11th Marcel Grossmann meeting
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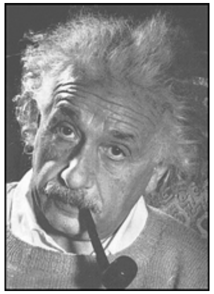
Kaluza-Klein black hole with gravitational charge in Einstein-Gauss-Bonnet gravity



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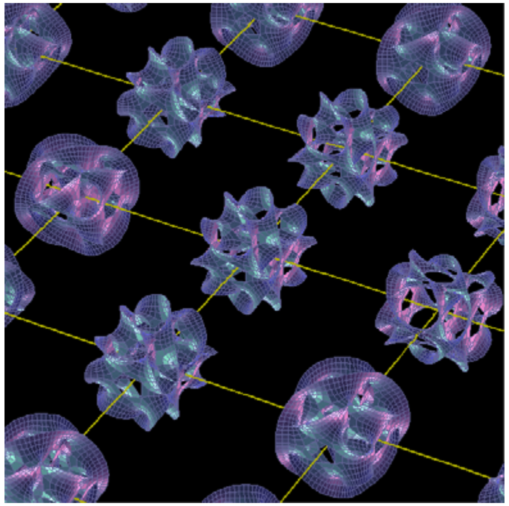
Introduction(1) : Unified theory and higher-dimensions

- Unified theory
  - superstgin-/M-theory is a promissing candidate
  - Higher-dimensions are required (n-dimensions: n=10 or 11)
    - Higher-dimensional spacetime has a rich structure
      - Ex. Black ring (horizon is $S^2 \times S^1$)
- 4-dimensional spacetime is observable
  - Small and compact extra-dimensions
    - Kaluza-Klein sapcetime: $M^n = M^4 \times M^{n-4}$
- Effects of extra dimensions on our observable universe should be investigated



Introduction(2)

- Superstring theory is a promising candidate of the unified theory
  - Non-perturbative aspects are not known completely so far
- Let us consider the n($\geqq$ 5)-dimensional action with the Gauss-Bonnet (GB) term (curvature corrections to GR)
  - Naturally arise in the low-energy limit of heterotic superstring theory (Gross & Sloan 1987)

- We present a new Kaluza-Klein vacuum black hole solution with negatively curved compact extra dimensions for n$\geqq$6

Einstein-Gauss-Bonnet gravity

Non-negative coupling constant

Gauss-Bonnet term

- n(>4)-dimensional action： $\kappa_n \equiv \sqrt{8\pi G_n},$

$$S = \int d^n x \sqrt{-g} \left[\frac{1}{2\kappa_n^2} (R - 2\Lambda + \alpha L_{GB}) \right] + S_{\rm matter}$$

$$L_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}.$$

- Basic equations:

$$\mathcal{G}^\mu{}_\nu \equiv G^\mu{}_\nu + \alpha H^\mu{}_\nu + \Lambda \delta^\mu{}_\nu = \kappa_n^2 T^\mu{}_\nu,$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R,$$

$$H_{\mu\nu} \equiv 2\Big[RR_{\mu\nu} - 2R_{\mu\alpha}R^\alpha{}_\nu - 2R^{\alpha\beta}R_{\mu\alpha\nu\beta} + R_\mu{}^{\alpha\beta\gamma}R_{\nu\alpha\beta\gamma}\Big] - \frac{1}{2} g_{\mu\nu} L_{GB}.$$

$$H_{\mu\nu} \equiv 0 \quad \text{for} \quad n \leq 4$$

Tensor decomposition to $M^4 \times K^{n-4}$

$$g_{\mu\nu} = \mathrm{diag}(g_{AB}, r_0^2\gamma_{ab}),\ A,B = 0,\cdots,3;\ \ a,b = 4,\cdots,n-1.$$

◎ g_{AB} is a general metric on M^4

◎ γ_{ab} is the unit metric of a constant curvature spacetime K^{n-4}, of which curvature is $\bar{k} = \pm 1, 0$

◎ r_0 is a constant warp factor

$$\begin{aligned}
\mathcal{G}^A{}_B &= \left[1 + \frac{2\bar{k}\alpha(n-4)(n-5)}{r_0^2}\right]\overset{(4)}{G}{}^A{}_B \\
&\quad + \left[\Lambda - \frac{\bar{k}(n-4)(n-5)}{2r_0^2} - \frac{\bar{k}^2\alpha(n-4)(n-5)(n-6)(n-7)}{2r_0^4}\right]\delta^A{}_B, \\
\mathcal{G}^a{}_b &= \delta^a{}_b\left[-\frac{1}{2}\overset{(4)}{R} + \Lambda - \frac{(n-5)(n-6)\bar{k}}{2r_0^2}\right. \\
&\quad \left. -\alpha\left\{\frac{\bar{k}(n-5)(n-6)}{r_0^2}\overset{(4)}{R} + \frac{1}{2}\overset{(4)}{L}_{GB} + \frac{(n-5)(n-6)(n-7)(n-8)\bar{k}^2}{2r_0^4}\right\}\right],
\end{aligned}$$

