Short distances, Black Holes, and TeV-Gravity

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Based on the work

I. Agullo, J. N-S and G.J. Olmo, Black hole radiance, short distances, and TeV gravity, hep-th/0604044
Physical Review Letters, to appear 2006

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- Motivation of the problem: Hawking radiation and transPlanckian energies

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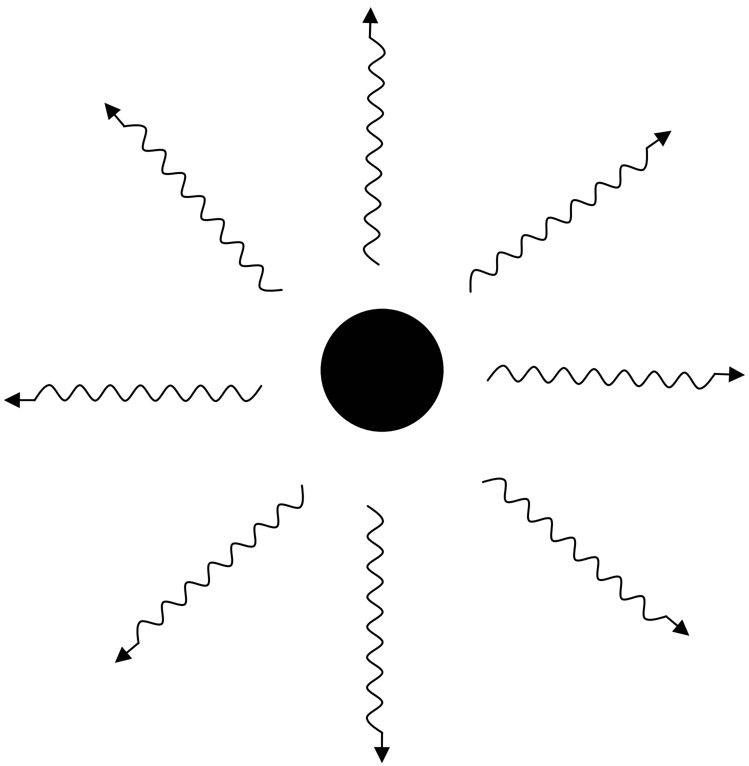
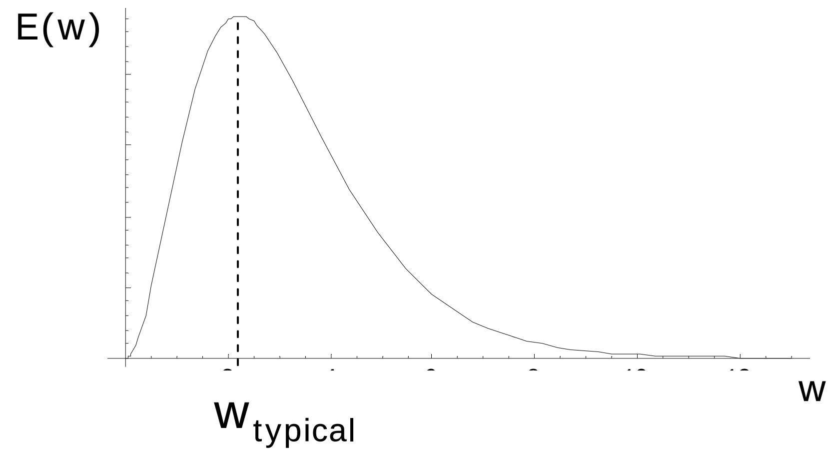
  - For mini black-holes in Tev-gravity scenarios

MOTIVATION

Hawking (1974): Black Holes radiate thermally





With a temperature at infinity:

$$T_H = \frac{\hbar c^3}{8\pi k_B G M}$$

“A Disturbing Point”

Any out-goint Hawking quanta with finite energy at infinity

$$E = \hbar\omega = k_B T_H$$

will have an exponentially increasing frequency when it is propagated backwards in time and measured at the horizon

$$E' = \hbar\omega' = \frac{k_B T_H}{\sqrt{1 - \frac{2GM}{c^2 r}}}$$

➡ **Hawking radiation involves transPlanckian frequencies!!**

t’Hooft, Jacobson, Unruh, ...

