Gravitational collapse in electrically charged Lovelock gravity: Hamiltonian formulation

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Outline

Origin and Motivation

- This method was introduced in Crisóstomo and Olea (2004).
- For radial collapse in spherically symmetric spacetimes it is much more convenient than Israel's (1966).
- It is also very convenient for theories with higher powers of the curvature in the action (Crisóstomo, del Campo, Saavedra (2004) and below).

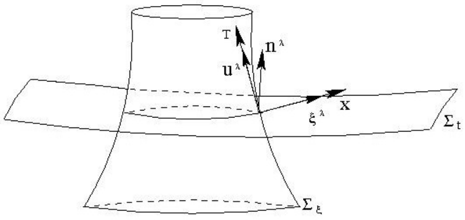

Figure: The schematic view of the hypersurfaces: spacelike hypersurface Σ_t; n^λ timelike normal Σ_t; u^λ tangent vector to the shell submanifold Σ_ξ, and ξ^λ normal vector to Σ_ξ. (From Crisóstomo and Olea (2004))

Einstein-Hilbert Action

We introduce it using the Einstein-Hilbert action:

$$I = -\kappa \int d^d x \sqrt{-{}^{(d)}g}(R - 2\Lambda), \tag{1}$$
$$\kappa = \frac{1}{2(d-2)\Omega_{(d-2)}G}, \; \Lambda = -\frac{(d-1)(d-2)}{2\ell^2}.$$

Hamiltonian action with ADM foliation:

$$I = \int dt d^{(d-1)}x (\pi^{ij}\dot{g}_{ij} - N^{\perp}\mathcal{H}_{\perp} - N^i \mathcal{H}_i), \tag{2}$$
$$\mathcal{H}_{\perp} = -\frac{1}{\kappa\sqrt{g}}\left(\pi_{ij}\pi^{ij} - \frac{1}{(d-2)}(\pi_i^i)^2\right) + $$
$$+\kappa\sqrt{g}\left({}^{(d-1)}R(g) - 2\Lambda\right) + \sqrt{g}T_{\perp\perp}, \tag{3}$$
$$\mathcal{H}_i = -2\pi^j_{i|j} + \sqrt{g}T_{\perp i}, \tag{4}$$

The energy momentum tensor is the perfect fluid tensor:

$$T_{\mu\nu} = [\sigma u_\mu u_\nu - \hat{\tau}(h_{\mu\nu} + u_\mu u_\nu)]\delta(X) \tag{5}$$

Radial Collapse

Spherically symmetric and static interior and exterior spacetimes:

$$ds^2_\pm = -N^2_\pm(r) f^2_\pm(r) dt^2_\pm + f^{-2}_\pm(r) dr^2 + r^2 d\Omega^2_{(d-2)}, \quad (6)$$

$$N_\pm = 1, \quad \text{choice of time matching condition.} \quad (7)$$

This *Ansatz* is replaced in the constraint $\mathcal{H}_\perp$:

$$\mathcal{H}_\perp = -\frac{\sqrt{g}}{2\Omega_{(d-2)}G}\left[\frac{(d-3)}{r^2}(1-f^2) - \frac{(f^2)'}{r} + \frac{(d-1)}{\ell^2}\right] + +\sqrt{g}\,T_{\perp\perp}. \quad (8)$$

After int., with $T^{\mu\nu} n_\mu n_\nu = T_{\perp\perp} = \alpha\sigma\delta(r-R)$, $\sigma > 0$:

$$-\Delta f^2(r) = (\Omega_{(d-2)} R\sigma)\left(\sqrt{f^2_+ + \dot{R}^2} + \sqrt{f^2_- + \dot{R}^2}\right) G. \quad (9)$$

We conclude that a collapse to a naked singularity is ruled out. *E.g.* interior horizon and exterior naked solution, $f^2_+ > 0$ everywhere and $f^2_-(R_h) = 0$ contradicts $-\Delta f^2(r) > 0$.

Radial Collapse

For Schw-AdS, with $m = \Omega_{(d-2)} R^{(d-2)} \sigma$, $\alpha_{\pm} = \sqrt{f_{\pm}^2 + \dot{R}^2}$:

$$\Delta M = M_+ - M_- = \frac{1}{2}(\alpha_+ + \alpha_-) m. \tag{10}$$

For Schwarzschild over empty spacetime ($M_- = 0$), where $a = \frac{M}{m}$, $f_-(R) = 1$, and $f_+(R) = 1 - 2M/R$:

$$(1 + \dot{R}^2)^{1/2} = a + \frac{M}{2\,a\,R}$$

This recovers the example in Israel (1966).