(4) means the geometric quantity on M^4

A no-go theorem on M^4

Theorem 1

If

(i) $r_0^2 = -2\bar{k}\alpha(n-4)(n-5)$

(ii) $\alpha\Lambda = -(n^2 - 5n - 2)/[8(n-4)(n-5)]$

are satisfied,

then $\mathcal{G}^A{}_B = 0$ *for* $n \geq 6$ *and* $\bar{k}$ *and* Λ *being non-zero.*

For small and positive α (required by superstring), $\Lambda<0$ and k=-1.
With appropriate identifications, negatively curved small compact extra-dimensions are obtained.

Corollary1: If (i) and (ii) are satisfied, a matter cannot exist on M^4

Hereafter we consider **the vacuum case with conditions (ii) and (ii)**, then the governing equation for g_{AB} is a **single** scalar equation on M4, $\mathcal{G}^a{}_b = 0$

Vacuum solution on M^4 with $M^4=M^2 \times K^2$

We seek a static solution on M^4 such as

$$g_{AB}dx^A dx^B = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Sigma^2_{2(k)}$$

$d\Sigma^2_{2(k)}$ is the unit metric on $\mathcal{K}^2$ and $k = \pm 1, 0.$

With conditions (i) and (ii), $\mathcal{G}^a{}_b = 0$ gives

$$\frac{1}{n-4}\overset{(4)}{R} + \frac{\alpha}{2}\overset{(4)}{L}_{GB} + \frac{2n-11}{\alpha(n-4)^2(n-5)} = 0.$$

General solution is

$$f(r) = k + \frac{r^2}{2(n-4)\alpha}\left[1 \mp \left\{1 - \frac{2n-11}{3(n-5)} + \frac{4(n-4)^2\alpha^{3/2}\mu}{r^3} - \frac{4(n-4)^2\alpha^2 q}{r^4}\right\}^{1/2}\right]$$

Two parameter family: μ and q
Two branches (plus- and minus- branch): no GR limit ($\alpha \to 0$) in both branches

A new Kaluza-Klein vacuum solution with negatively curved extra-dimensions!

Properties of the new Kaluza-Klein vacuum solution (1)

For $r \to \infty$,

$$f(r) \approx k \mp \frac{\alpha^{1/2}\mu\sqrt{3(n-4)(n-5)}}{r} \pm \frac{\alpha q\sqrt{3(n-4)(n-5)}}{r^2} + \frac{r^2}{2(n-4)\alpha}\underbrace{\left(1 \mp \sqrt{\frac{n-4}{3(n-5)}}\right)}_{\text{positive}}$$

Asymptotically Reissner-Nortstrom-anti-deSitter for k=1
in both branches (topological black hole for k≠1)
μ : mass parameter
q: charge-like parameter (no Maxwell field, however)
we call q ``gravitational charge'' (can be + and -)

Comparison with the charged solution with $M^n = M^2 \times K^{n-2}$

n(≧5)-dimensional solution with the Maxwell field (Cai 2002)

$$F_{rt} = \frac{Q}{r^{n-2}}$$

$$ds^2 = -g(r)dt^2 + \frac{1}{g(r)}dr^2 + r^2 d\Sigma^2_{n-2(k)}$$

$$g(r) = k + \frac{r^2}{2(n-3)(n-4)\alpha}\left[1 \mp \left\{1 + \frac{8(n-3)(n-4)\alpha\Lambda}{(n-1)(n-2)} + \frac{8(n-3)(n-4)\kappa_n^2\alpha M}{(n-2)V^k_{n-2}r^{n-1}} - \frac{(n-4)\alpha\kappa_n^2 Q^2}{(n-2)\pi g_c^2 r^{2(n-2)}}\right\}^{1/2}\right],$$

g_c: coupling constant of the Maxwell field, V^k_{n-2}: area constant

Our solution corresponds to n=4 case, although it does not admit

$$f(r) = k + \frac{r^2}{2(n-4)\alpha}\left[1 \mp \left\{1 - \frac{2n-11}{3(n-5)} + \frac{4(n-4)^2\alpha^{3/2}\mu}{r^3} - \frac{4(n-4)^2\alpha^2 q}{r^4}\right\}^{1/2}\right]$$

