NEW NON-UNIFORM BLACK STRING SOLUTIONS

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- **Introduction**
- **Black Strings in** $D = 5, 6$
- **Topology Changing Transition?**
- **Black Strings in EMD Theory**
- **Conclusions**

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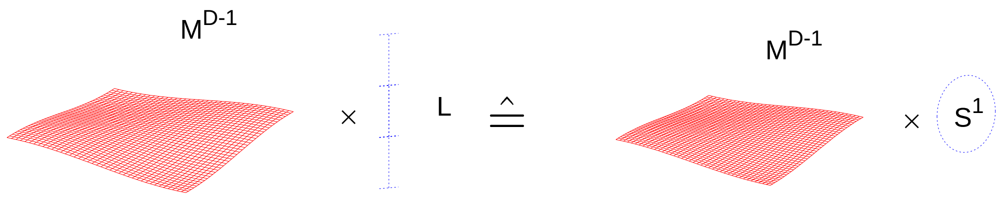
*Supported by DFG

Introduction

- **vacuum solutions** in $D \geq 5$ space-time dimensions
- $(D-1)$ space-time $\times$ 1 extra (compact) dimension



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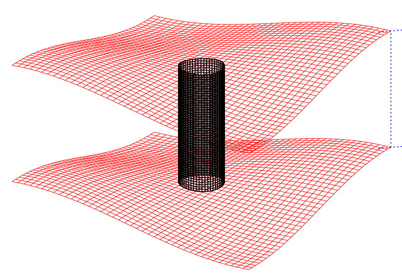
uniform black string

- $D-1$ black hole
- translational invariance
- horizon $S^{D-3} \times S^1$
- exist for all L, M (mass)
- unstable for small M (Gregory-Laflamme)



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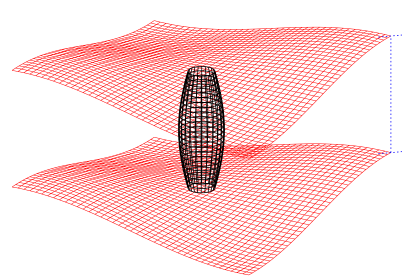
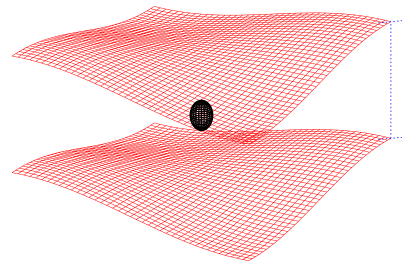
non-uniform black string

- perturbative solutions (Gubser, Sorkin)
- depend on extra coord.
- horizon $S^{D-3} \times S^1$
- exist above GL point
- exact (num) solutions (Wiseman, Kol)



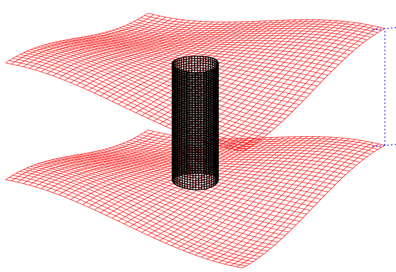
caged black hole

- horizon S^{D-2} (Sorkin, Kol, Piran; Kudoh, Wiseman)
- exist for small masses



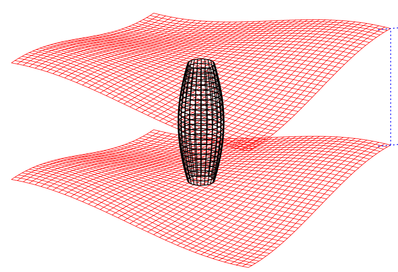
Topology changing transition?

uniform black string



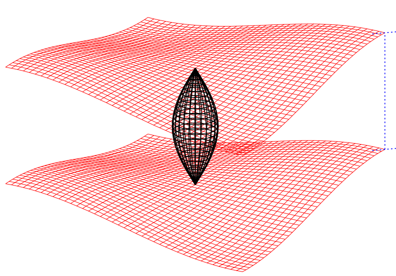
$S^{D-3} \times S^1$

non-uniform black string



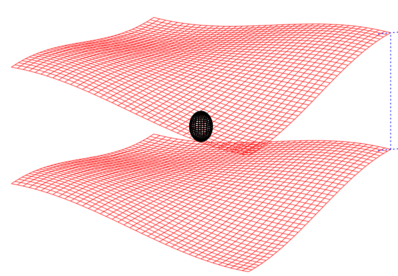
$S^{D-3} \times S^1$

pinch off



$\rightarrow S^{D-2}$

caged black hole



S^{D-2}

Ansatz

$$ds^2 = -e^{2A(r,z)} f(r) dt^2 + e^{2B(r,z)} \left[\frac{dr^2}{f(r)} + dz^2 \right] + e^{2C(r,z)} r^2 d\Omega^2_{D-3}$$

periodic coordinate z, length L

$$r^2 = \sum_{i=1}^{D-2} (x^i)^2, \quad x^D = t, \quad f(r) = 1 - \left(\frac{r_0}{r}\right)^{D-4}$$

