LG (Landau-Ginzburg) in GL (Gregory-Laflamme)

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Outline:

- Gregory-Laflamme instability of black strings
- Phase transition and its order, critical dimension
- Landau-Ginzburg theory of phase transitions
- Application to black string . T^1
- Arbitrary torus compactification T^p

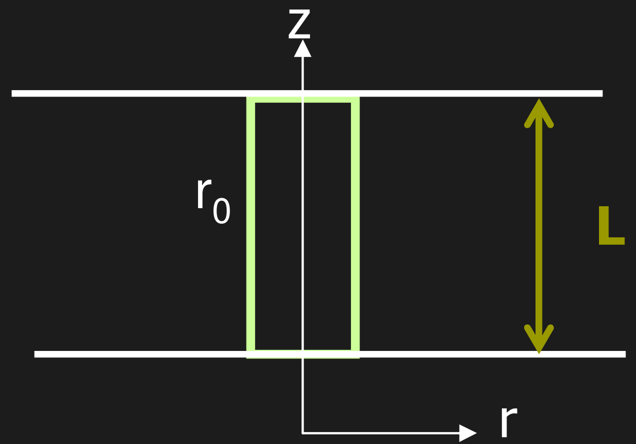
  The critical dimension depends only on number of extended dimensions

Gregory-Laflamme instability of uniform black strings

Spacetime topology - cylinder $R^{D-2,1} \times S^1$ **D=d+1**

Uniform B-Str (UBS):
wrapped along compact z-direction

$\mathrm{Schw}_d + dz^2$



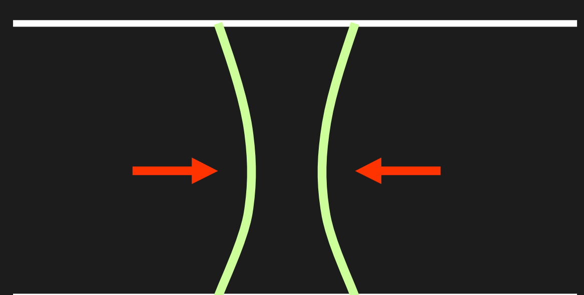
A single dimensionless control-parameter

$$\mu \equiv \frac{G_N m}{L^{D-3}}$$

for

$$\mu < \mu_{GL}$$

Classical growing mode – Gregory&Laflamme Instability (1993)

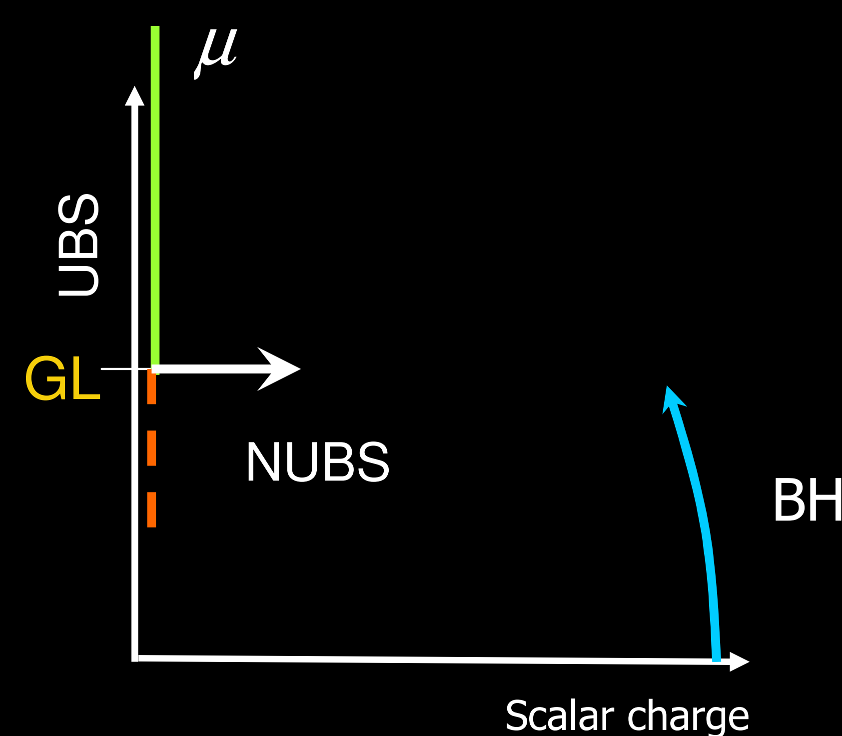


Non-uniform B-Str (NUBS)