Hawking derivation via Bogolubov coefficients

$$\phi = \sum_j (a_j^{in} f_j^{in} + a_j^{in\dagger} f_j^{in*})$$

$$\phi = \sum_j (a_j^{out} f_j^{out} + a_j^{out\dagger} f_j^{out*})$$

Two vacuum states

$$a_i^{in}|in\rangle = 0 \quad a_i^{out}|out\rangle = 0$$

One can expand one set of modes in terms of the other:

$$f_j^{out} = \sum_i (\alpha_{ij} f_i^{in} + \beta_{ij} f_i^{in*})$$

→ **BOGOLUBOV TRANSFORMATION**

Hawking derivation via Bogolubov coefficients

Particle Number:

$$< in|N_j^{out}|in > \;=\; \hbar^{-1} < in|a_j^{out\dagger} a_j^{out}|in >= \sum_i |\beta_{ji}|^2$$

One can apply this scheme to the formation process of a black hole with metric:

$$ds^2 = -(1 - \frac{2GM}{c^2 r})c^2 dt^2 + \frac{1}{(1 - \frac{2GM}{c^2 r})} dr^2 + r^2 d\Omega^2$$

`in' modes $\quad f_{lm}^{in}(w) \longrightarrow_{t \to -\infty} e^{-i\omega v}\, Y_{lm} \qquad v = t + r$

'out' modes $\quad f_{lm}^{out}(w) \longrightarrow_{t \to \infty} e^{-i\omega u}\, Y_{lm} \qquad u = t - r$

Hawking derivation via Bogolubov coefficients

$$\langle in|N_{\omega_1\omega_2}|in\rangle = \int_0^{+\infty} dw' \beta_{w_1w'}\beta^*_{w_2w'}$$

$$\beta_{ij} = -(f_i^{out}, f_j^{in*}) = i\int_{I^-} dvr^2 d\Omega (f_i^{out}\overleftrightarrow{\partial}_v f_j^{in})$$

$$f_{lm}^{out}(\omega) \longrightarrow_{t\to-\infty} e^{-i\omega(v_H - 4M\ln\frac{|v-v_H|}{4M})} Y_{lm}$$

$$\beta_{\omega\omega'} = -\frac{1}{2M}\sqrt{\frac{\omega'}{\omega}}(4M)e^{-i(\omega+\omega')v_H}(-i4M\omega'+\epsilon)^{-1-i4M\omega}\Gamma(1+i4M\omega)$$

$$\int_0^{+\infty} dw' \beta_{w_1w'}\beta^*_{w_2w'} = \frac{1}{e^{8\pi M w_1}-1}\delta(w_1-w_2)$$

To obtain this result we have to assume that QFT is valid on all scales ¡¡¡

Impossing a cutoff

We can imposse a cutoff in w' - frequencies:

$$\int_0^{\Lambda} dw' \beta_{w_1 w'} \beta^*_{w_2 w'}$$

Hawking radiation is no longer thermal, even at small emission frequencies **(in comparison with the black hole size)**

In tension with results of string theory¡¡¡

Strominger-Vafa, Callan-Maldacena, ….