- **Einstein equations:** $G_t^t = 0, \quad G_r^r + G_z^z = 0, \quad G_\theta^\theta = 0$
- **Constraints:** $G_r^z = 0, \quad G_r^r - G_z^z = 0$
- **all other equations hold trivially**

- **uniform black strings:** $A = 0, \quad B = 0, \quad C = 0$

- **Horizon at** $r = r_0$

  Transform $r = \sqrt{r_0^2 + \tilde{r}^2} \quad \rightsquigarrow \quad$ **Horizon at** $\tilde{r} = 0$

- **Boundary conditions**

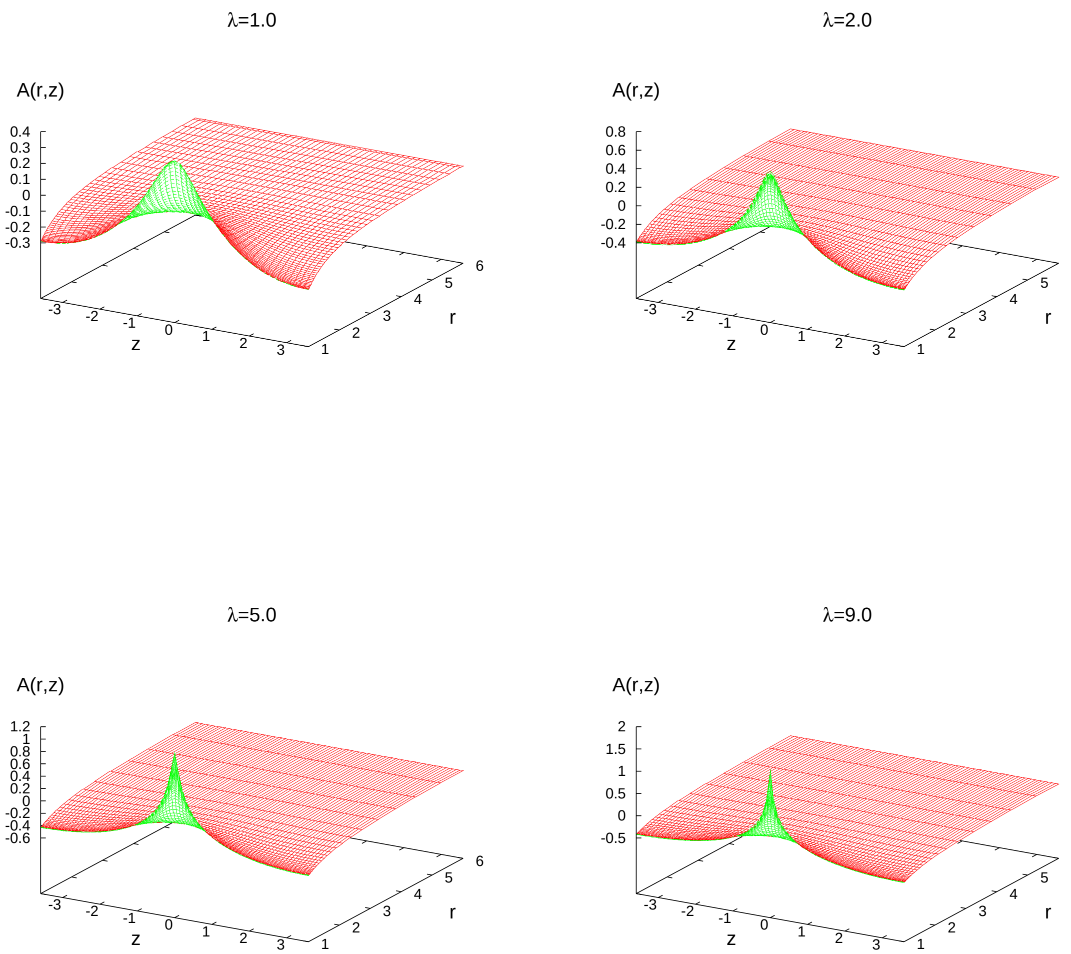
$\tilde{r} = 0 \qquad A - B = d_0, \quad \partial_{\tilde{r}} A = \partial_{\tilde{r}} C = 0$ (**also** $\partial_{\tilde{r}} B$ **required**)

