Ultrarelativistic boost of the black ring

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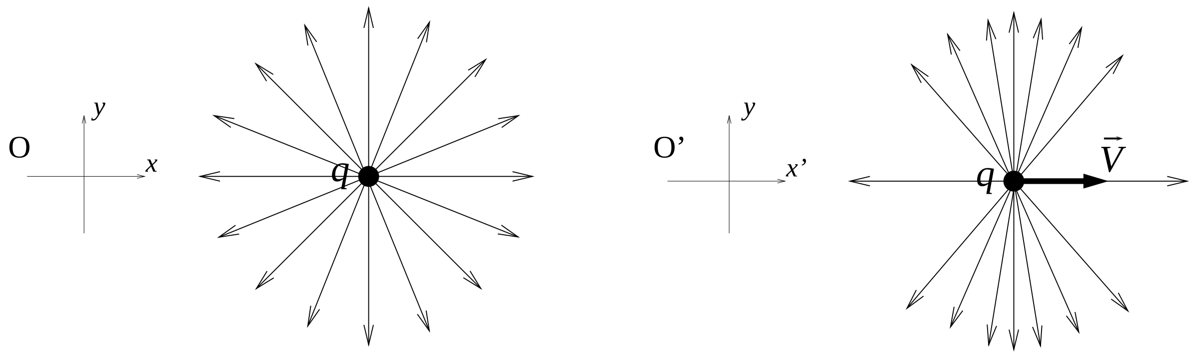
Boosting the Coulomb field

Take the Coulomb field of a point charge

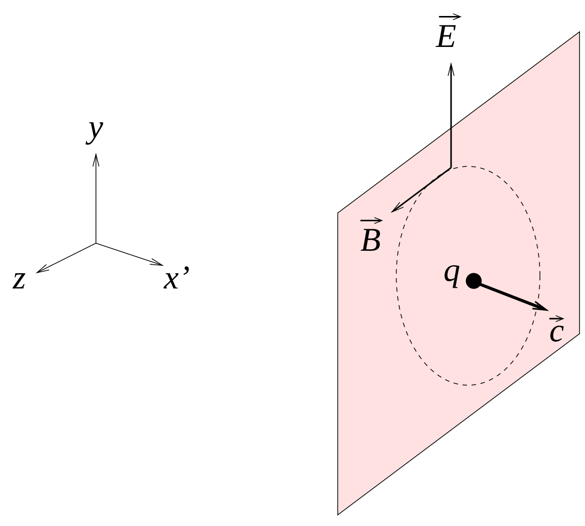
$$F = \frac{q}{r^2} \mathrm{d}r \wedge \mathrm{d}t$$

boost it: $t \to \gamma(t - Vx)$, $x \to \gamma(x - Vt)$



In the singular limit $V \to 1$ the e.m. field is "null", and the wave front is a 2-plane moving at the speed of light

$$F = \frac{2q}{\rho} \delta(x - t)(\mathrm{d}x - \mathrm{d}t) \wedge \mathrm{d}\rho$$



[Robinson-Rózga'85]

The field of a fast moving particle in GR

- Pirani, 1959: the gravitational field of a fast moving particle resembles a “plane” gravitational wave

- Aichelburg & Sexl, 1971:
  Schwarzschild “to the speed of light”

  $$\mathrm{d}s^2 = 2\mathrm{d}u\mathrm{d}v + \mathrm{d}\rho^2 + \rho^2\mathrm{d}\phi^2 - (4\sqrt{2}p\ln\rho^2)\delta(u)\mathrm{d}u^2$$

  impulsive pp-wave: $\Psi_4 \sim \delta(u)$, $R_{uu} \sim \delta(u)\delta^{(2)}(\rho)$

- Various generalizations in $D = 4$
  e.g. boost of Kerr, based on Kerr-Schild
  [Balasin-Nachbagauer'95]

- B.H. production at colliders in ADD TeV gravity:

  AS is a model for “incoming states”

  [Eardley-Giddings'02, . . .]

