Perturbative Stability and Absorption Cross-Section in Higher-Derivative Heterotic String Black Holes

Filipe Moura

Instituto Superior Técnico, Lisbon

Collaboration with Ricardo Schiappa (hep-th/0605001)

Leading α' corrections

- Effective action in the Einstein frame

$$\frac{1}{2\kappa^2}\int\sqrt{-g}\left[\mathcal{R}-\frac{4}{d-2}\left(\partial^\mu\phi\right)\partial_\mu\phi+\mathrm{e}^{\frac{4}{2-d}\phi}\frac{\lambda}{2}\mathcal{R}^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}\right]\mathrm{d}^dx,$$

$$\lambda=\frac{\alpha'}{2},\frac{\alpha'}{4}\text{ (bosonic, heterotic).}$$

- Field equations

$$\mathcal{R}_{\mu\nu} + \lambda\mathrm{e}^{\frac{4}{2-d}\phi}\left(\mathcal{R}_{\mu\rho\sigma\tau}\mathcal{R}_{\nu}{}^{\rho\sigma\tau}-\frac{1}{2(d-2)}g_{\mu\nu}\mathcal{R}_{\rho\sigma\lambda\tau}\mathcal{R}^{\rho\sigma\lambda\tau}\right)=0;$$

$$\nabla^2\phi - \frac{\lambda}{4}\mathrm{e}^{\frac{4}{2-d}\phi}\left(\mathcal{R}_{\rho\sigma\lambda\tau}\mathcal{R}^{\rho\sigma\lambda\tau}\right)=0.$$

General setup

- Metric of the type

$$d\,s^2 = -f(r)\,d\,t^2 + f^{-1}(r)\,d\,r^2 + r^2\,d\,\Omega^2_{d-2};$$

- Variation of the metric

$$h_{\mu\nu} = \delta g_{\mu\nu};$$

- Variation of the Riemann tensor:

$$\begin{aligned}\delta\mathcal{R}_{\rho\sigma\mu\nu} &= \frac{1}{2}\Big(\mathcal{R}_{\mu\nu\rho}{}^{\lambda}h_{\lambda\sigma} - \mathcal{R}_{\mu\nu\sigma}{}^{\lambda}h_{\lambda\rho} \\ &- \nabla_\mu\nabla_\rho h_{\nu\sigma} + \nabla_\mu\nabla_\sigma h_{\nu\rho} - \nabla_\nu\nabla_\sigma h_{\mu\rho} + \nabla_\nu\nabla_\rho h_{\mu\sigma}\Big)\,.\end{aligned}$$

Perturbations on the $(d-2)$-sphere

- General tensors of rank at least 2 on the $(d-2)$-sphere can be uniquely decomposed in their *tensorial, vectorial and scalar* components.
- One can in general consider perturbations to the metric and any other physical field of the system under consideration.

Tensorial perturbations of the metric

- We consider only the tensorial part of $h_{\mu\nu}$:

$$h_{ij} = 2r^2 H_T(r,t)\mathcal{T}_{ij}\left(\theta^i\right), \; h_{ia} = 0, h_{ab} = 0$$

with

$$\left(\gamma^{kl} D_k D_l + k_T\right)\mathcal{T}_{ij} = 0, \; D^i \mathcal{T}_{ij} = 0, \; g^{ij}\mathcal{T}_{ij} = 0.$$

- D_i: $(d-2)$-sphere covariant derivative, associated to the metric γ_{ij}.
- $\mathcal{T}_{ij}$ are the eigentensors of D^2 on S^{d-2}
- $-k_T = 2 - \ell\left(\ell + d - 3\right)$ are the eigenvalues of D^2 on S^{d-2}, where $\ell = 2, 3, 4, \ldots$.

