Perturbatively non-uniform charged black strings: a new stable phase

Umpei MIYAMOTO
(Waseda U, Japan)

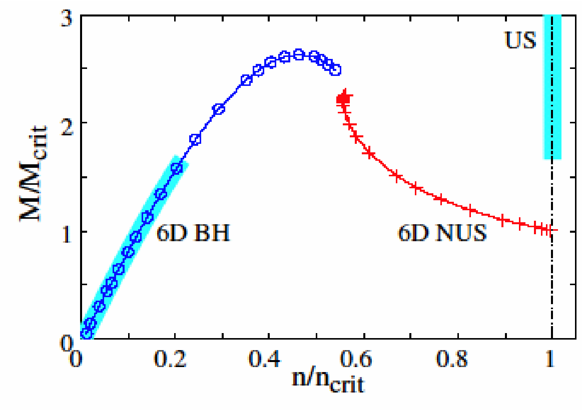
Hideaki KUDOH
(UCSB, US)

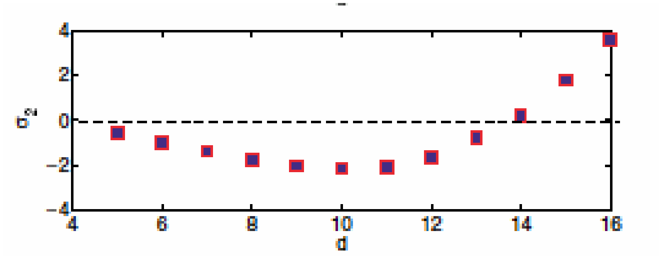
MG11@Berlin, July 2006

Introduction

Intro: Caged (vacuum) BH-BS

- BS suffers from Gregory-Laflamme instability
  - Final fate has not been known
    [Horowitz, Maeda; Choptuik et al.]
- Phase structure in 5, 6 dims.
  - Perturbation
    - NUBS branch [Gubser]
    - BH branch [Gorbonos, Kol]
  - Fully non-linear
    [Wiseman; Kudoh, Wiseman; Kleihaus et al.]
- Critical dimension; D*=13.5
  [Sorkin; Kudoh, UM]





$$S_{NU} > S_U \text{ for } D > D^* = 13.5$$

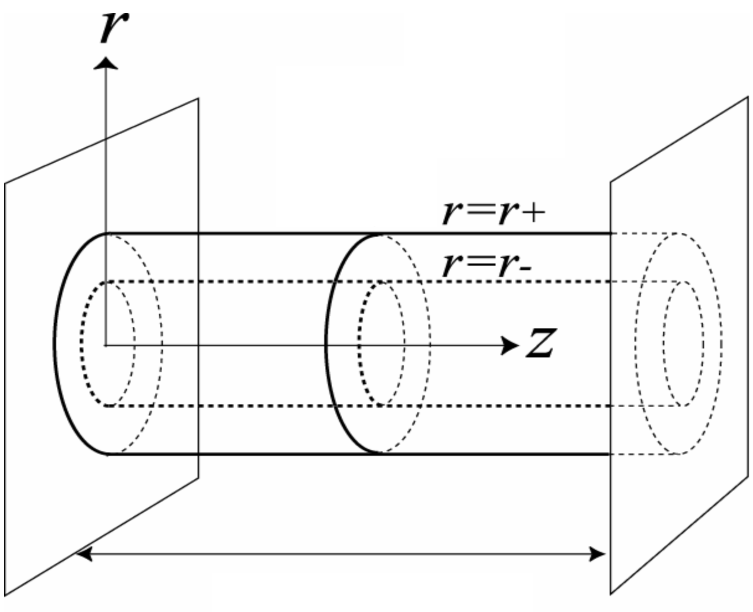
Motivation: Why charged BH-BS?