  Effect of higher dimensions. [Cf. Yoshino's talk in `S1`]

- $D > 4$ “Kerr”: boost of Myers-Perry [Yoshino'05]

Boost of Schwarzschild-Tangherlini

Easy generalization of the case $D = 4$ [Aichelburg-Sexl'71]

Decompose the Schwarzschild-Tangherlini metric as

$$\mathrm{d}s^2 = \mathrm{d}s_0^2 + \mu\Delta \text{ (= flat space + ``something''),}$$

$$\mathrm{d}s_0^2 = -\mathrm{d}t^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{(D-2)}, \quad \Delta = \frac{\mathrm{d}t^2}{r^{D-3}} + \frac{\mathrm{d}r^2}{r^{D-3} - \mu}$$

introduce cartesian and null coordinates

$$r^2 = x_1^2 + x_2^2 + x_3^2 + \ldots + x_{D-1}^2, \quad t = \frac{-u+v}{\sqrt{2}}, \quad x_1 = \frac{u+v}{\sqrt{2}}$$

perform a standard Lorentz boost

$$u \to \epsilon^{-1}u, \quad v \to \epsilon v, \qquad \gamma \equiv (1 - V^2)^{-1/2} = \frac{\epsilon + \epsilon^{-1}}{2}$$

take the $\boxed{\text{ultrarelativistic limit } V \to 1}$, i.e.

$$\epsilon \to 0, \qquad \text{with the mass rescaling} \qquad \mu = \epsilon p$$

use the distributional identity

$$\lim_{\epsilon \to 0} \frac{1}{\epsilon} f(\epsilon^{-1}u) = \delta(u) \int_{-\infty}^{+\infty} f(z)\mathrm{d}z$$

you end up with flat space + impulse moving along x_1:

$$\mathrm{d}s^2 = \overbrace{2\mathrm{d}u\mathrm{d}v + \mathrm{d}x_2^2 + \mathrm{d}x_3^2 + \ldots}^{\mathrm{d}s_0^2} + \frac{\tilde{p}\,\delta(u)\mathrm{d}u^2}{(x_2^2 + x_3^2 + \ldots)^{(D-4)/2}}.$$

The black ring [Emparan-Reall'02]

In the coordinates of [Emparan'04]:

$$ds^2 = -\frac{F(y)}{F(x)}\left(dt + C(\nu,\lambda)L\frac{1+y}{F(y)}d\psi\right)^2 + L^2\frac{F(x)}{(x-y)^2} \times \left[-\frac{G(y)}{F(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\phi^2\right],$$

$$F(\zeta) = 1 + \lambda\zeta, \qquad G(\zeta) = (1-\zeta^2)(1+\nu\zeta),$$

$$C^2(\nu,\lambda) \sim \lambda - \nu, \quad 0 < \nu \le \lambda < 1,$$

$$y \in (-\infty, -1], \quad x \in [-1, 1], \quad \Delta\phi = 2\pi\frac{\sqrt{1-\lambda}}{1-\nu} = \Delta\psi$$

- solution of $D = 5$ *vacuum* GR ($R_{\mu\nu} = 0$)
- asymptotically flat ($x, y \to -1$)
- stationary
- regular event horizon $S^1 \times S^2$ at $y = -1/\nu$
- Petrov type I_i (Myers-Perry is D) [Pravda-Pravdová'05]

$$M = \frac{3\pi L^2}{4}\frac{\lambda}{1-\nu}, \qquad J = \frac{\pi L^3}{2}\frac{\sqrt{\lambda(\lambda-\nu)(1+\lambda)}}{(1-\nu)^2}$$

$$\lambda = \frac{2\nu}{1+\nu^2} \longrightarrow \text{ equilibrium condition.}$$

Boost of the black ring

We used the "standard algorithm":

$$\mathrm{d}s^2 = \mathrm{d}s_0^2 + \Delta,$$

$$\lambda = \epsilon p_\lambda, \qquad \nu = \epsilon p_\nu, \qquad (p_\lambda = 2p_\nu \quad \text{at equilibrium})$$

$$\begin{aligned}
\mathrm{d}s_0^2 = & -\mathrm{d}t^2 + \frac{L^2}{(x-y)^2} \\
& \times \left[(y^2-1)\mathrm{d}\psi^2 + \frac{\mathrm{d}y^2}{y^2-1} + \frac{\mathrm{d}x^2}{1-x^2} + (1-x^2)\mathrm{d}\phi^2\right] \\
= & -\mathrm{d}t^2 + \mathrm{d}\xi^2 + \xi^2\mathrm{d}\psi^2 + \mathrm{d}\eta^2 + \eta^2\mathrm{d}\phi^2 \\
= & -\mathrm{d}t^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}y_1^2 + \mathrm{d}y_2^2,
\end{aligned}$$