Tensorial perturbations of $\mathcal{R}_{\rho\sigma\mu\nu}$

$$
\begin{aligned}
\delta\mathcal{R}_{ijkl} &= \left[(2f-1)\,H_T + f\partial_r H_T\right]\left(g_{il}\mathcal{T}_{jk} - g_{ik}\mathcal{T}_{jl} - g_{jl}\mathcal{T}_{ik} + g_{jk}\mathcal{T}_{il}\right) \\
&+ r^2 H_T\left(D_i D_l \mathcal{T}_{jk} - D_i D_k \mathcal{T}_{jl} - D_j D_l \mathcal{T}_{ik} + D_j D_k \mathcal{T}_{il}\right); \\
\delta\mathcal{R}_{itjt} &= \left[-r^2\partial_t^2 H_T + \frac{1}{2} f f' r^2 \partial_r H_T + f f' r H_T\right]\mathcal{T}_{ij}; \\
\delta\mathcal{R}_{irjr} &= \left[-r\frac{f'}{f} H_T - -\frac{1}{2} r^2 \frac{f'}{f}\partial_r H_T - 2r\partial_r H_T - r^2\partial_r^2 H_T\right]\mathcal{T}_{ij}; \\
\delta\mathcal{R}_{ij} &= \frac{r^2}{f}\left(\partial_t^2 H_T\right)\mathcal{T}_{ij} - r^2 f\left(\partial_r^2 H_T\right)\mathcal{T}_{ij} - r^2 f'\left(\partial_r H_T\right)\mathcal{T}_{ij} \\
&- 2rf' H_T \mathcal{T}_{ij} + (2-d)\,rf\left(\partial_r H_T\right)\mathcal{T}_{ij} \\
&+ (2d-6)\,(1-f) H_T \mathcal{T}_{ij} + \ell\,(\ell+d-3)\,H_T \mathcal{T}_{ij}; \\
\delta\mathcal{R}_{rtrt} &= 0;\ \delta\mathcal{R}_{r\mu} = \delta\mathcal{R}_{t\mu} = 0;\ \ \delta\mathcal{R} = 0.
\end{aligned}
$$

Perturbations of the field equations

$$\begin{aligned}\delta\nabla^2\phi &- \frac{\lambda}{4}\mathrm{e}^{\frac{4}{2-d}\phi}\delta\left(\mathcal{R}_{\rho\sigma\lambda\tau}\mathcal{R}^{\rho\sigma\lambda\tau}\right)+\frac{\lambda}{d-2}\mathrm{e}^{\frac{4}{2-d}\phi}\mathcal{R}_{\rho\sigma\lambda\tau}\mathcal{R}^{\rho\sigma\lambda\tau}\delta\phi=0,\\ \delta\mathcal{R}_{ij} &+ \lambda\mathrm{e}^{\frac{4}{2-d}\phi}\left[\delta\left(\mathcal{R}_{i\rho\sigma\tau}\mathcal{R}_j^{\ \rho\sigma\tau}\right)-\frac{1}{2(d-2)}\mathcal{R}_{\rho\sigma\lambda\tau}\mathcal{R}^{\rho\sigma\lambda\tau}h_{ij}\right.\\ &- \left.\frac{1}{2(d-2)}g_{ij}\delta\left(\mathcal{R}_{\rho\sigma\lambda\tau}\mathcal{R}^{\rho\sigma\lambda\tau}\right)\right]+\frac{4}{d-2}\mathcal{R}_{ij}\delta\phi=0.\end{aligned}$$

- Spherical symmetry, $\partial_k\phi=0,\ (a,b=r,t):$

$$\begin{aligned}\delta\nabla^2\phi &= g^{ab}\partial_a\partial_b\delta\phi-g^{ab}\Gamma_{ab}^{\ \ c}\partial_c\delta\phi+g^{ij}\partial_i\partial_j\delta\phi-g^{ij}\Gamma_{ij}^{\ \ k}\partial_k\delta\phi\\ &- g^{ij}\Gamma_{ij}^{\ \ a}\partial_a\delta\phi.\end{aligned}$$

- Using $\delta\left(\mathcal{R}_{\rho\sigma\lambda\tau}\mathcal{R}^{\rho\sigma\lambda\tau}\right)=0$, we can set $\delta\phi=0$.