$\tilde{r} \to \infty \qquad A = B = C = 0 \qquad (M^{D-1} \times S^1)$

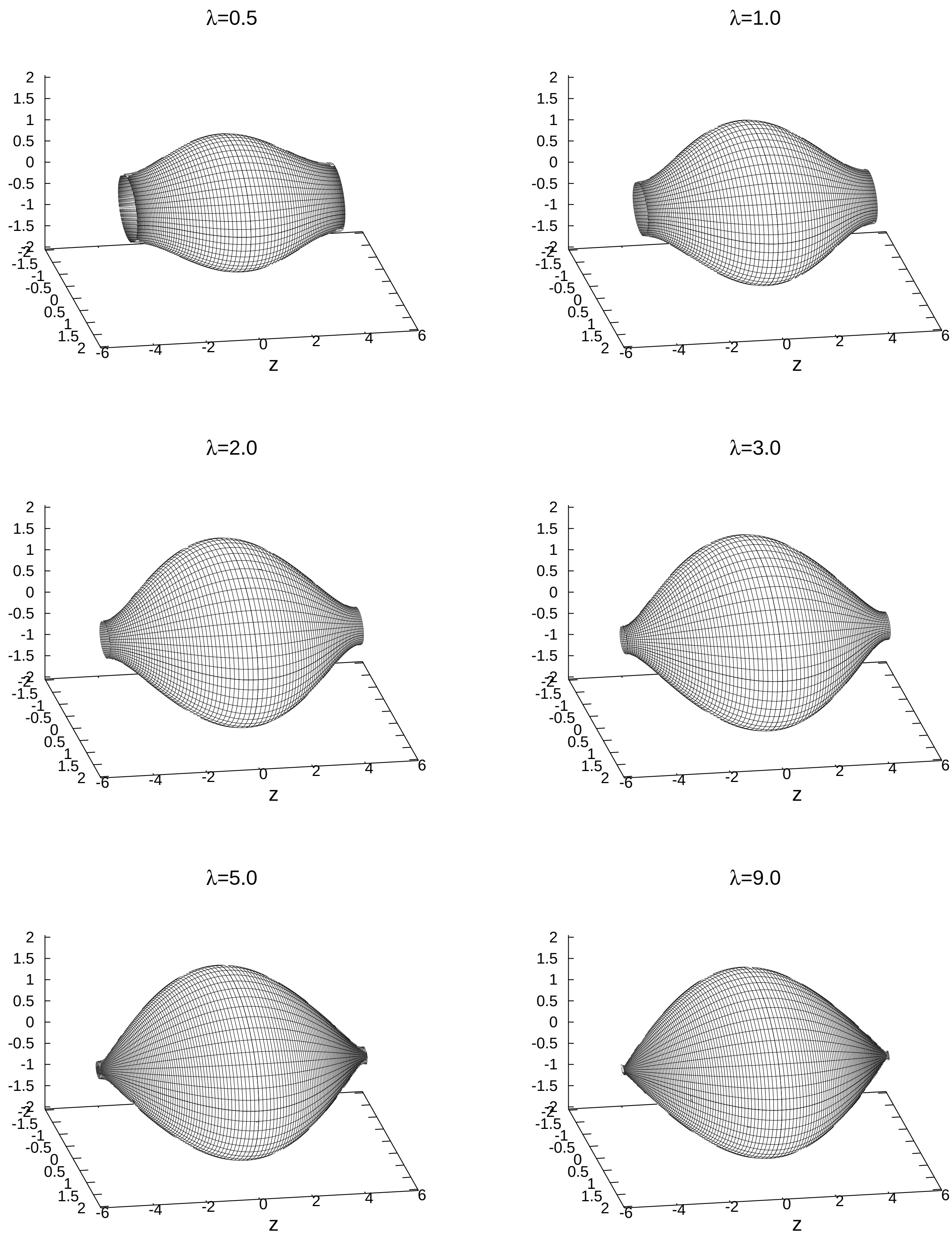
$z = 0, L/2 \qquad \partial_z A = \partial_z B = \partial_z C = 0$

- $d_0 = const.$, $e^{d_0} \sim T$ **Hawking temperatur**

$$\lambda = \frac{1}{2}\left(\frac{\mathcal{R}_{\max}}{\mathcal{R}_{\min}} - 1\right)$$



Embedded Horizon



Physical Quantities

- **Mass** M**, Tension** $\mathcal{T}$**, relative tension** $n = \mathcal{T}/M$
- **Temperature** $T = e^{d_0} T_0$
- **Entropy** $S = \frac{S_0}{L} \int_0^L e^{B_H + (D-3)C_H} dz$

