Charged Rotating Black Holes in Higher Dimensions

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Outline

- Introduction
- Einstein-Maxwell Black Holes
- Einstein-Maxwell-Dilaton Black Holes
- Einstein-Maxwell-Chern-Simons Black Holes
- Conclusions

Introduction

- 4D Einstein-Maxwell (EM) black holes

	Static	Rotating
Uncharged	Schwarzschild (M)	Kerr (M, J)
Charged	Reissner-Nordström (M, Q, P)	Kerr-Newman (M, J, Q, P)

- D>4 Einstein-Maxwell black holes

	Static	Rotating
Uncharged	Tangherlini (M)	Myers-Perry (M, J_i)
Charged	Tangherlini (M, Q)	?

Introduction

- Aim: Higher dimensional Abelian black holes asymptotically flat and with regular horizon
- Black rings are allowed for D>4 Emparan&Reall 2002
- D>4 EM +
  - nothing (pure EM theory)
  - dilaton (EMD theory)
  - Chern-Simons term (EMCS theory; just for odd D)

Einstein-Maxwell Black Holes

Einstein-Maxwell action

$$S = \int \frac{1}{16\pi G_D}\sqrt{-g}(R - F_{\mu\nu}F^{\mu\nu})d^Dx$$

- Maxwell field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Einstein equations

$$G_{\mu\nu} = 2T_{\mu\nu}$$

with stress-energy tensor

$$T_{\mu\nu} = F_{\mu\rho}F_\nu{}^\rho - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}$$

Maxwell equations

$$\nabla_\mu F^{\mu\nu} = 0$$

Einstein-Maxwell Black Holes

- General black holes: characterized by mass M N=[(D-1)/2] angular momenta J_i and charge Q (no magnetic charge for D>4)

- No analytical charged rotating solutions for D>4

- Numerical approach: too complicated in the general case

- Restricted case: odd dimensional black holes with equal-magnitude angular momenta

- Simplification=field equations reduce to a system of 5 ODE's Kunz, Navarro-Lérida, Viebahn 2006

- Similar procedure for EMD and EMCS theories

Einstein-Maxwell Black Holes (odd D)

- Ansätze (D=2N+1)

$$ds^2 = -f dt^2 + \frac{m}{f}\left[dr^2 + r^2 \sum_{i=1}^{N-1}\left(\prod_{j=0}^{i-1}\cos^2\theta_j\right)d\theta_i^2\right]$$
$$+\frac{n}{f}r^2 \sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k\left(\varepsilon_k d\varphi_k - \frac{\omega}{r}dt\right)^2$$
$$+\frac{m-n}{f}r^2\left\{\sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k d\varphi_k^2\right.$$
$$\left.-\left[\sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k\varepsilon_k d\varphi_k\right]^2\right\}$$

$$A_\mu dx^\mu = a_0 dt + a_\varphi \sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k\varepsilon_k d\varphi_k$$

$\theta_0 \equiv 0,\ \theta_i \in [0,\pi/2],\ i = 1,\ldots,N-1,\ \theta_N \equiv \pi/2,$
$\varphi_k \in [0,2\pi],\ k = 1,\ldots,N,$ and $\varepsilon_k = \pm 1$

Einstein-Maxwell Black Holes (odd D)

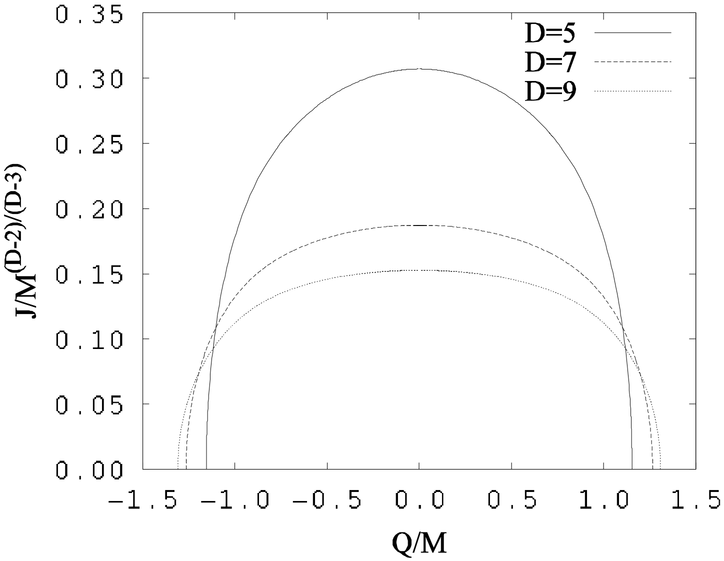
- Regular horizon at r=r_H with $f(r_H)=0$
- Killing vector null at the horizon $\chi = \partial_t + \Omega \sum_{k=1}^{N} \varepsilon_k \partial_{\varphi_k}$
  Ω=horizon angular velocity
- Removing a_0 : first integral

$$\frac{r^{D-2} m^{(D-5)/2}}{f^{(D-3)/2}} \sqrt{\frac{mn}{f}} \left(\frac{da_0}{dr} + \frac{\omega}{r} \frac{da_\varphi}{dr} \right) = -\frac{4\pi G_D}{A(S^{D-2})} Q$$

- Mass formula Gauntlett, Myers, Townsend 1999

$$\frac{D-3}{D-2} M = \frac{\kappa A_{\rm H}}{8\pi G_D} + N\Omega J + \frac{D-3}{D-2} \Phi_{\rm H} Q$$