- Gravity inevitably couples to other fields
  - Gauge fields, dilaton etc
- Gubser-Mitra Conjecture
  - Correlation between thermodynamical and GL instabilities
    [Reall; Freiss et al.]
  - GL instability depends on extremality/coupling
    [GL; Hirayama et al.; Gubser]
- Almost nothing is known in non-linear regime
  - Phase structure around the GM point
    [Harmark, Obers; Kudoh, UM]
  - KK BH (exact) solutions
    [Ishihara et al.; Maeda, Dadhich]
- New type of critical phenomena
  - Lowered critical dim. (<13.5)
  - or Critical charge

Model & results

Background; Magnetic BS [Horowitz, Strominger '91]

$$I_D = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left[R - \frac{2}{(D-3)!} F_{D-3}^2 \right] \qquad D \geq 5$$



$$ds^2 = -f_+ dt^2 + \frac{dr^2}{f_+ f_-} + f_- dz^2 + r^2 d\Omega_{D-3}^2$$

$$f_\pm(r) = 1 - \left(\frac{r_\pm}{r}\right)^{D-4}$$

$$F_{D-3} = Q \varepsilon_{D-3}$$

$$C_Q := \left(\frac{\partial M}{\partial T}\right)_Q$$

< 0 for $Q<Q_c$ (unstable)
> 0 for $Q>Q_c$ (stable)

Static perturbations

Static axisymm.

$$ds^2 = -e^{2a(r,z)} f_+ dt^2 + \frac{e^{2b(r,z)}}{f_+ f_-} dr^2 + e^{2b(r,z)} f_- dz^2 + e^{2c(r,z)} r^2 d\Omega_{D-3}^2$$

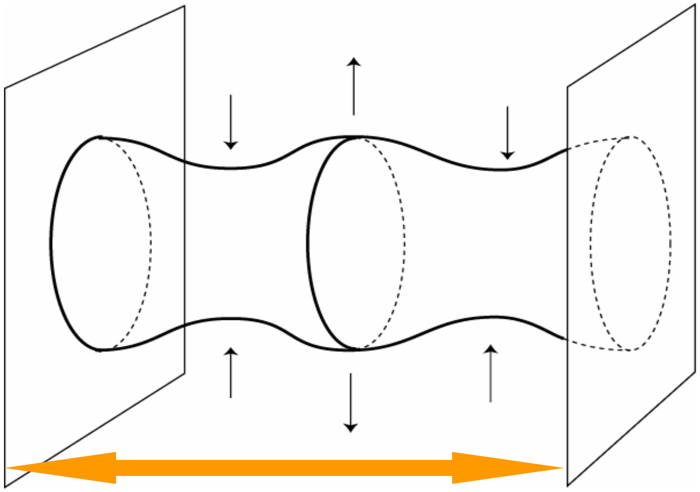
Fourier decomposition

$$X = a, b, c$$

$$X(r,z) = \sum_{n=0}^{\infty} \epsilon^n X_n(r) \cos(nKz)$$

$$X_n = \sum_{p=0}^{\infty} \epsilon^{2p} X_{n,p}(r) \qquad K = \sum_{q=0}^{\infty} \epsilon^{2q} k_q$$