Lovelock Action

(Lovelock (1970))

$$I = \kappa \int_{\mathrm{M}} \sum_{p=0}^{[d/2]} \alpha_p \, \epsilon_{a_1 \ldots a_d} R^{a_1 a_2} \wedge \cdots \wedge R^{a_{2p-1} a_{2p}} \wedge \wedge e^{a_{2p+1}} \wedge \cdots \wedge e^{a_d} + S_{\mathrm{m}} . \tag{11}$$

- $R^{ab} = d\omega^{ab} + \omega^a_c \wedge \omega^{cb}$, where the ω^{ab} is the spin connection, and the e^a is a local frame one-form.
- S_{m} matter action, in this case

$$S_{\mathrm{m}} = -\frac{1}{4\epsilon\Omega_{d-2}} \int_{\mathrm{M}} \sqrt{-g}\, F_{\mu\nu} F^{\mu\nu} d^d x . \tag{12}$$

- The first three terms of the action are the cosmological constant, the Einstein term, and the Gauss-Bonnet term.

Coefficents and Constants

- In order for there to be one cosmological constant, we have to choose the α_p. A possible choice is (Crisóstomo, Troncoso, Zanelli (2000)):

$$\alpha_p = \begin{cases} \frac{\ell^{2(p-k)}}{(d-2p)} \begin{pmatrix} k \\ p \end{pmatrix}, & p \leq k \\ 0, & p > k \end{cases} . \qquad (13)$$

with $1 \leq k \leq [(d-1)/2]$.

- This set of theories possesses only two constants:

$$\kappa = \frac{1}{2(d-2)!\Omega_{(d-2)} G_k} , \qquad \Lambda = -\frac{(d-1)(d-2)}{2\ell^2} .$$

- Top k and odd d: Chern-Simons type (in 2+1: BTZ).
- Top k and even d: Born-Infeld type (in 3+1: EH with Λ).

Solutions: metric functions

The static and spherically symmetric solutions of these equations are given, in the same *Ansatz* as before (Crisóstomo, Troncoso, Zanelli (2000)):

$$f^2(r) = 1 + \frac{r^2}{\ell^2} - \chi\, g_k(r)\,, \tag{14}$$

$$\chi = (\pm 1)^{k+1}$$

$$g_k(r) = \left(\frac{2G_k M + \delta_{d-2k,1}}{r^{d-2k-1}} - \frac{\epsilon\, G_k}{(d-3)} \frac{Q^2}{r^{2(d-k-2)}} \right)^{1/k} . \tag{15}$$

- For odd k, black hole.
- For even k, also naked singularity.
- There are 2 singularities, $r = 0$ and $r = r_e > 0$: spacetime exists eff. only for $r > r_e$ (except for $k = 1$).

Integration

For the same *Ansatz*, the total $\perp$ constraint is, with $T_{\perp\perp} = \sigma\gamma\delta(r - R(\tau))$:

$$\mathcal{H}_\perp = -\kappa\frac{(d-2)!}{r^{d-2}}\sqrt{g}\frac{d}{dr}\left\{r^{d-1}\sum_p (d-2p)\,\alpha_p\left(\frac{1-f^2}{r^2}\right)^p\right\} + \\ +\sqrt{g}\,T_{\perp\perp} + \mathcal{H}_\perp^{(\mathrm{em})}\,. \quad (16)$$

For Lovelock, after integration, where $\gamma_\pm = \sqrt{f_\pm^2 + \dot{R}^2}$, $m = \Omega_{(d-2)}R^{(d-2)}\sigma$:

$$(M_+ - M_-) - \frac{\epsilon(Q_+^2 - Q_-^2)}{2(d-3)}\frac{1}{R^{d-3}} = \frac{1}{2}m(\gamma_+ + \gamma_-)\,. \quad (17)$$

Let us denote the lhs as $K(R)$.

Shell equations

Squaring (17), with ($\sigma > 0$, $K(R) > 0$):

$$\dot{R}^2 = \left(\frac{K}{m} \pm \frac{m}{4\,K} \left(f_+^2 - f_-^2 \right) \right)^2 - f_\pm^2 \geq 0 \tag{18}$$

From this one implicitly determines the bounce radius R_b.
Replacing $\dot{R}^2$ in the shell eq. we find a minimum radius of validity, R_c.
Deriving (18) w.r.t. the proper time of the shell τ, $\hat{\tau}$ surf. tension:

$$\begin{aligned} m\ddot{R} &= \frac{(Q_+^2 - Q_-^2)\gamma_+\gamma_- m}{2\,K\,R^{d-2}} - (d-2)R^{d-3}\Omega_{d-2}\hat{\tau}\gamma_+\gamma_- \\ &- \frac{m^2}{2\,K} \times \left(\gamma_- \frac{df_+^2}{dR} + \gamma_+ \frac{df_-^2}{dR} \right) . \end{aligned} \tag{19}$$

The first term provides a way of avoiding collapse to naked singularities in Chern-Simons (Crisóstomo, del Campo, Saavedra (2000)).