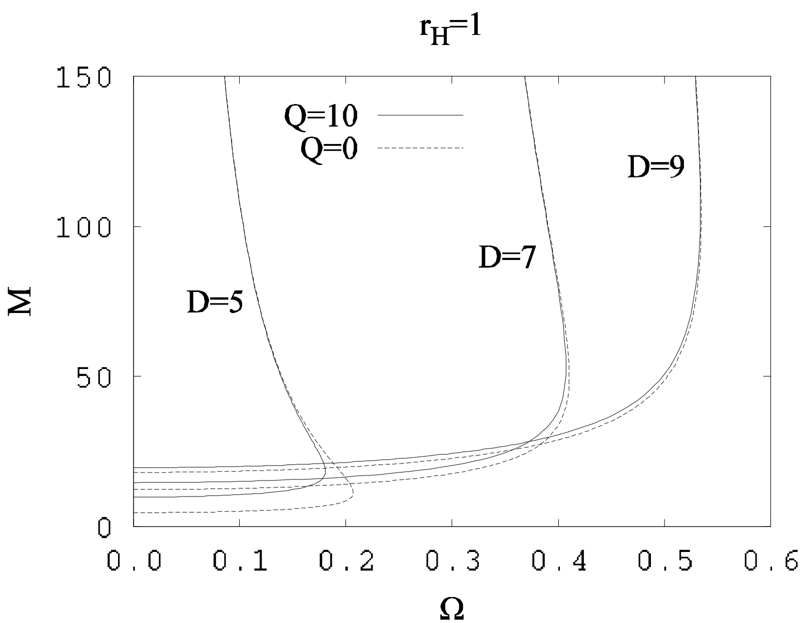
Einstein-Maxwell Black Holes (odd D)



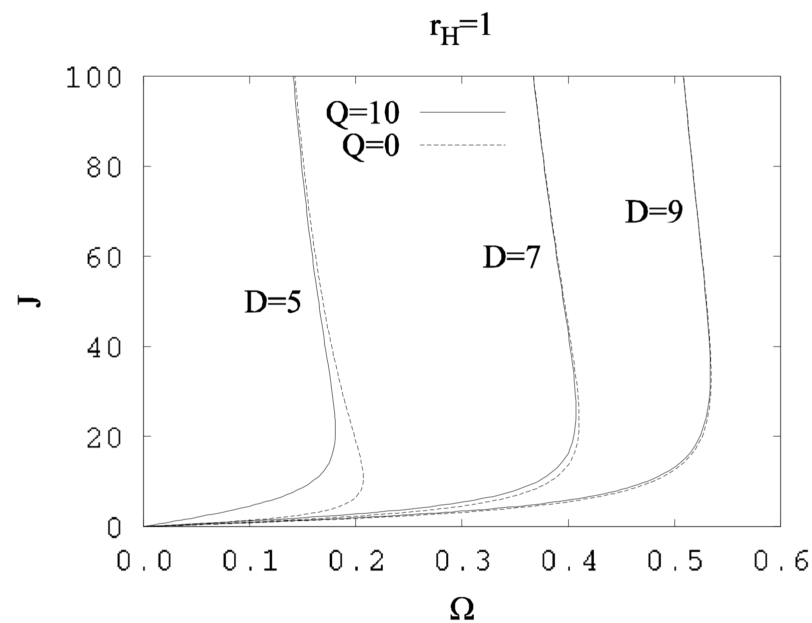
Domain of existence

(scaled quantities)