Perturbed graviton field equation

$$\left[1 - 2\lambda \frac{f'(r)}{r}\right] \left(\partial_t^2 H_T - f^2(r)\, \partial_r^2 H_T\right)$$
$$- f(r) \left[(d-2)\frac{f(r)}{r} + f'(r) + \frac{2\lambda}{r}\left(2(d-4)\frac{f(r)\,(1-f(r))}{r^2}\right.\right.$$
$$\left.\left. -2\frac{f(r) f'(r)}{r} - \left(f'(r)\right)^2\right)\right] \partial_r H_T$$
$$+ \frac{f(r)}{r}\left[\frac{\ell\,(\ell+d-3)}{r} - 2f'(r) + 2(d-3)\frac{1-f(r)}{r}\right.$$
$$\left. + \frac{\lambda}{r}\left(4\ell\,(\ell+d-3)\,\frac{1-f(r)}{r^2} + 2\,(d-3)\,\frac{(1-f(r))^2}{r^2} - r^2\,\frac{(f''(r))^2}{d-2}\right)\right] H_T = 0.$$

The Master Equation

The perturbation equation can be written as a "master equation"

$$\frac{\partial^2\Phi}{\partial x^2} - \frac{\partial^2\Phi}{\partial t^2} =: V_T\Phi.$$

- $d\,x/d\,r = 1/f$ ("tortoise" coordinate);
- $\Phi = k(r)H_T$ ("master" variable);
- V_T : potential for tensor-type gravitational perturbations. In classical EH gravity it is the same as the potential for scalar fields (Ishibashi, Kodama);
- $k(r) = \frac{1}{\sqrt{f}}\exp\left(\int \frac{\frac{f'}{f}+\frac{d-2}{r}+\frac{4}{r^3}(d-4)\lambda(1-f)-\frac{4}{r^2}\lambda f'-\frac{2}{rf}\lambda f'^2}{2-\frac{4}{r}\lambda f'}dr\right).$

The string-corrected tensor potential

$$
\begin{aligned}
V_{\mathsf{T}}[f(r)] &= f(r)\left(\frac{\ell\,(\ell+d-3)}{r^2}+\frac{(d-2)\,(d-4)\,f(r)}{4r^2}+\frac{2\,(d-3)\,(1-f(r))}{r^2}+\frac{(d-6)\,f'(r)}{2r}\right)\\
&+ \frac{\lambda}{r^2}\left[\left(\frac{2\ell\,(\ell+d-3)}{r}+\frac{(d-4)\,(d-5)\,f(r)}{r}+\frac{(d-3)\,(1-f(r))}{r}+(d-4)\,f'(r)\right)\right.\\
&\times \left(2\frac{1-f(r)}{r}+f'(r)\right)+\Big(3(d-3)-(4d-13)f(r)\Big)\frac{f'(r)}{r}-\\
&- \left.4\left(f'(r)\right)^2+(d-4)\,f(r)f''(r)-\frac{\left(rf''(r)\right)^2}{d-2}\right]f(r).
\end{aligned}
$$

This is the potential for tensor–type gravitational perturbations of any kind of static, spherically symmetric $\mathcal{R}^2$ string–corrected black hole in d–dimensions.

Study of the stability

- Solutions of the form $\Phi(x,t) = e^{i\omega t}\phi(x)$;
- The master equation is then written in the Schrödinger form,

$$\left[-\frac{d^2}{dx^2} + V\right]\phi(x) =: A\phi(x) = \omega^2\phi(x);$$

- A solution to the field equation is then stable if the operator A has no negative eigenvalues (Ishibashi, Kodama; Dotti, Gleiser).

"S-deformation" approach

Stability means positivity (for every possible ϕ) of the following inner product:

$$\begin{aligned}\langle \phi, A\phi \rangle &= \int_{-\infty}^{+\infty} \overline{\phi}(x) \left[-\frac{d^2}{dx^2} + V \right] \phi(x) \, dx \\ &= \int_{-\infty}^{+\infty} \left[\left| \frac{d\phi}{dx} \right|^2 + V \left| \phi \right|^2 \right] \, dx \\ &= \int_{-\infty}^{+\infty} \left[\left| D\phi \right|^2 + \widetilde{V} \left| \phi \right|^2 \right] \, dx\end{aligned}$$

with $D = \frac{d}{dx} + S$, $\widetilde{V} = V + f \frac{dS}{dr} - S^2$.

"S-deformation" approach (cont.)