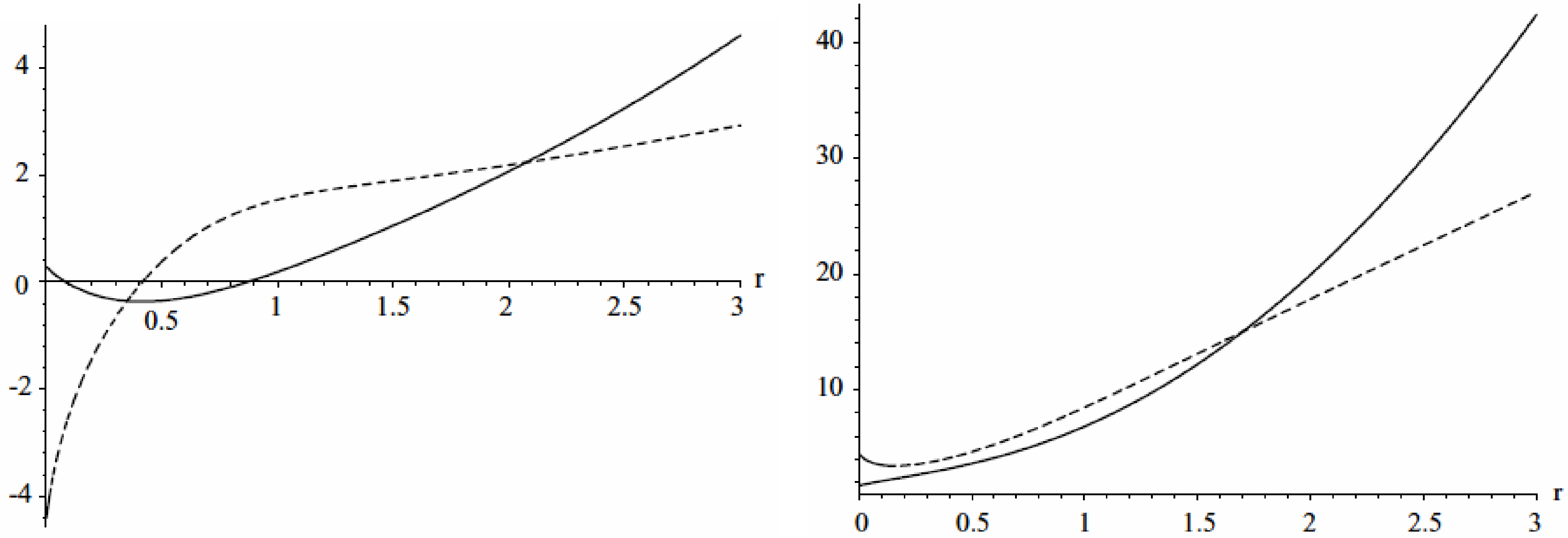
Q^2 corresponds to q (can be negative, however)

Properties of the new Kaluza-Klein vacuum solution (2)

- Central singularity at r=0
  - Kretschmann invariant ~ $1/r^4$
- ``Branch'' singularity at $r=r_b>0$, where inside the square root in f(r) vanishes
  - Metric is complex for $r<r_b$
  - Kretschmann invariant ~ $1/(r-r_b)^3$
    - Divergent behavior is weak
- There exist two horizons at most
  - Black hole event horizon and inner horizon
  - The solution have rich global structures depending on the parameters μ and q

Properties of the new Kaluza-Klein vacuum solution (3)

f(r) (solid) and df/dr (dashed) with k=1, α =0.1, n=6, μ =2, and q=-0.5



Minus-branch (two-horizons):
Black hole with an inner horizon

Plus-branch (no horizon):
Naked singularity

Summary

- Einstein-Gauss-Bonnet gravity with a cosmological constant Λ
- A new Kaluza-Klein vacuum black hole solution is obtained for $n \geqq 6$
  - $M^n = M^2 \times K^2 \times H^{n-4}$ with a constant warp factor r_0 of H^{n-4}
    - Special relations between α, Λ, k and r_0 are required
    - $\alpha > 0$ (required by superstring) gives
      - $\Lambda < 0$ and $k = -1$ (extra dimensions H^{n-4} are negatively curved)
  - Two parameters (μ and q)
    - Q: charge-like new parameter (gravitational charge) however no Maxwell field
      - Q<0 corresponds to the imaginary charge
  - Two branches (plus- and minus-branches): no GR limit ($\alpha \to 0$)
  - M^4 is asymptotically RN-AdS
    - Two horizons at most
    - Two classes of singularities
      - Central (r=0) and branch (r>0) singularities
- The global structure depending on μ and q will be reported