Mass

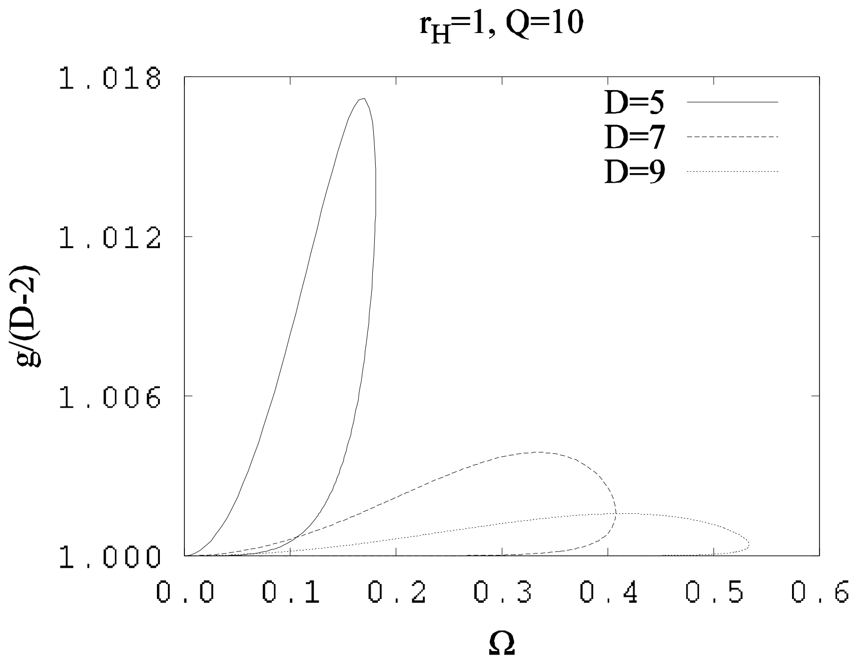


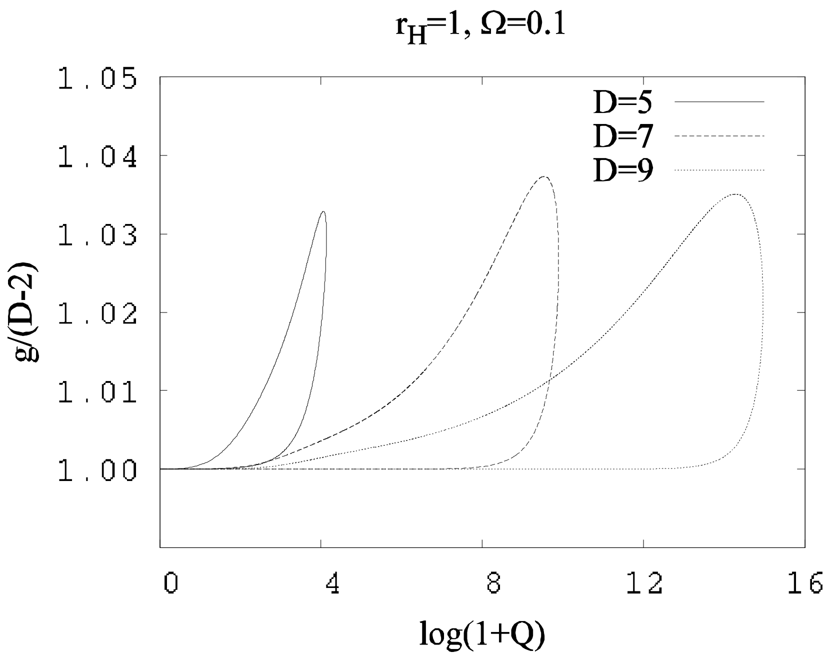
Angular momentum



Einstein-Maxwell Black Holes (odd D)

- Gyromagnetic ratio $$g = \frac{2M\mu_{\mathrm{mag}}}{QJ}$$
- g=2 for D=4 but …
- Perturbative value g=(D-2) Aliev 2006





Einstein-Maxwell-Dilaton Black Holes

Einstein-Maxwell-Dilaton action

$$S = \int d^D x \sqrt{-g} \left(R - \frac{1}{2}\Phi_{,\rho}\Phi^{,\rho} - \frac{1}{4} e^{-2h\Phi} F_{\rho\sigma}F^{\rho\sigma} \right)$$

(units $16 \pi G_D = 1$) h=dilaton coupling constant

Field equations

$$G_{\rho\sigma} = \frac{1}{2}\left[\partial_\rho\Phi\partial_\sigma\Phi - \frac{1}{2} g_{\rho\sigma}\partial\tau\Phi\partial^\tau\Phi + e^{-2h\Phi}\left(F_{\rho\tau}F_\sigma{}^\tau - \frac{1}{4} g_{\rho\sigma}F_{\tau\beta}F^{\tau\beta} \right) \right]$$

$$\nabla_\rho \left(e^{-2h\Phi} F^{\rho\sigma} \right) = 0$$

$$\nabla^2\Phi = -\frac{h}{2} e^{-2h\Phi} F_{\rho\sigma}F^{\rho\sigma}$$

- Analytical solutions!!! Kaluza-Klein black holes
  Kunz, Maison, Navarro-Lérida, Viebahn 2006

Einstein-Maxwell-Dilaton Black Holes (Kaluza-Klein)

- Myers-Perry solution as seed
- Fixed dilaton coupling constant

$$L = \begin{pmatrix} \cosh\alpha & \sinh\alpha \\ \sinh\alpha & \cosh\alpha \end{pmatrix}$$

$$h = \frac{D-1}{\sqrt{2(D-1)(D-2)}}$$

$$ds^2 = \left(1 + \frac{mr^{2-\varepsilon}}{\Pi F}\sinh^2\alpha\right)^{\frac{1}{D-2}} \Bigg\{ -dt^2 + \frac{\Pi F}{\Pi - mr^{2-\varepsilon}}dr^2 + \varepsilon r^2 d\nu^2 +$$

$$\sum_{i=1}^{N}(r^2 + a_i^2)(d\mu_i^2 + \mu_i^2 d\varphi_i^2) + \frac{mr^{2-\varepsilon}}{\Pi F + mr^{2-\varepsilon}\sinh^2\alpha}\left(\cosh\alpha dt - \sum_{i=1}^{N} a_i\mu_i^2 d\varphi_i\right)^2 \Bigg\}$$

$$A_\rho dx^\rho = \frac{mr^{2-\varepsilon}\sinh\alpha}{\Pi F + mr^{2-\varepsilon}\sinh^2\alpha}\left(\cosh\alpha dt - \sum_{i=1}^{N} a_i\mu_i^2 d\varphi_i\right)$$

$$\Phi = -\frac{1}{2(D-2)\iota}\log\left(1 + \frac{mr^{2-\varepsilon}}{\Pi F}\sinh^2\alpha\right)$$

$$N \equiv \left[\frac{D-1}{2}\right] \qquad \varepsilon \equiv \frac{1}{2}(1 + (-1)^D)$$

$$F \equiv 1 - \sum_{i=1}^{N}\frac{a_i^2\mu_i^2}{r^2 + a_i^2}\,, \qquad \Pi \equiv \prod_{i=1}^{N}(r^2 + a_i^2) \qquad \sum_{i=1}^{N}\mu_i^2 + \varepsilon\nu^2 = 1$$

Einstein-Maxwell-Dilaton Black Holes (Kaluza-Klein)