- Taking $S = -\frac{f}{k}\frac{dk}{dr}$ we are left with

$$\langle \phi, A\phi \rangle = \int_{-\infty}^{+\infty} |D\phi|^2 \; dx + \int_{-\infty}^{+\infty} \frac{Q(r)}{f} |\phi|^2 \; dx,$$

with

$$\begin{aligned} Q &= f \left[\frac{\ell\,(\ell + d - 3)}{r^2} - \frac{2f'}{r} + 2(d-3)\frac{1-f}{r^2} \right. \\ &+ \frac{\lambda}{r^2} \left(\frac{2\ell\,(\ell + d - 3)}{r} \left(2\frac{1-f}{r} + f' \right) + \frac{2(d-3)\,(1-f)}{r} \left(\frac{1-f}{r} + 2f' \right) \right. \\ &- \left. \left. 4\,(f')^2 - \frac{(rf'')^2}{d-2} \right) \right]. \end{aligned}$$

Stability condition

- The second term of $\langle \phi, A\phi \rangle$ can be written as

$$\int_{R_H}^{+\infty} Q(r) \frac{|\phi|^2}{f(r)} dr.$$

- For $r > R_H$, $f(r) > 0$.
- This condition keeps valid with α' corrections as long as the black hole in consideration is *large*, i.e. $R_H \gg \sqrt{\lambda}$, which is true in string perturbation theory.
- This way the perturbative stability of a given black hole solution, with respect to tensor–type gravitational perturbations, follows if and only if one has $Q(r) > 0$ for $r \geq R_H$.

The Callan-Myers-Perry black hole

- The only free parameter is $\boldsymbol{\mu}$ (secondary hair);
- Horizon $R_H := (2\boldsymbol{\mu})^{\frac{1}{d-3}}$;
- $f(r) = \left(1 - \left(\frac{R_H}{r}\right)^{d-3}\right)\left[1 - \lambda \frac{(d-3)(d-4)}{2} \frac{R_H^{d-5}}{r^{d-1}} \frac{r^{d-1} - R_H^{d-1}}{r^{d-3} - R_H^{d-3}}\right]$;
- α'-corrected ADM black hole mass:

$$m = \left(1 + \frac{(d-3)(d-4)}{2} \frac{\lambda}{R_H^2}\right) \frac{(d-2)\, A_{d-2}}{\kappa^2} \boldsymbol{\mu}$$

- dilaton vanishes classically and only gets α'-corrections.

Stability of solutions with secondary hair

- For *any* string theory corrected, spherically symmetric, static solution, which has *no* dilaton field at the classical level (as is the case of the CMP solution), one has

$$Q(r) = \frac{f(r)}{1-2\lambda\frac{f'(r)}{r}}\left[\frac{\ell\left(\ell+d-3\right)}{r^2}+4\lambda\ell\left(\ell+d-3\right)\frac{1-f(r)}{r^4}\right]$$
$$\simeq f(r)\,\frac{\ell\left(\ell+d-3\right)}{r^2}\left[1+\frac{2\lambda}{r}\left(2\frac{1-f(r)}{r}+f'(r)\right)\right].$$

- One will have $Q(r)\geq 0$ for $r\geq R_H$, in any spacetime dimension, as long as

$$2\frac{1-f(r)}{r}+f'(r)\bigg|_{\lambda=0}>0.$$

Stability of solutions with secondary hair

- At the classical level, the solution is unique (Myers, Perry) and one has

$$2\frac{1-f(r)}{r}+f'(r)\bigg|_{\lambda=0}=\frac{2\boldsymbol{\mu}(d-1)}{r^{d-2}},$$

which is positive for any $r > R_H$.

- This proves stability under tensor–type gravitational perturbations of any spherically symmetric static solution with no dilaton at $\lambda = 0$ for any $d > 4$.

Scattering Theory

- The equation describing gravitational perturbations to a black hole solution allows for a study of scattering in this spacetime geometry.
- Classical result in EH gravity: for *any* spherically symmetric black hole in arbitrary dimension, the absorption cross–section of minimally–coupled massless scalar fields equals the area of the black hole horizon (Das, Gibbons, Mathur, 1997).
- Universality of the low–frequency absorption cross–sections of generic black holes in EH gravity?
- Not much work has been done on trying to extend such result with the inclusion of higher–derivative corrections.

Scattering of tensor–type gravitational waves

- We work in the low-frequency regime, $R_H \omega \ll 1$.
- This allows us to use the technique of matching solutions near the event horizon to solutions at asymptotic infinity (Natário, Schiappa).
- Leading contribution: s–wave, with $\ell = 0$.