K: GL critical wave number

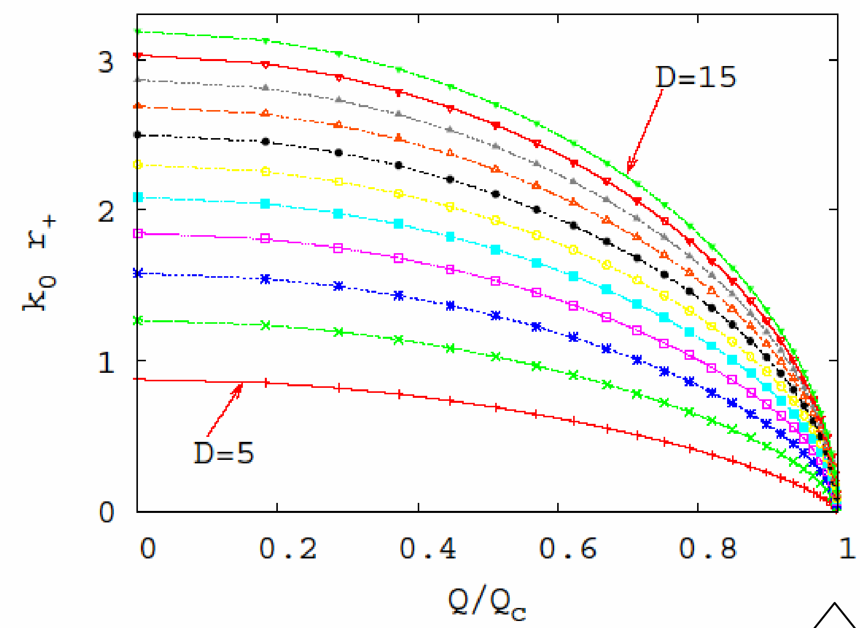


$$\frac{2\pi}{K}$$

Result (1): GL critical mode

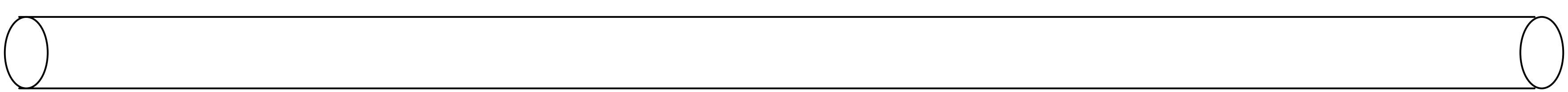
1st order pert.

$$a_1''(r) = \dots$$
$$c_1''(r) = \dots k_0, Q, D$$



- GLI disappear at Qc
  - Realization of GMC
  - Arbitrarily thin BS near Qc

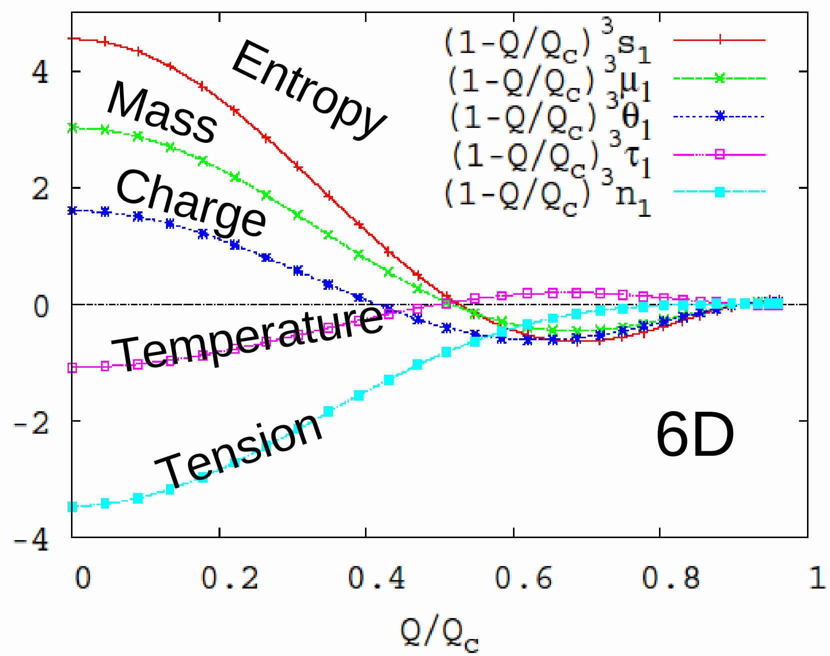
$$\frac{2\pi}{k_0} \to +\infty$$



Result (2): Thermodynamics (i)

- 2nd & 3rd order ➔ corrections to thermodynamical quant.
$$(\delta M, \delta S, \delta T, \delta n, \delta Q, \delta \Phi_H)$$