- Some quantities

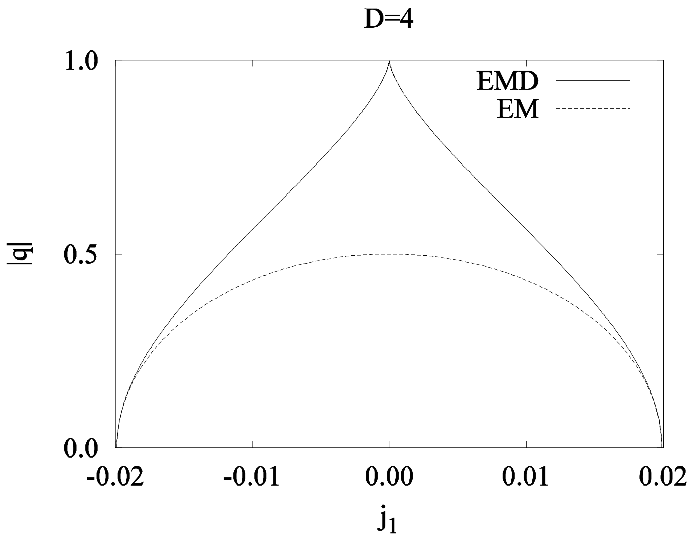
$$M = m\left(1 + (D-3)\cosh^2\alpha\right) A(S^{D-2})$$
$$J_i = 2m\, a_i \cosh\alpha A(S^{D-2}) \ , \quad i = 1, \ldots, N$$
$$Q = (D-3)m \sinh\alpha \cosh\alpha A(S^{D-2})$$
$$g_i = \frac{2\mathcal{M}_i M}{Q J_i} = (D-3) + \frac{1}{\cosh^2\alpha} = g$$

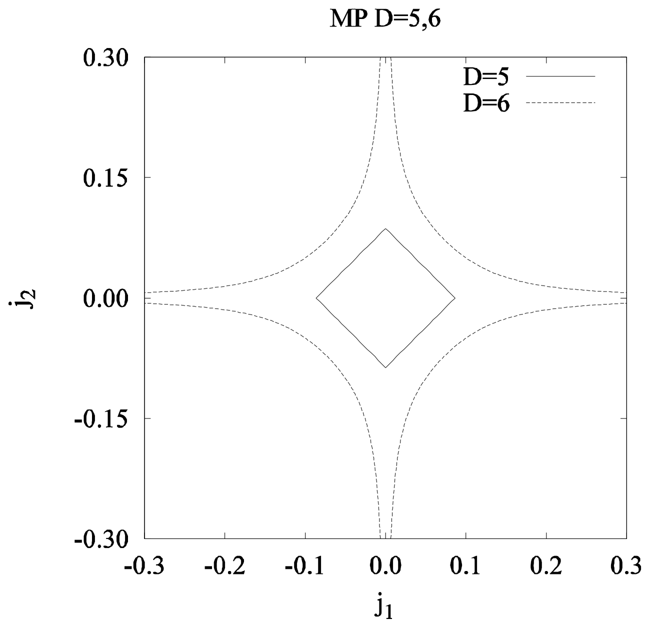
- Horizon: $\Delta|_{\mathrm{H}} = 0$ with $\Delta \equiv \Pi - m r^{2-\varepsilon}$
- Surface gravity:

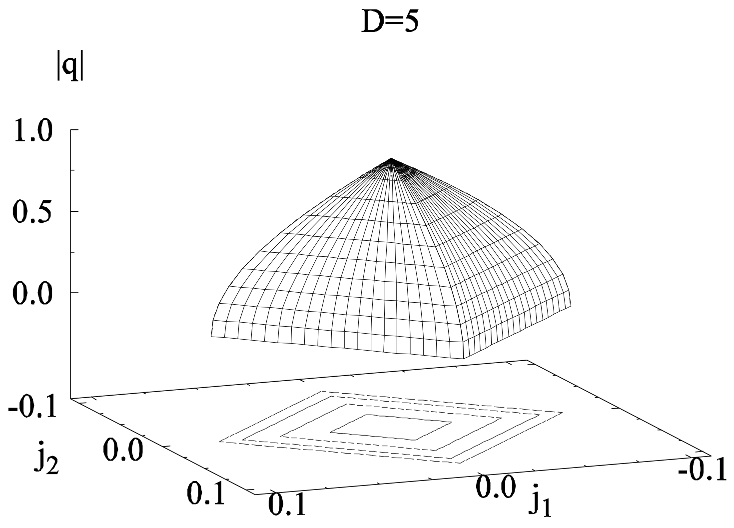
$$\kappa_{\mathrm{sg}} = \left.\frac{\Delta_{,r}}{2m r_{\mathrm{H}}^{2-\varepsilon}\cosh\alpha}\right|_{r=r_{\mathrm{H}}}$$