$\mu = \mu_{GL}$ marginally tachyonic mode

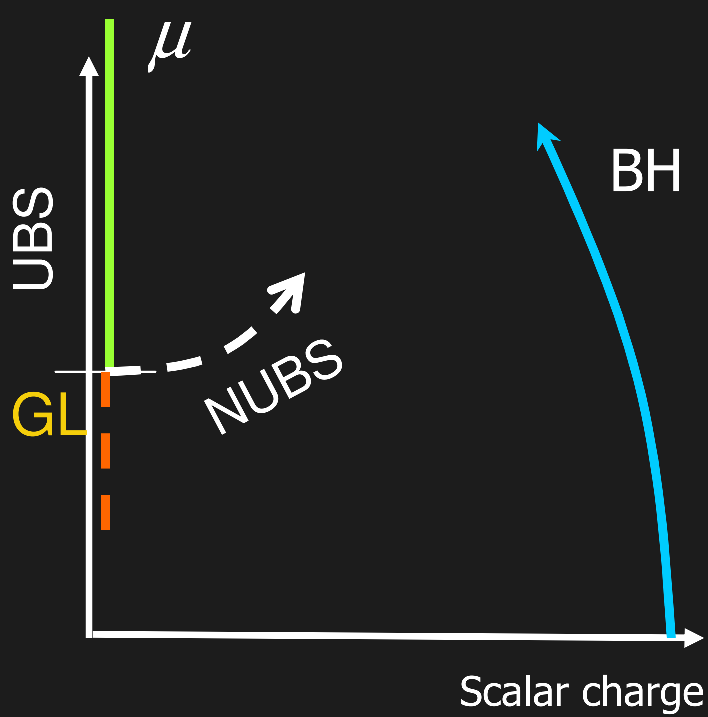
The instability indicates a phase transition. One may wish to construct the branch of solutions emerging from the GL point and to find the order of the phase transition.

A phase diagram [Kol ‘02]



Gubser ’01 developed a method to follow the zero mode and to construct the NUBS branch perturbatively around the critical string. Going to the 3rd order is necessary in order to compute the thermodynamics.

D<13

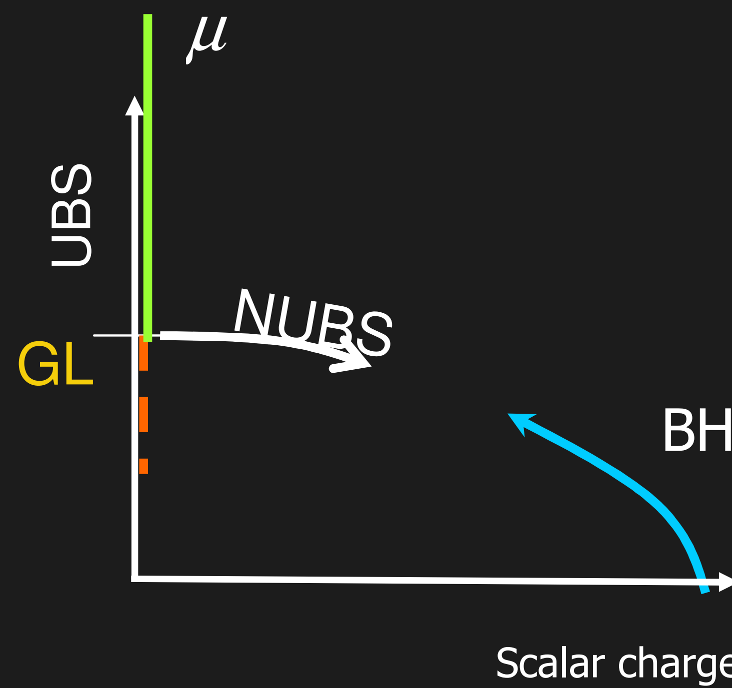


$\mu_{NUBS} > \mu_{UBS}$

$S_{NUBS}(\mu) < S_{UBS}(\mu)$

1st order phase transition

D>13



$\mu_{NUBS} < \mu_{UBS}$

$S_{NUBS}(\mu) > S_{UBS}(\mu)$

2nd order phase transition

ES '04

Next we wish to discuss the phase transition from the Ginzburg-Landau perspective for thermodynamics.