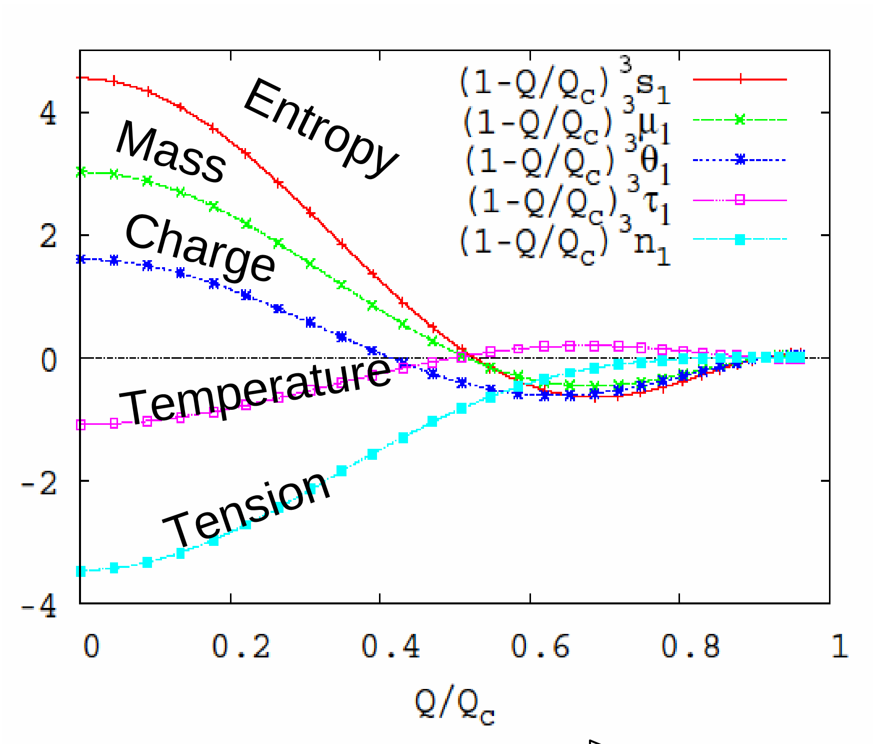
$$\frac{\delta S}{S} \simeq s_1 \epsilon^2$$
$$\frac{\delta M}{M} \simeq \mu_1 \epsilon^2$$
$$\frac{\delta Q}{Q} \simeq \vartheta_1 \epsilon^2$$
$$\frac{\delta T}{T} \simeq \tau_1 \epsilon^2$$
$$\frac{\delta n}{n} \simeq n_1 \epsilon^2$$



- Critical charge Q* appears!!

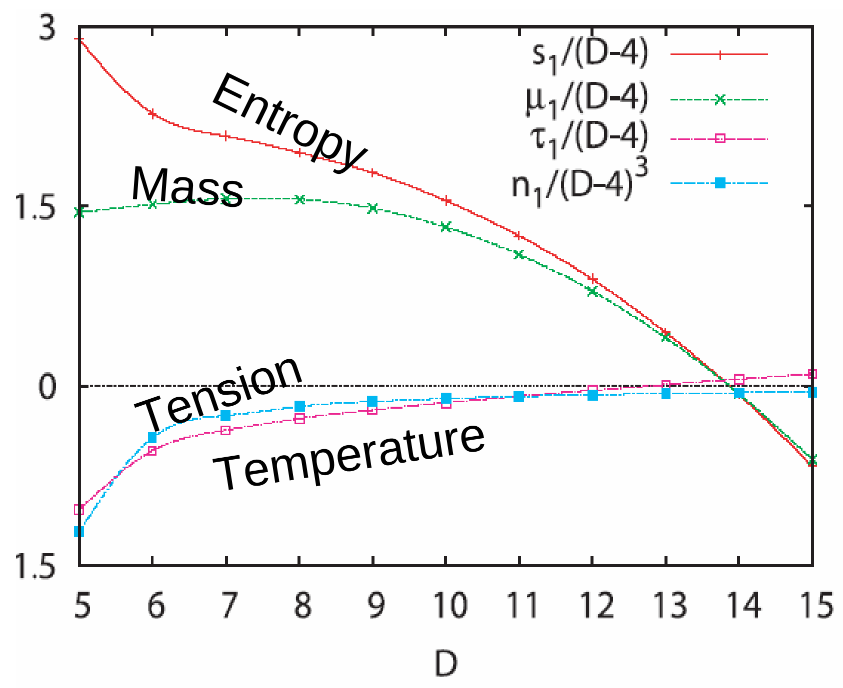
Similarity btwn charge and dims.