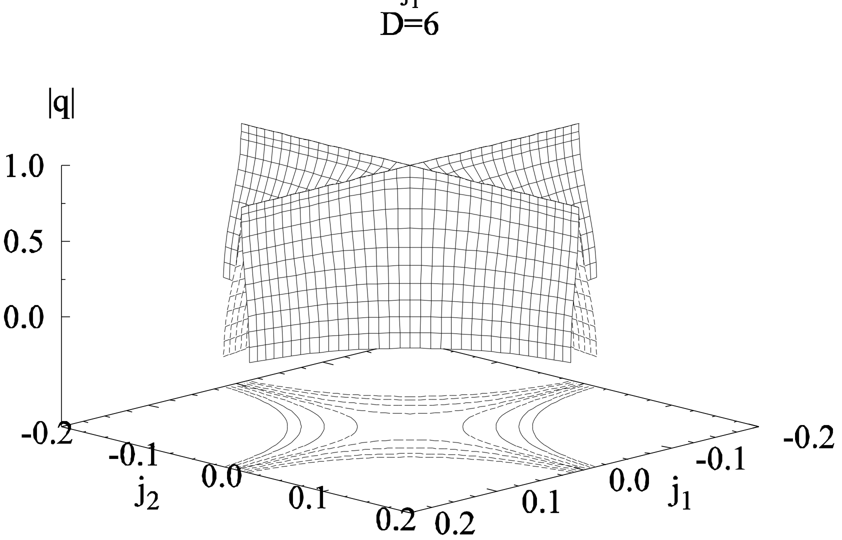
- Domains of existance: Extremal solutions $\kappa_{sg}=0$
- Scaled quantities: $q \equiv Q/M \quad j_i \equiv J_i/M^{(D-2)/(D-3)}$

Einstein-Maxwell-Dilaton Black Holes (Kaluza-Klein)

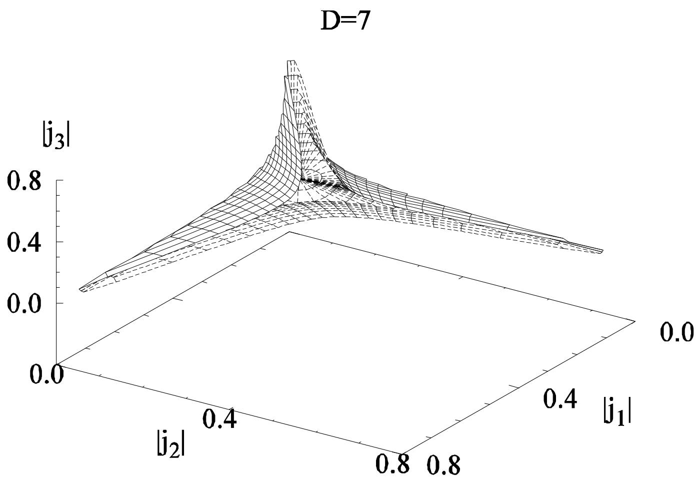


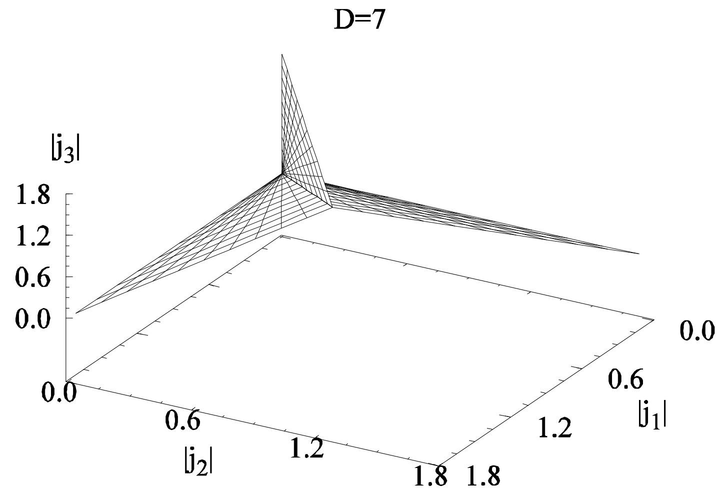






Einstein-Maxwell-Dilaton Black Holes (Kaluza-Klein)