$$\xi = L\tfrac{\sqrt{y^2-1}}{x-y}, \qquad \eta = L\tfrac{\sqrt{1-x^2}}{x-y},$$

$$x_1 = \eta\cos\phi,\ x_2 = \eta\sin\phi, \quad y_1 = \xi\cos\psi,\ y_2 = \xi\sin\psi,$$

$$\begin{aligned}
\Delta = & \ \lambda\frac{x-y}{1+\lambda x}\mathrm{d}t^2 - 2(1-\lambda)C(\lambda,\nu)L\frac{1+y}{1+\lambda x}\mathrm{d}t\,\mathrm{d}\psi \\
& + \frac{\lambda-\nu}{1-\nu}\frac{L^2}{1+\lambda y}\left[-\lambda\frac{1+\lambda}{1-\lambda}\frac{(1+y)^2}{1+\lambda x} + \frac{y^2-1}{x-y}\right]\mathrm{d}\psi^2 + \frac{L^2}{(x-y)^2} \\
& \times\left[\nu\frac{x+1}{1-\nu}(y^2-1)\mathrm{d}\psi^2 + \frac{\lambda(1-\nu)(x-y)+(\lambda-\nu)(1+y)}{(1-\lambda)(1+\nu y)}\frac{\mathrm{d}y^2}{y^2-1}\right. \\
& \left. + \frac{\lambda-\nu}{1-\lambda}\frac{\mathrm{d}x^2}{(1-x)(1+\nu x)} + \nu\frac{x+1}{1-\nu}(1-x^2)\mathrm{d}\phi^2\right] = \ldots\ldots\ldots\ldots
\end{aligned}$$

1) Boost along x_1 – *orthogonal* to the ring plane (y_1, y_2):

$$\mathrm{d}s^2 = 2\mathrm{d}u\mathrm{d}v + \mathrm{d}x_2^2 + \mathrm{d}y_1^2 + \mathrm{d}y_2^2 + H_\perp(x_2, y_1, y_2)\delta(u)\mathrm{d}u^2,$$

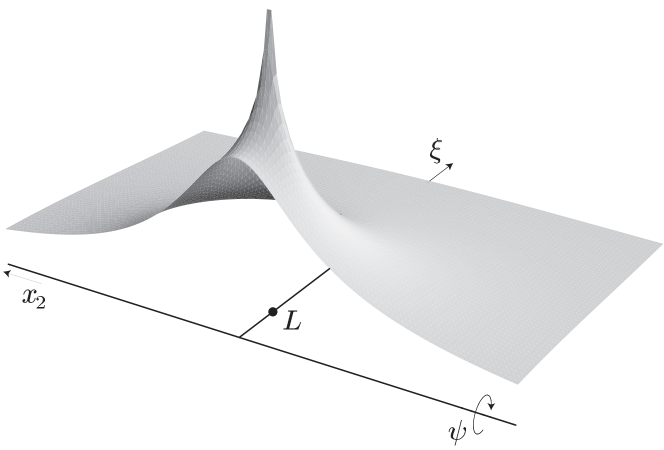
$$H^e_\perp = \frac{3\sqrt{2}\, p_\lambda L^2}{\sqrt{(\xi+L)^2 + x_2^2}}\, K(k),$$

$$\xi = \sqrt{y_1^2 + y_2^2}, \quad k = \sqrt{\frac{4\xi L}{(\xi+L)^2 + x_2^2}}.$$

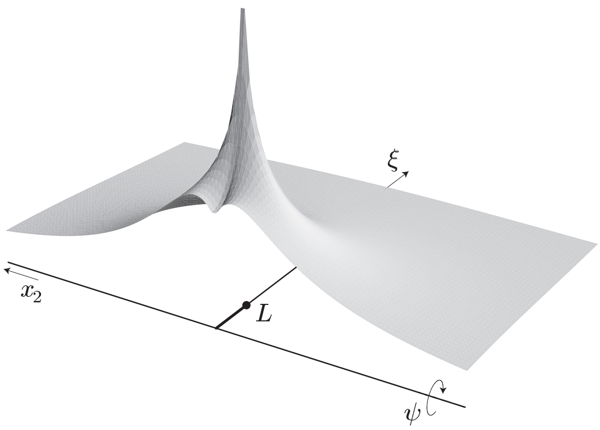
$$K(k) \equiv \int_0^{\pi/2} \frac{\mathrm{d}\alpha}{\sqrt{1 - k^2 \sin^2\alpha}}$$

(Newtonian potential of a uniform ring of radius L).