Charged BS in D=6



Chage

Neutral BS in D-dims



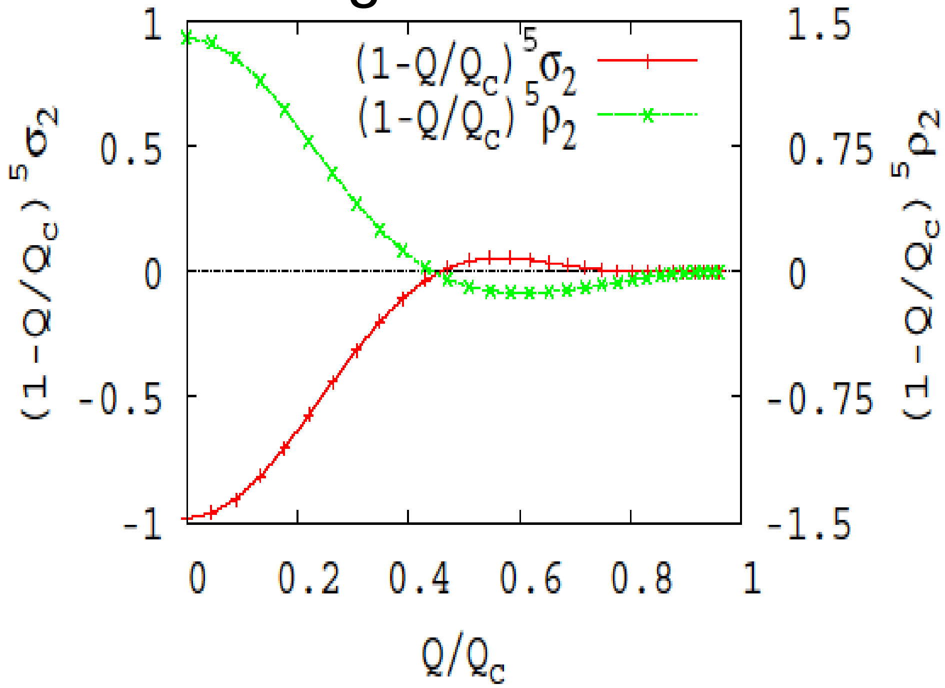
Dims.

[Sorkin '04]

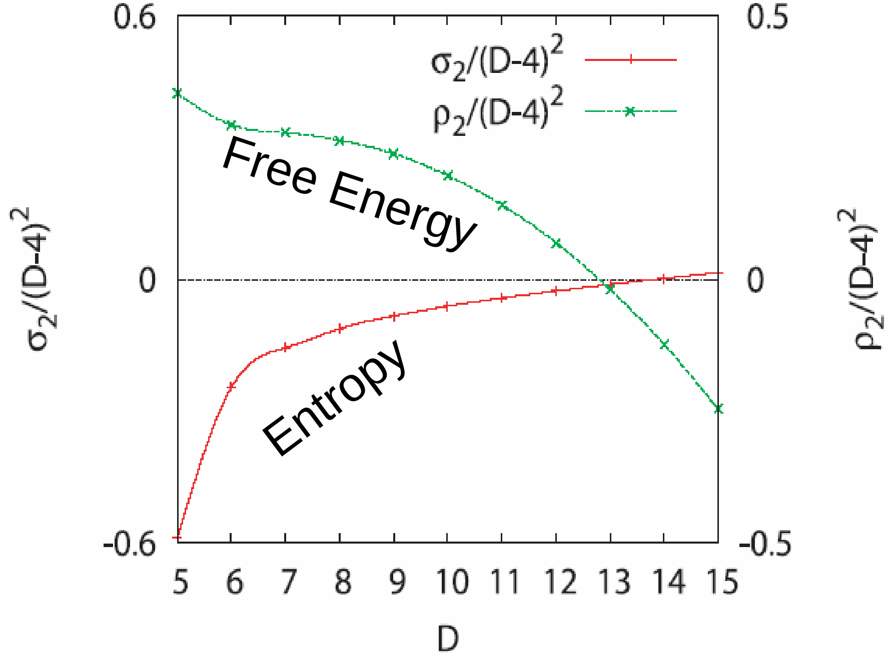
Result (3): Thermodynamics (ii)