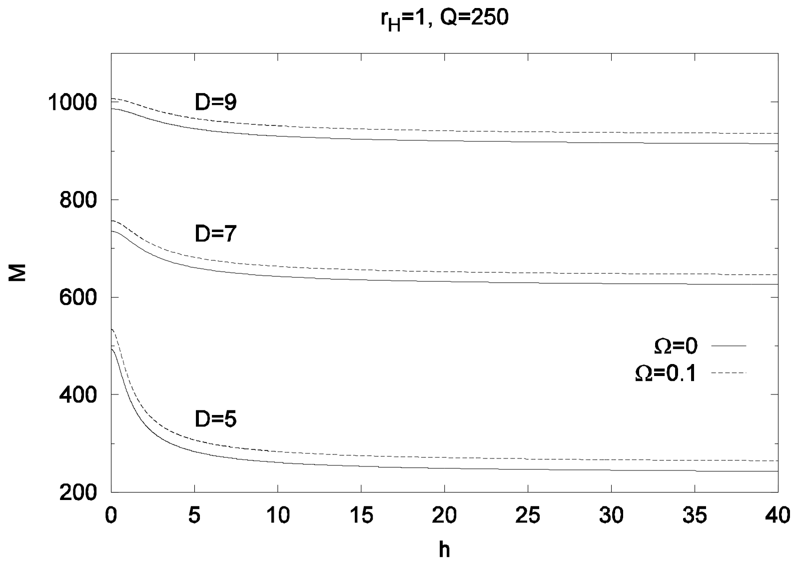


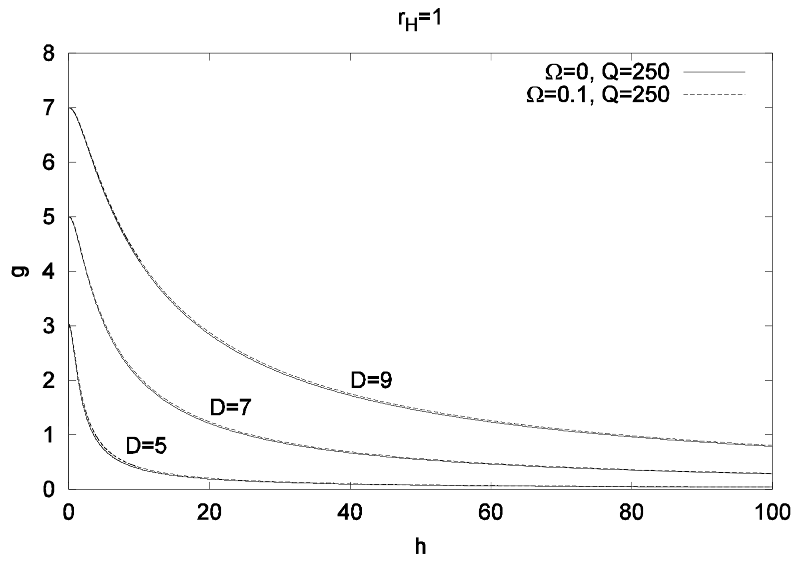


- Similar pattern for D>6

Einstein-Maxwell-Dilaton Black Holes (odd D)

- Restricted case: odd D, equal-magnitude angular momenta
- Same ansatz as in EM theory + $\Phi=\Phi(r)$
- No constraint on the dilaton coupling constant





Einstein-Maxwell-Chern-Simons Black Holes

- Just for odd D(=2N+1): Chern-Simons term AF^N

Einstein-Maxwell-Chern-Simons action

$$S = \int \frac{1}{16\pi G_D}\sqrt{-g}\Big(R - F_{\mu\nu}F^{\mu\nu} + \frac{8}{D+1}\tilde{\lambda}\epsilon^{\mu_1\mu_2\ldots\mu_{D-2}\mu_{D-1}\mu_D} F_{\mu_1\mu_2}\ldots F_{\mu_{D-2}\mu_{D-1}} A_{\mu_D}\Big)\, d^D x$$

Einstein equations

$$G_{\mu\nu} = 2\left(F_{\mu\rho}F_\nu{}^\rho - \frac{1}{4} g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right)$$

Maxwell equations

$$\nabla_\nu F^{\mu_1\nu} = \tilde{\lambda}\epsilon^{\mu_1\mu_2\mu_3\ldots\mu_{D-1}\mu_D} F_{\mu_2\mu_3}\ldots F_{\mu_{D-1}\mu_D}$$

Kunz, Navarro-Lérida 2006

Einstein-Maxwell-Chern-Simons Black Holes

- Black hole solutions: regular horizon $r=r_H$
- Restricted case: same ansatz as for EM black holes
- First integral of the system of ODE's

$$\frac{r^{D-2}m^{(D-5)/2}}{f^{(D-3)/2}}\sqrt{\frac{mn}{f}}\left(\frac{da_0}{dr}+\frac{\omega}{r}\frac{da_\varphi}{dr}\right)-\varepsilon_D 2^{D-2}(N-1)!\tilde{\lambda}a_\varphi^N=-\frac{4\pi G_D}{A(S^{D-2})}Q$$

$$D=2N+1 \quad \varepsilon_D=(-1)^{\frac{1}{2}N(N+1)}$$

- Mass formula

$$\frac{D-3}{D-2}M=\frac{\kappa A_H}{8\pi G_D}+N\Omega J+\frac{D-3}{D-2}\Phi_H Q+\tilde{\lambda}(D-5)I$$

- Scaling

$$r_H \to \gamma r_H, \quad a_\varphi \to \gamma a_\varphi, \quad \tilde{\lambda} \to \gamma^{N-2}\tilde{\lambda}$$
$$Q \to \gamma^{D-3}Q, \quad M \to \gamma^{D-3}M, \quad J \to \gamma^{D-2}J$$

Einstein-Maxwell-Chern-Simons Black Holes (D=5)

- Redefinition: $\lambda = 2\sqrt{3}\tilde{\lambda}$
- Cases:

  $\lambda=0$: Einstein-Maxwell theory

  $\lambda=1$: bosonic sector of minimal D=5 supergravity

  $\lambda>1$

- Analytical solutions: only for $\lambda=1$
  Breckenridge, Myers, Peet, Vafa 1997
- Good for testing the numerical scheme
  (restricted case: $|J_1|=|J_2|$)
- Very high accuracy!!!

Einstein-Maxwell-Chern-Simons Black Holes (D=5)

- Domain of existence (extremal solutions)