Ginzburg-Landau theory of phase transitions

Consider an expansion of the free energy F around the critical temperature T_c in powers of an order parameter λ

$$F(T,\lambda) = F_0(T) + (T - T_c)A\,\lambda^2 + B(T_c)\,\lambda^3 + C(T_c)\,\lambda^4 + \ldots$$

We assume existence of a marginally unstable mode at T_c;
A>0 since for $T>T_c$ the free energy has as minimum

However in some cases symmetries set B=0 identically;
In our case this will be the parity $\lambda \rightarrow -\lambda$ symmetry, such that
$F(\lambda) = F(-\lambda) \Rightarrow F(\lambda^2)$

$$F(T,\lambda) = F_0(T) + A(T-T_c)\lambda^2 + \cancel{B(T-T_c)\lambda^3} + C(T-T_c)\lambda^4 + \dots$$

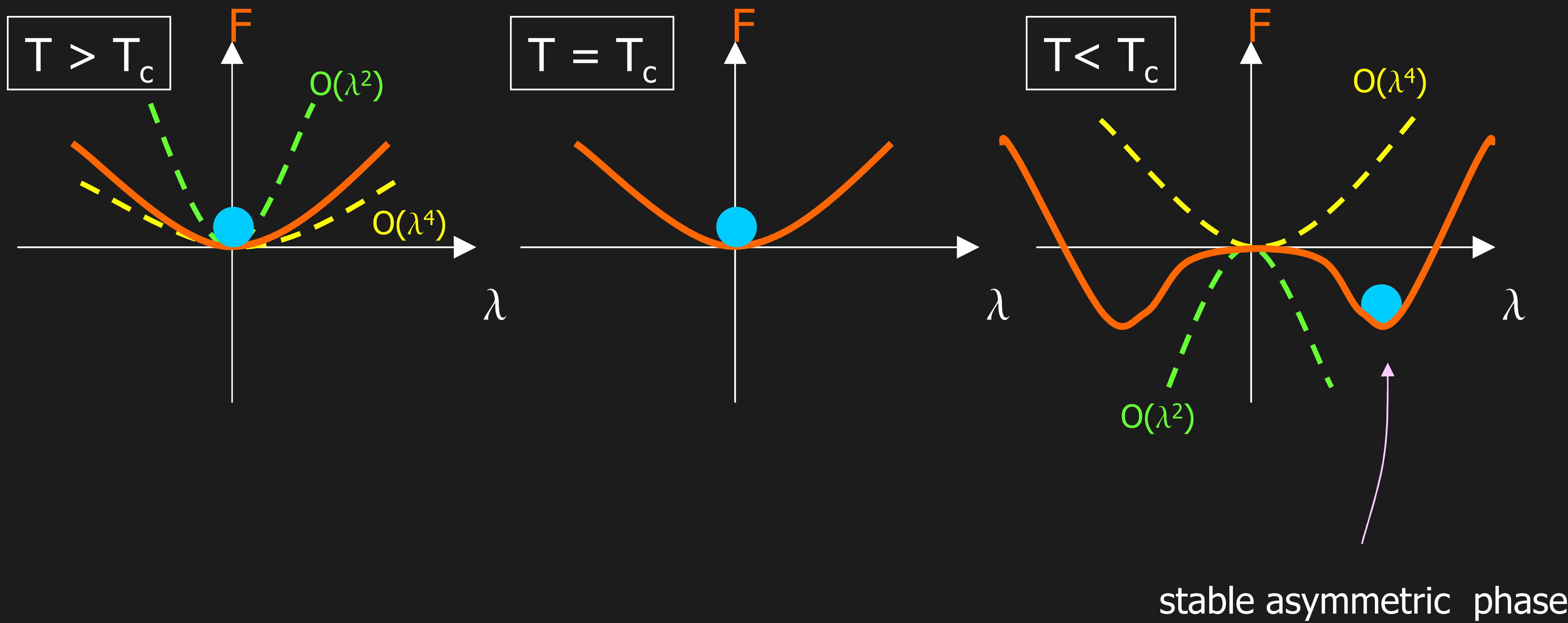
Extrema of the free energy $dF/d\lambda=0$ are $\lambda=0$ and $$\lambda_b^2 = -\frac{A(T-T_c)}{2C}$$