Near-horizon solution (I)

- The potential $V_{\mathsf{T}}[f(r)]$ vanishes.
- The master equation reduces to a simple free–field equation

$$\left(\frac{d^2}{dx^2} + \omega^2\right)\left(k(r)H_T(r)\right) = 0$$

whose solutions are incoming plane–waves in the tortoise coordinate:

$$\begin{aligned} k(r) &\simeq iR_H^{\frac{d+1}{2}}\left(1 + \frac{(d+1)(d-4)}{4}\,\frac{\lambda}{R_H^2}\right) + \mathcal{O}\left(r - R_H\right), \\ H_T(x) &= A_{\mathsf{near}}e^{i\omega x}. \end{aligned}$$

Near-horizon solution (II)

Very close to the event horizon, $r \simeq R_H$, one has

$$V_{\mathsf{T}}(r) \simeq \frac{(d-2)(d-3)^2}{2}\left(1-\frac{(d-1)(d-4)^2}{d-2}\,\frac{\lambda}{R_H^2}\right)\frac{r-R_H}{R_H^3}+\mathcal{O}\left((r-R_H)^2\right),$$
$$x(r) \simeq \frac{R_H}{d-3}\left(1+\frac{(d-1)(d-4)}{2}\,\frac{\lambda}{R_H^2}\right)\log\left(\frac{r-R_H}{R_H}\right)+\mathcal{O}\left(r-R_H\right)$$

and then

$$H_T(r) \simeq A_{\mathsf{near}}\left(1+i\frac{R_H\omega}{d-3}\left(1+\frac{(d-1)(d-4)}{2}\,\frac{\lambda}{R_H^2}\right)\log\left(\frac{r-R_H}{R_H}\right)\right)$$

Asymptotic infinity solution (I)

- The asymptotic region of the CMP black hole is basically flat Minkowski spacetime.
- At asymptotic infinity, again $V_{\mathsf{T}}[f(r)]$ vanishes.
- The master equation reduces to a free–field equation whose solutions are incoming or outgoing plane–waves in the tortoise coordinate.
- In the original radial coordinate,

$$H_T(r) = (r\omega)^{(3-d)/2} \left[A\, J_{(d-3)/2}\,(r\omega) + B\, N_{(d-3)/2}\,(r\omega) \right] .$$

Asymptotic infinity solution (II)

- At low–frequencies, with $r\omega \ll 1$, such solution becomes

$$H_T(r) \simeq A_{\text{asymp}} \frac{1}{2^{\frac{d-3}{2}}\Gamma\left(\frac{d-1}{2}\right)} + B_{\text{asymp}} \frac{2^{\frac{d-3}{2}}\Gamma\left(\frac{d-3}{2}\right)}{\pi\left(r\omega\right)^{d-3}} + \mathcal{O}\left(r\omega\right).$$

- In order to compute the absorption cross–section, one now needs to relate A_{near}, A_{asymp} and B_{asymp}.

Intermediate region solution (I)

- $V_{\mathsf{T}}(r) \gg \omega^2$, $r\omega \ll 1$ (low–frequency constraint), $\frac{r-R_H}{R_H} \gg (R_H\omega)^2$.
- Solution perturbative in $\lambda : H_T(r) = H_0(r) + \lambda H_1(r)$.
- $H_0(r)$ satisfies

$$\left[-f(r)\frac{d}{dr}\left(f(r)\frac{d}{dr}\right) + f(r)\left(\frac{(d-2)(d-4)f(r)}{4r^2} + \frac{(d-2)f'(r)}{2r}\right)\right](k_0(r)H_0(r)) = 0$$

with $k_0(r) = iR_H^{3/2} r^{\frac{d-2}{2}}$.

- The most general solution is

$$H_0(r) = A^0_{\text{inter}} + B^0_{\text{inter}} \log\left(1 - \frac{R_H^{d-3}}{r^{d-3}}\right).$$

Intermediate region solution (II)

- Solving for $H_1(r)$ requires splitting into homogeneous and non-homogeneous parts:

$$H_1(r) = A^1_{\text{inter}} + B^1_{\text{inter}} \log\left(1 - \frac{R_H^{d-3}}{r^{d-3}}\right) + H_1^{\text{NH}}(r).$$

- After a very long analysis one concludes that

$$H_T(r) = A_{\text{inter}} + B_{\text{inter}} \log\left(1 - \frac{R_H^{d-3}}{r^{d-3}}\right) + \lambda H_1^{\text{NH}}(r).$$