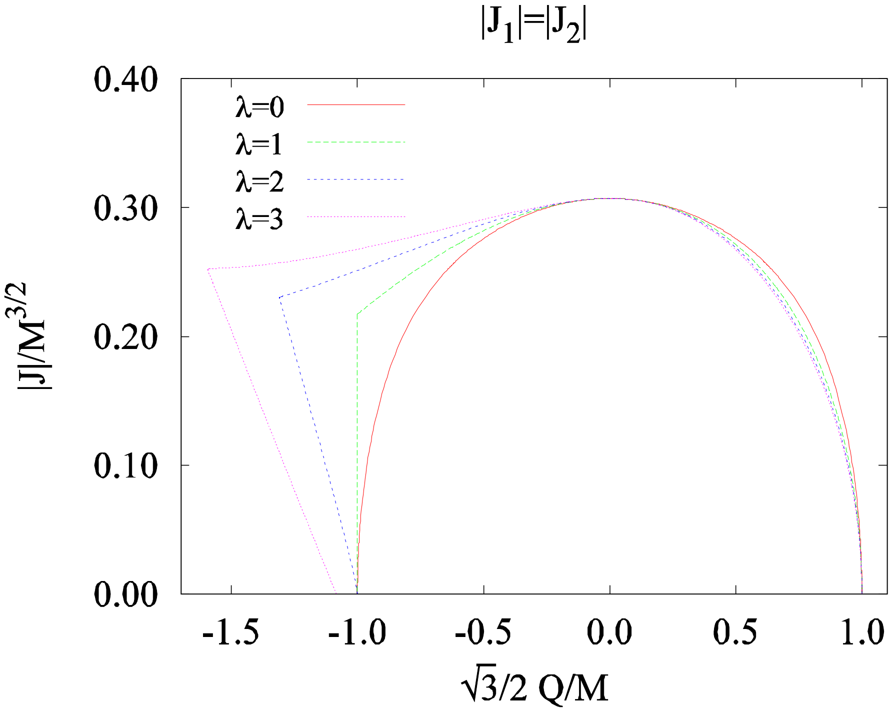
Extremal λ=1 EMCS

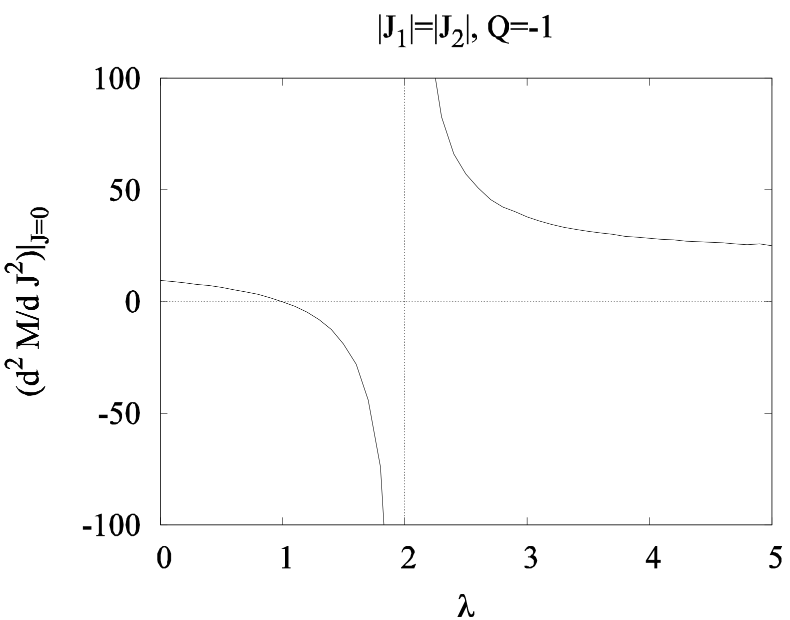
(supersymmetric branch)

- Mass saturates $M \geq \frac{\sqrt{3}}{2}|Q|$
- Angular momentum satisfies

$$|J|^2 \leq \frac{1}{6\sqrt{3}\pi}|Q|^3$$

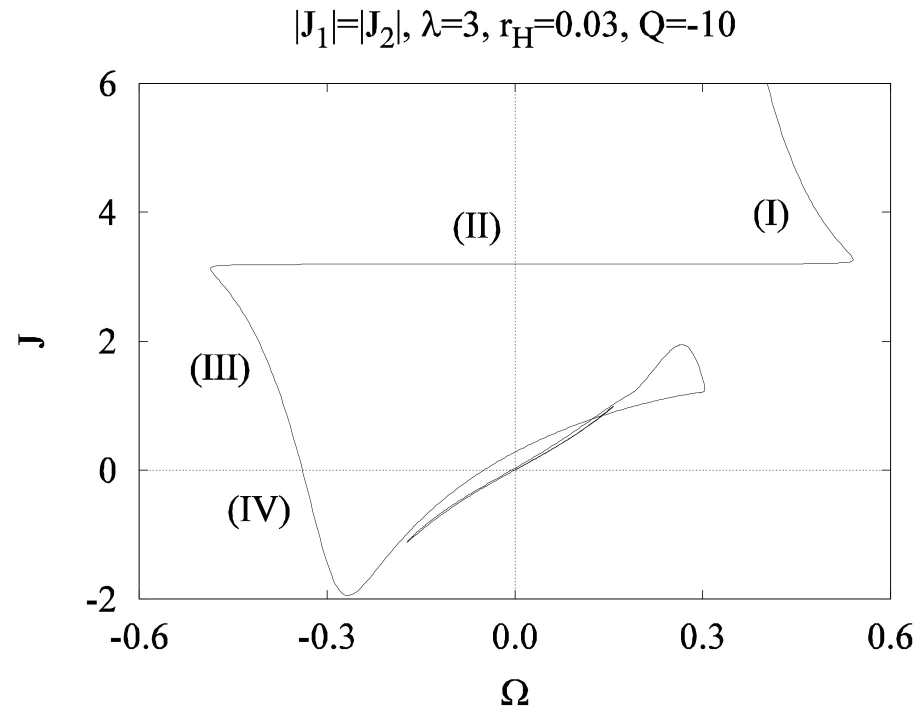
- Vanishing horizon angular velocity





- Instability beyond λ=1 (up to λ=2)
  supersymmetry marks a borderline between stability and instability
- λ=2 is a special case
  infinite set of extremal black holes with the same charges?