$$\frac{S_{\rm NU}-S_{\rm U}}{S_{\rm U}} \simeq \sigma_2\,\epsilon^4, \quad \text{for same } (M,Q) \qquad \frac{F_{\rm NU}-F_{\rm U}}{F_{\rm U}} \simeq \rho_2\,\epsilon^4, \quad \text{for same } (T,Q)$$

Charged BS in D=6



Neutral BS in D-dims



Critical charge appears!!

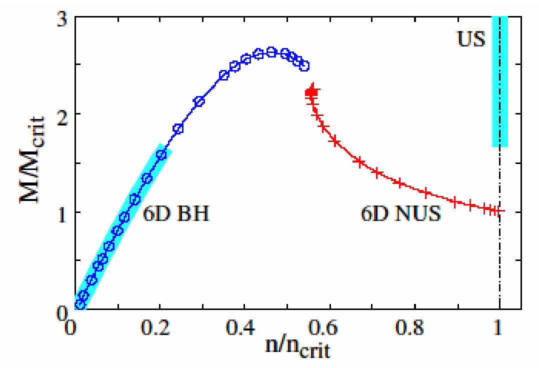
$$S_{NU} > S_U \;\; \& \;\; F_{NU} < F_U \;\; \text{for} \;\; Q > Q^* \simeq 0.5 Q_c$$

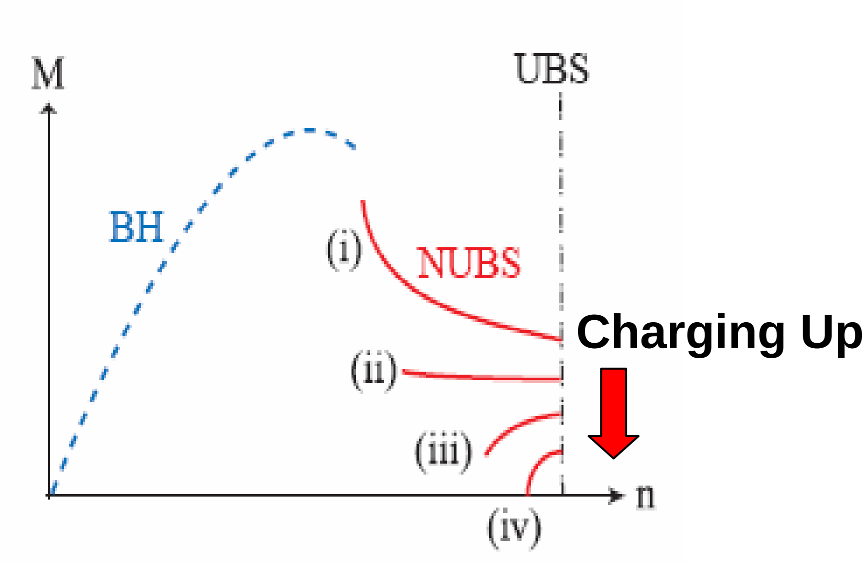
Summary & discussion

Summary

- Non-uniform magnetic NUBS is constructed perturbatively (up to 3rd order).
- New critical phenomenon due to charge:
  - Charge plays the similar role of dims.
  - Thermodynamically favored NUBS is possible even in lower dims. $(5 \leq D \leq 13 < 13.5)$

Implication & applications





Applications:

- Dilaton, electric charge
- Fully non-linear calculation [Kudoh, UM in progress]
- Dynamical evolution
  - Charge is easier than 14D?