Intermediate region solution (III)

- Although we do not know the exact expression for $H_1^{\mathsf{NH}}(r)$, we know that it approximately behaves near the black hole horizon as

$$\begin{aligned} H_1^{\mathsf{NH}}(r) &\simeq \frac{(d-1)(d-4)}{2R_H^2}\left[-(d+2)B^0_{\mathsf{inter}} + 4\left(A^0_{\mathsf{inter}} + B^0_{\mathsf{inter}}\log\left(\frac{r-R_H}{R_H}\right)\right.\right. \\ &+ \left.\left. B^0_{\mathsf{inter}}\log(d-3)\right)\right]\frac{r-R_H}{R_H} \\ &+ \mathcal{O}\left(\left(\frac{r-R_H}{R_H}\right)^2\log\left(\frac{r-R_H}{R_H}\right)\right), \end{aligned}$$

and that at asymptotic infinity this term can be neglected.

- It is this solution that allows us to interpolate between the solutions near the event horizon and at asymptotic infinity.

Intermediate region solution (IV)

- Near the horizon,

$$H_T(r) \simeq A_{\text{inter}} + B_{\text{inter}} \log\left(\frac{r - R_H}{R_H}\right) + B_{\text{inter}} \log(d-3) + \cdots$$

- At asymptotic infinity, one finds

$$H_T(r) \simeq A_{\text{inter}} - B_{\text{inter}} \frac{R_H^{d-3}}{r^{d-3}} + \cdots$$

Calculation of the fluxes

- Matching coefficients:

$$A_{\text{as}} = 2^{\frac{d-3}{2}} \Gamma\left(\frac{d-1}{2}\right) A_{\text{int}} = 2^{\frac{d-3}{2}} \Gamma\left(\frac{d-1}{2}\right) A_{\text{near}},$$

$$B_{\text{as}} = -\frac{\pi\left(R_H\omega\right)^{d-3}}{2^{\frac{d-3}{2}}\Gamma\left(\frac{d-3}{2}\right)} B_{\text{int}} = -\frac{i\pi\left(R_H\omega\right)^{d-2}}{2^{\frac{d-3}{2}}(d-3)\Gamma\left(\frac{d-3}{2}\right)}\left(1+\frac{(d-1)(d-4)}{2}\,\frac{\lambda}{R_H^2}\right) A_{\text{near}}.$$

- Near the black hole event horizon the flux per unit area is

$$J_{\text{near}} = \frac{1}{2i}\left(H_T^\dagger(x)\frac{dH_T}{dx} - H_T(x)\frac{dH_T^\dagger}{dx}\right) = \omega\left|A_{\text{near}}\right|^2.$$

- The flux per unit area at infinity is

$$J_{\text{as}} = \frac{1}{2i}\left(H_T^\dagger(r)\frac{dH_T}{dr} - H_T(r)\frac{dH_T^\dagger}{dr}\right) = \frac{2}{\pi} r^{2-d}\omega^{3-d}\left|A_{\text{as}}B_{\text{as}}\right|.$$

The absorption cross–section

- General formula: $\sigma = \frac{\int r^{d-2} J_{\text{asymp}} d\Omega_{d-2}}{J_{\text{near}}}$.
- In our case,

$$\sigma_{\mathsf{T}}^{\ell=0} = A_H \left(1 + \frac{(d-1)(d-4)}{2} \frac{\lambda}{R_H^2} \right).$$

- σ receives α' corrections with respect to the EH result, although it is *still* proportional to the area of the event horizon;
- σ is increased due to the string theoretic corrections.
- Possible general formula:

$$\sigma_{\mathsf{T}}^{\ell=0} = \frac{d-3}{R_H f'(R_H)} A_H.$$

Conclusions

- We extended the perturbation theory to $\mathcal{R}^2$ stringy gravity;
- We studied the stability of black hole solutions under tensor type gravitational perturbations, and proved the perturbative stability of the CMP solution for any space-time dimension;
- We applied the master equation to compute the absorption cross–section for low frequency gravitational waves for the CMP black hole. We showed that this cross–section is still proportional to the area of the black hole horizon, in spite of receiving α' corrections;
- We proposed a generalization for the cross–section which could be valid to all orders in α'.