 1, \ldots, N$, and $\varepsilon_k = \pm 1$ denotes the sense of rotation in the k-th orthogonal plane of rotation. An adequate parametrization for the gauge potential is given by

$$A_\mu dx^\mu = a_0 dt + a_\varphi \sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k\varepsilon_k d\varphi_k \ . \tag{2}$$

The dilaton field ϕ is a function of r only, like all the other unknown functions f, m, n, ω, a_0, and a_φ.

To obtain asymptotically flat solutions, the unknowns have to satisfy $f|_{r=\infty} = m|_{r=\infty} = n|_{r=\infty} = 1$, $\omega|_{r=\infty} = 0$, $a_0|_{r=\infty} = a_\varphi|_{r=\infty} = 0$, $\phi|_{r=\infty} = 0$. The horizon is located at $r_{\rm H}$, and is characterized by the condition $f(r_{\rm H}) = 0$.[9,10] Requiring the horizon to be regular, the following boundary conditions must hold $f|_{r=r_{\rm H}} = m|_{r=r_{\rm H}} = n|_{r=r_{\rm H}} = 0$, $\omega|_{r=r_{\rm H}} = r_{\rm H}\Omega$, $\Phi_{\rm H} = (a_0 + \Omega a_\varphi)|_{r=r_{\rm H}}$, $da_\varphi/dr|_{r=r_{\rm H}} = 0$, $d\phi/dr|_{r=r_{\rm H}} = 0$, where Ω is (related to) the horizon angular velocity and $\Phi_{\rm H}$ is the horizon electrostatic potential.

Subject to the above set of boundary conditions, the system of ordinary differential equations resulting from the substitution of the Ansätze in the field equations possesses regular black hole solutions in EM, EMD, and EMCS theories. In general, those solutions share most of the properties of their uncharged counterparts, namely, the Myers-Perry solutions.[3] However, in EMCS theory new surprising phenomena appear for certain parameter ranges, to which we now turn.

To describe these new features we focus on $D = 5$ EMCS black holes, denoting the CS coupling constant by λ. For an appropriate normalization, $\lambda = 1$ corresponds to the supergravity value. Depending on the value of λ, new types of black holes appear, which can be classified by their total angular momentum J and horizon angular velocity Ω. Black holes with $\lambda \geq 1$ may possess a static horizon, while their

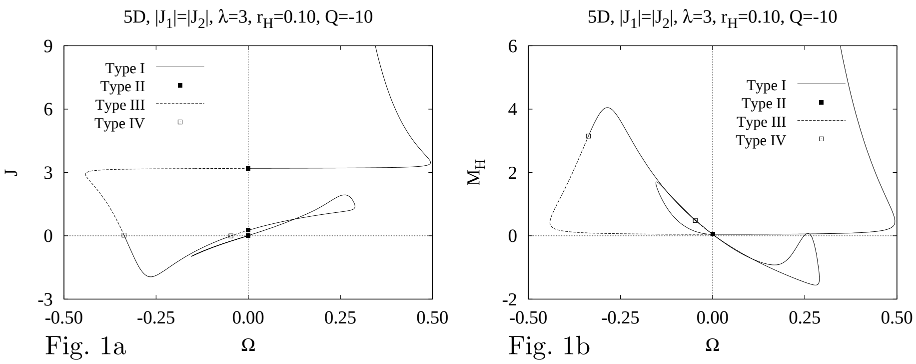

Fig. 1 Properties of $5D$ non-extremal $\lambda = 3$ EMCS black holes with charge $Q = -10$ and horizon radius $r_{\rm H} = 0.1$. a) Angular momentum J, b) horizon mass $M_{\rm H}$ versus horizon angular velocity Ω.

total angular momentum is non-vanishing (type II). When $\lambda > 1$, counterrotating configurations appear, for which the horizon angular velocity and the total angular momentum have opposite signs, $\Omega J < 0$ (type III). When $\lambda \geq 2$, black holes with rotating horizon but vanishing total angular momentum appear (type IV). For $\lambda \geq 2$ thus four types of rotating black holes are present (Fig. 1a).

In addition, EMCS black holes with negative horizon mass (Fig. 1b) arise, their total mass remaining always positive. Moreover, when $\lambda > 2$ EMCS black holes are not completely characterized by their global charges, giving rise to a violation of uniqueness, and their gyromagnetic ratio may take any real value, including zero. Further details of these intriguing new features may be found in Ref. [10].

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