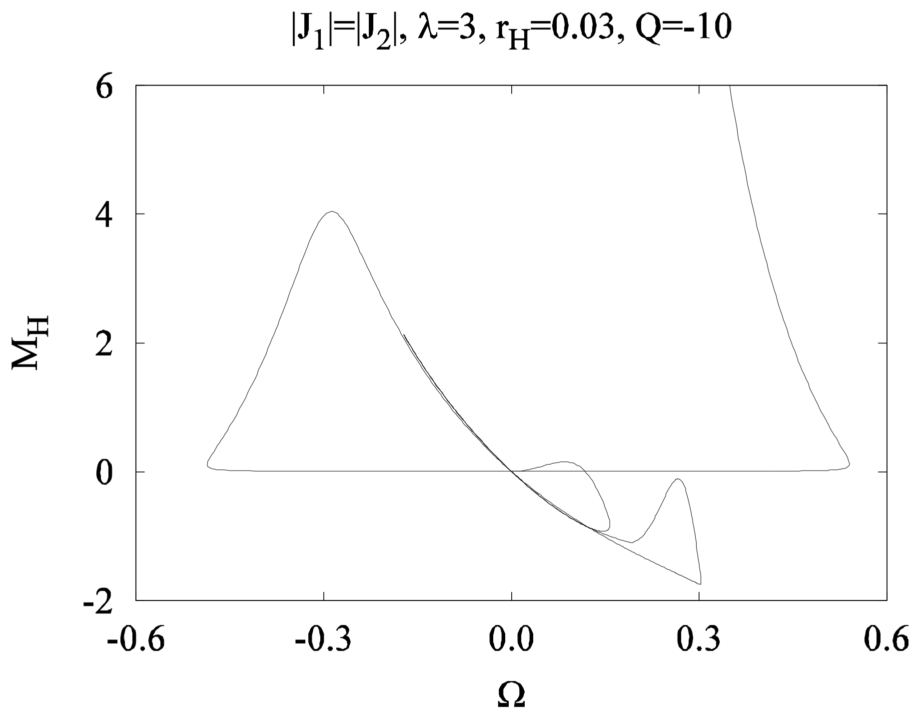
Einstein-Maxwell-Chern-Simons Black Holes (D=5)



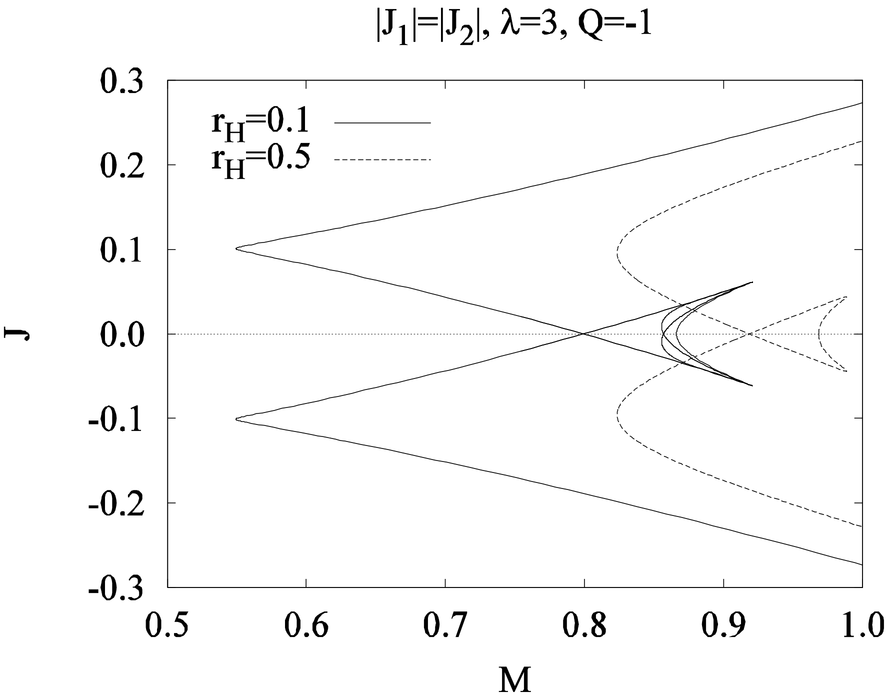
Four types of black holes

- Type I: Corrotating $\Omega J \geq 0$ and $\Omega=0 \Leftrightarrow J=0$
- Type II: Static horizon $\Omega=0$ but non-vanishing $J \neq 0$ ($\lambda \geq 1$ and $\lambda=1 \Rightarrow$ extremal)
- Type III: Counterrotating $\Omega J < 0$ ($\lambda > 1$)
- Type IV: Rotating horizon $\Omega \neq 0$ but $J=0$ ($\lambda \geq 2$ and $\lambda=2 \Rightarrow$ extremal)

Einstein-Maxwell-Chern-Simons Black Holes (D=5)



The horizon mass may be negative !!!



Black holes are not uniquely determined by M, J_i, Q (non-uniqueness even for horizons of spherical topology)

Conclusions

- Abelian higher dimensional charged rotating BH's
- Restricted case: odd D + equal-magnitude angular momenta $\Rightarrow$system of ODE's
- EM theory: non-constant gyromagnetic ratio for D>4
- Analytical Kaluza-Klein solutions in EMD theory
- (Odd-D) EMCS theory: D=5 is a special case
- λ=1: supersymmetry marks a borderline between stability and instability
- Four types of black holes (for λ>2)
- Non-uniqueness (for λ>2)