Properties

- Shell restricted by $\dot{R}^2 > 0$, in which domain is governed by the equation of motion.
- There is a minimum radius R_c for the validity of the $\dot{R}^2$ equation.
- It is the new term provided by the electric charge that is the source of the dynamical barrier preventing collapse.
- There is also a condition for collapse to a black hole, R_h being the horizon radius:

$$\frac{4}{m^2}\left(1+\frac{R_{\mathrm{h}}^2}{\ell^2}\right)^{2k-1}\left(\frac{R_{\mathrm{h}}^{d-2k-1}}{2\,G_k}\right)^2 \geq 1\,. \tag{20}$$

- The method does not apply when a shell crosses a horizon, we must return to Israel (Gao and Lemos (2006)).

Graphic Examples

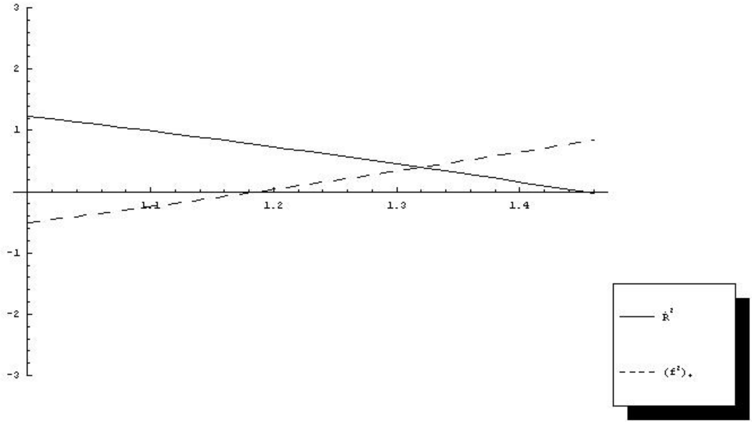

Figure: $d = 10,\ k = 4,\ \ell = 1,\ \chi = 1,\ M = 20,\ Q = 1.5,\ m = 15;\ G_4 = 1 = \epsilon$
Collapses to black hole, acceleration negative up to horizon.

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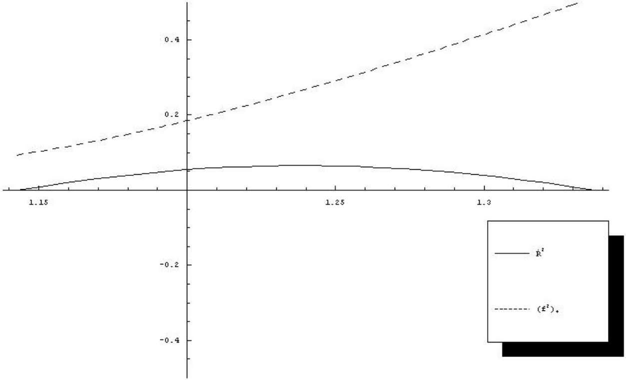

Figure: $d = 10,\ k = 4,\ \ell = 1,\ \chi = 1,\ M = 20,\ Q = 15,\ m = 15;\ G_4 = 1 = \epsilon$
Overcharged black hole.

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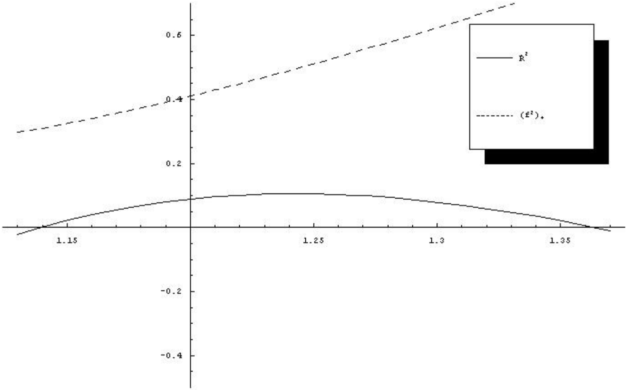

Figure: $d = 11,\ k = 5,\ \ell = 1,\ \chi = 1,\ M = 20,\ Q = 15,\ m = 15;\ G_5 = 1 = \epsilon$
Overcharged black hole spacetime.

Conclusion and References

Conclusions:

- This method is more economic in radial spherical collapse than Israel's, specially for higher powers of the curvature. It just requires the knowledge of the inside and outside metric functions.
- We were able to show through this method that the electric charge provides a mechanism for cosmic censorship for Chern-Simons type theories.

References:

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- J. Crisóstomo, S. del Campo, and J. Saavedra, Phys. Rev. D **70**, 064034 (2004).