so a new branch exists if $\mathrm{sign}(T-T_c) = -\,\mathrm{sign}(C)$

There are two possibilities: $C>0$ or $C<0$

$$\delta F(T;\lambda) = A(T - T_c)\lambda^2 + C\lambda^4 + \ldots$$

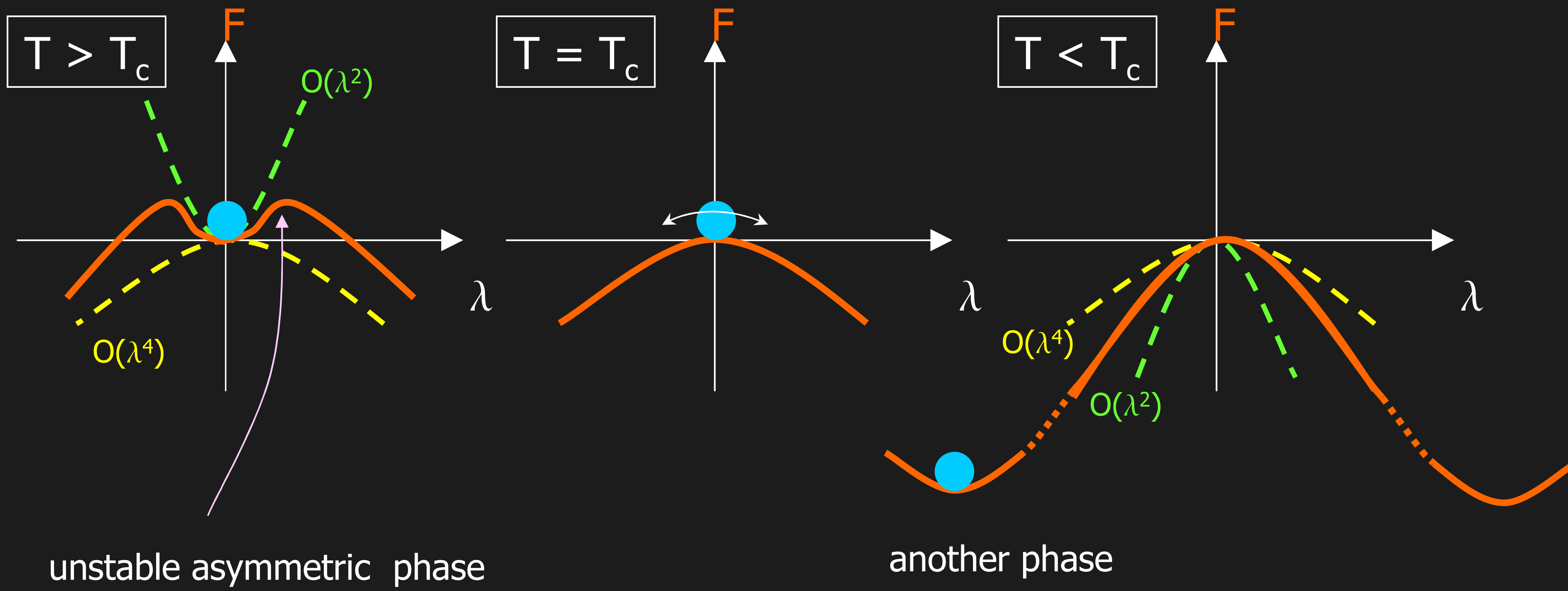
C>0



Second order phase transition

$$\delta F(T;\lambda) = A(T - T_c)\lambda^2 + C\lambda^4 + \ldots$$

C<0



First order phase transition

$$F(T,\lambda) = F_0(T) + (T - T_c) A \lambda^2 + C(T_c) \lambda^4 + \ldots$$

In summary:

C<0 first order phase transition
C>0 second order phase transition
(C=0 higher order)

Thermodynamics is encoded by F

Relation to black holes: York-Gibbons-Hawking action

$$I(\beta) = -\beta F = \frac{1}{16\pi G_D}\int dV_D R + \frac{1}{8\pi G_D}\int_{\partial} dV_{D-1}\left[K - K^0\right]$$

In Euclidean signature β is the period of the imaginary time.
It is related to the temperature $T=\hbar/\beta$

The program: We will compute variation of this action around the GL point, following the change of the uniform black string background induced by the marginally unstable GL mode.

$$h_{ab}(r,z) = \lambda H_{\alpha\beta}(r) e^{ik_{GL}z}$$

Non-uniform black-strings

The most general static black string metric on cylinder is

$$ds^2 = e^{2A(r,z)} f(r)\, dt^2 + e^{2B(r,z)} \left[f(r)^{-1} dr^2 + dz^2 \right] + e^{2C(r,z)} r^2 d\Omega_{d-2}$$

$f(r) = 1 - 1/r^{d-3}$ horizon is at $r_0=1$

A,B,C=0 give a *uniform BS*

We denote the fields collectively by x
and consider the expansion

$$x = x^{(0)} + \lambda X^{(1)} + \lambda^2 X^{(2)} + \ldots$$

$$X^{(1)} = X_{GL}$$

$$X^{(2)} = X_{BR}$$

The expansion of the free energy (or action) to the fourth order in λ

$$F(x) = F_0 + F_2(X,X) + F_3(X,X,X) + F_4(X,X,X,X) + \ldots$$
$$+ \delta k \frac{\partial F}{\partial k} + \ldots$$

Since X are decomposed into GL mode and BR, we get

$$F(\lambda; X; \beta) = F_0(\beta) + A\, \delta\beta\, \lambda^2 + \underbrace{F_2\left(X_{BR}, X_{BR}\right) + \lambda^2 G(X_{BR}) + F_{4GL}\lambda^4}$$

$$C\lambda^4 = \left[F_{4GL} - F_2\left(X_{BR}\right)\right]\lambda^4$$

Because of the (schematic) equation: $\delta F / \delta X = 0 \;\Rightarrow\; 2F_2 X_{BR} + G = 0$

The computation steps:

1. Solve Einstein equations for X_{GL}: $\mathbf{L}X_{GL}=0$

2. Solve Einstein equations for X_{BR} : $\mathbf{L}X_{BR} = \mathrm{Src}(X_{GL}{}^2)$

3. Substitute into the action $F(T,\lambda)=F_0(T)+(T-T_c)A\,\lambda^2+C(T_c)\,\lambda^4+\ldots$
   and by integration compute the coefficients A and C

4. Calculate thermodynamical variables M and S

Results

In either Gubser's or LG methods there are 2 "bottom line" numbers $\{M,S\}$ and $\{A,C\}$.

They are related by usual thermodynamics relations
$S = -dF/dT$ and $M = F + TS$

Numerical values of S, M computed directly (Gubser's) or derived from A and C (LG) are comparable within 5% in the verified range of dimensions $4 \leq d \leq 14$

√ LG works in black string case

Results for T^p

- It's enough to explore symmetric torus. We take the square one: convenient

- In this case it's enough to take p=2; general p follows

- The transition order depends only on d

- It follows that (for d>4) the diagonal direction is disfavored relative to turning on a "single tachyon"

Hence in a case of 1st order p.t. the decay will proceed (initially) through a single tachyon, while for a 2nd order p.t. the system will re-settle into a slightly non-uniform along one of the directions brane. (spontaneous symmetry breaking)