Alternative approach via two-point functions

[I. Agullo, J. N-S and G.J. Olmo, PRL (2006)]

$$\langle in|N_{ij}|in\rangle = \langle in|a^{out\dagger}_i a^{out}_j|in\rangle = \int_\Sigma d\Sigma_1^\mu d\Sigma_2^\nu [f_i^{out}(x_1)\overleftrightarrow{\partial}_\mu][f_j^{out*}(x_2)\overleftrightarrow{\partial}_\nu]\langle in| : \phi(x_1)\phi(x_2) : |in\rangle$$

$$\langle in|N_\omega|in\rangle = -\frac{1}{2\pi w}\int_{-\infty}^{+\infty} dz \ e^{-i2wz} \left[\frac{\kappa^2 e^{-\kappa z^-}}{(e^{-\kappa z^-}-1)^2} - \frac{1}{(z^-)^2}\right] = \frac{1}{e^{2\pi w \kappa^{-1}}-1}$$

$z = u_2 - u_1$ at $\mathcal{I}^+$

Thermal spectrum

We can compute explicitly the contribution of ultra-short distances $z = u_2 - u_1$ to the thermal spectrum.

Contribution of ultra-short distances to the Planckian spectrum

The contribution to the spectrum coming from distances $z \leq \epsilon$ is:

$$I(w,\kappa,\epsilon) = \frac{-1}{2\pi w}\int_{-\epsilon}^{+\epsilon} dz e^{-iwz}[\frac{\kappa^2 e^{-\kappa z}}{(e^{-\kappa z}-1)^2} - \frac{1}{z^2}]$$

This integral can be solved analytically

➡ **We can evaluate the contribution of $z \leq l_P$ to the spectrum.**

l_P =1.6 10^{-35}m is the Planck's length

Contribution of ultra-short distances to the Planckian spectrum

We obtain:

1st case $M = 3M \odot (\kappa = 8{,}9 \times 10^{-38}\,\% \frac{1}{l_P})$

At ω_{tipica} the contribution of $z \leq l_P$ is $\sim 10^{-38}\,\%$

At $96\omega_{tipica}$ the contribution of $z \leq l_P$ is of order of the total spectrum itself.

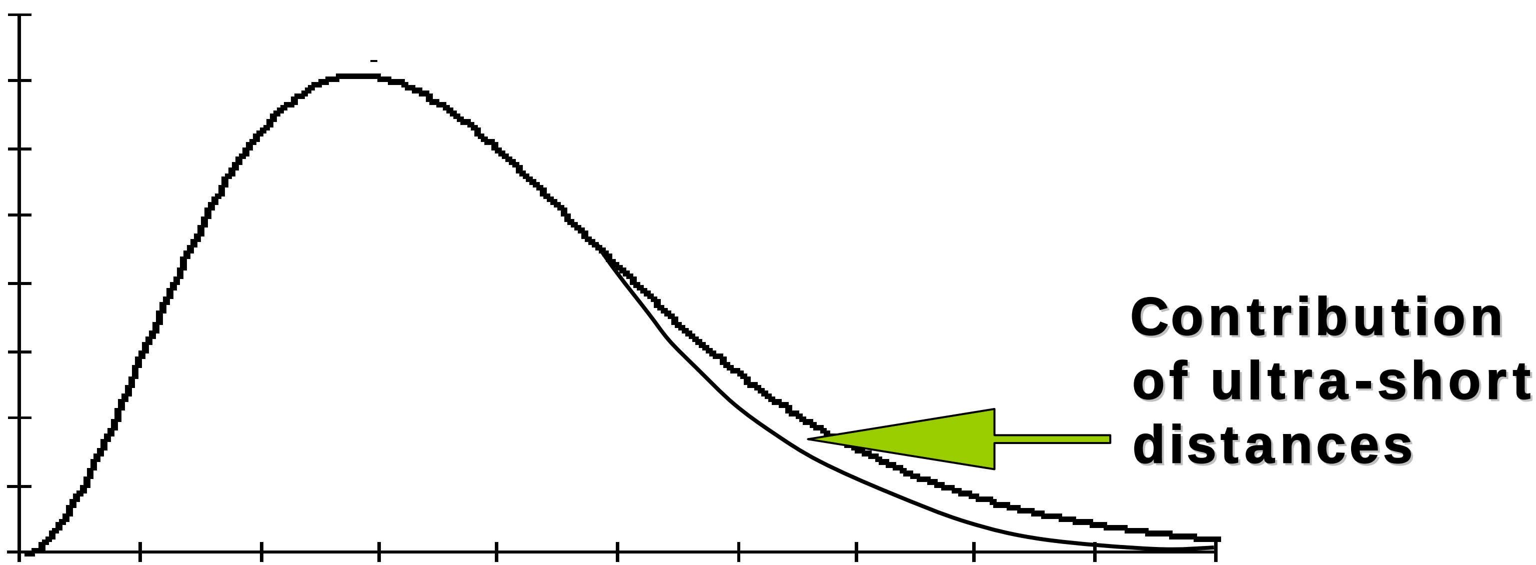
2nd case $M = 10^{-15} g(\kappa = 5{,}5 \times 10^{-21}\,\% \frac{1}{l_P})$