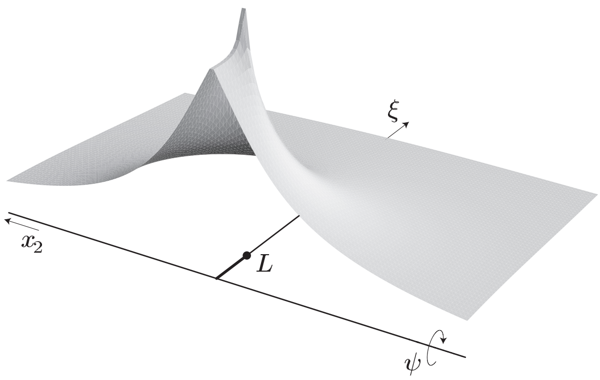
Vacuum, singular ring at $u = 0$, $\xi = L$, $x_2 = 0$:



($p_\lambda = 2p_\nu$, equilibrium)



($p_\lambda < 2p_\nu$, underspinning)



($p_\lambda > 2p_\nu$, overspinning)

2) Boost along y_1, i.e. *parallel* to the ring plane (y_1, y_2):

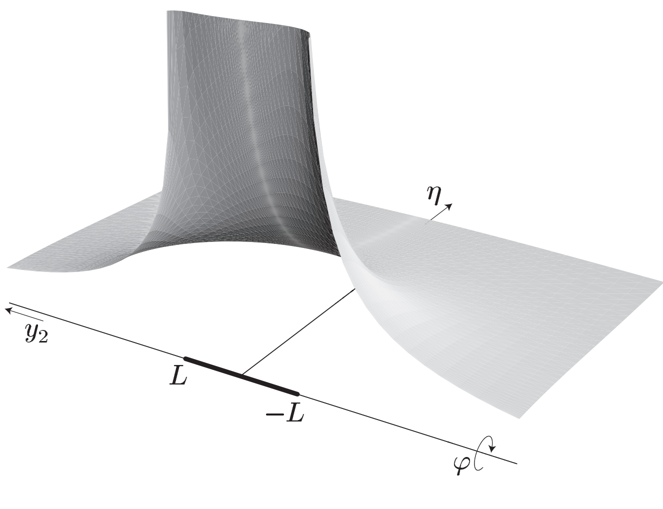
$$ds^2 = 2\mathrm{d}u\mathrm{d}v + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}y_2^2 + H_{||}(x_1, x_2, y_2)\delta(u)\mathrm{d}u^2,$$

$$H^e_{||} = p_\lambda L\left[\frac{3\sqrt{2}L}{a} + \frac{2y_2}{a}\left(1 - \frac{L^2+\eta^2}{a^2-y_2^2}\right)\right]K(k)$$
$$+ p_\lambda L\left[\frac{\eta^2+L^2}{ay_2}\frac{a^2+y_2^2}{a^2-y_2^2}\,\Pi(\rho, k) - \pi\,\mathrm{sgn}(y_2)\right].$$

$$\eta = \sqrt{x_1^2 + x_2^2}, \qquad a = \left[(\eta^2 + y_2^2 - L^2)^2 + 4\eta^2 L^2\right]^{1/4},$$

$$k = \frac{\left(a^2 - \eta^2 - y_2^2 + L^2\right)^{1/2}}{\sqrt{2}a}, \qquad \rho = -\frac{(a^2 - y_2^2)^2}{4a^2 y_2^2}.$$

Vacuum, singular rod at $u = 0$, $\eta = 0$, $|y_2| \le L$:



Same method: boost of supersymmetric black rings (subtleties with electric charge).

Related to supergravity "gyratons" of [Frolov-Lin'06].