At ω_{tipica} the contribution of $z \leq l_P$ is $\sim 10^{-19}\,\%$

At $52\omega_{tipica}$ the contribution of $z \leq l_P$ is of order of the total spectrum itself.

Main results

- At ω_{tipica} the contribution of $z \leq l_P$ to the spectrum is of order of κl_P
- The contribution of $z \leq l_P$ increases exponentially for $\omega > \omega_{tipica}$



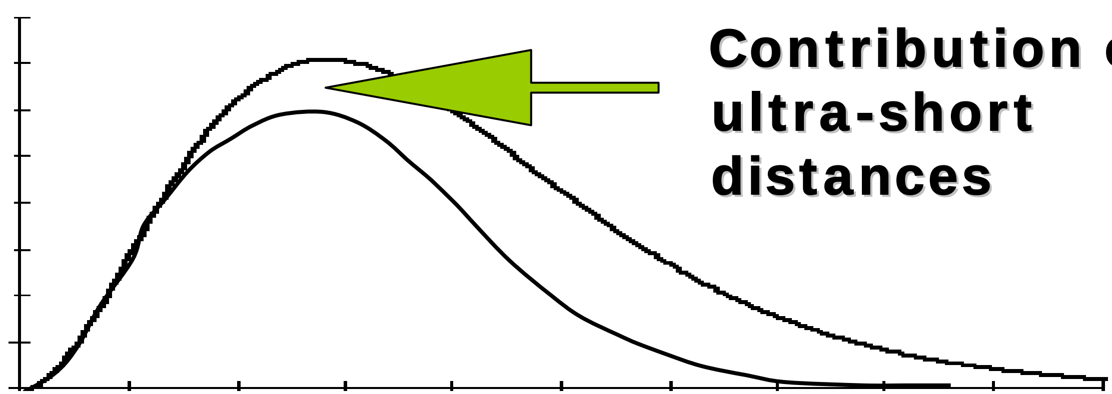
For microscopic black holes in TeV-gravity

M_{Planck} = 1 TeV [Giddings-Thomas, Dimopoulos-Lansberg]

We obtain (for n extra-dimensions)

M=5TeV	$n=2$	$n=4$	$n=6$
% contribution at ω_{tipico}	21 %	26 %	28 %
ω/ω_{tipico} where $I(w,\kappa,l_P) \sim \frac{1}{e^{2\pi\omega\kappa^{-1}}-1}$	3,3	3,1	3,0

M=10TeV	$n=2$	$n=4$	$n=6$
% contribution at ω_{tipico}	17 %	22 %	26 %
ω/ω_{tipico} where $I(w,\kappa,l_P) \sim \frac{1}{e^{2\pi\omega\kappa^{-1}}-1}$	3,6	3,3	3,1



Contribution of ultra-short distances

Conclusions

- We have introduced an alternative approach to analyze the tranPlanckian problem
- In this approach it is possible to evaluate explicitly the contribution of "ultra-short" distances to the spectrum
- At ω_{tipica} Hawking radiation is due mainly to physics coming from the same scale of the black hole.
- However, the contribution of short-distances increases exponencially with the emission frequency.