Smarr relation $TS = \dfrac{D-3-n}{D-2} M$

1^{st} **law** $dM = TdS + \mathcal{T} dL$

- **Asymptotics** $(D = 5)$

$$g_{tt} \to -1 + \frac{c_t}{r}, \qquad g_{zz} \to 1 + \frac{c_z}{r}$$

$$M = \frac{\Omega_2 L}{16\pi G} (2c_t + c_z), \quad n = \frac{c_t - 2c_z}{2c_t - c_z}$$

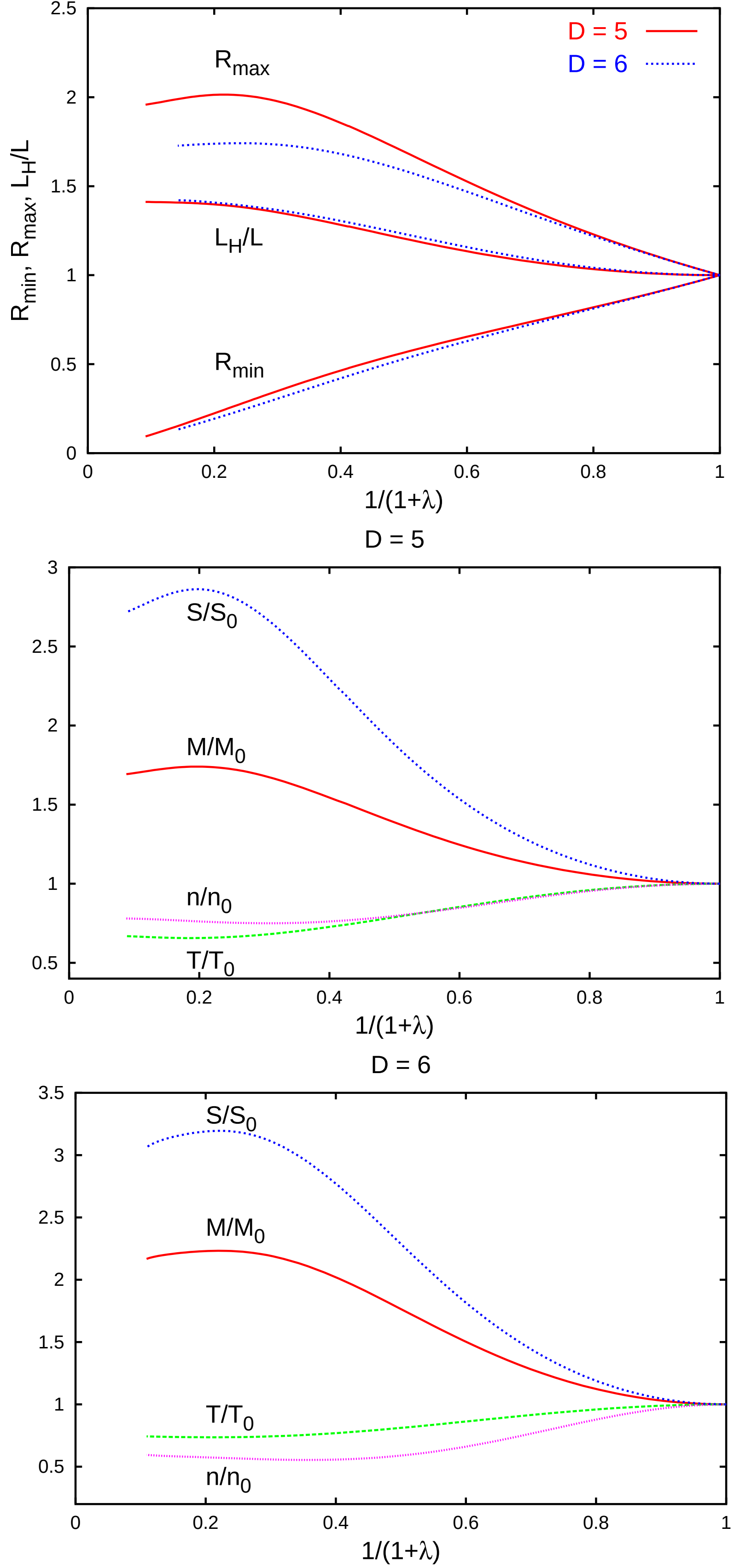
- c_z **is error prone,**

  use instead 1^{st} **law:** $M = M_0 + \int_{S_0}^{S} T(s') ds'$

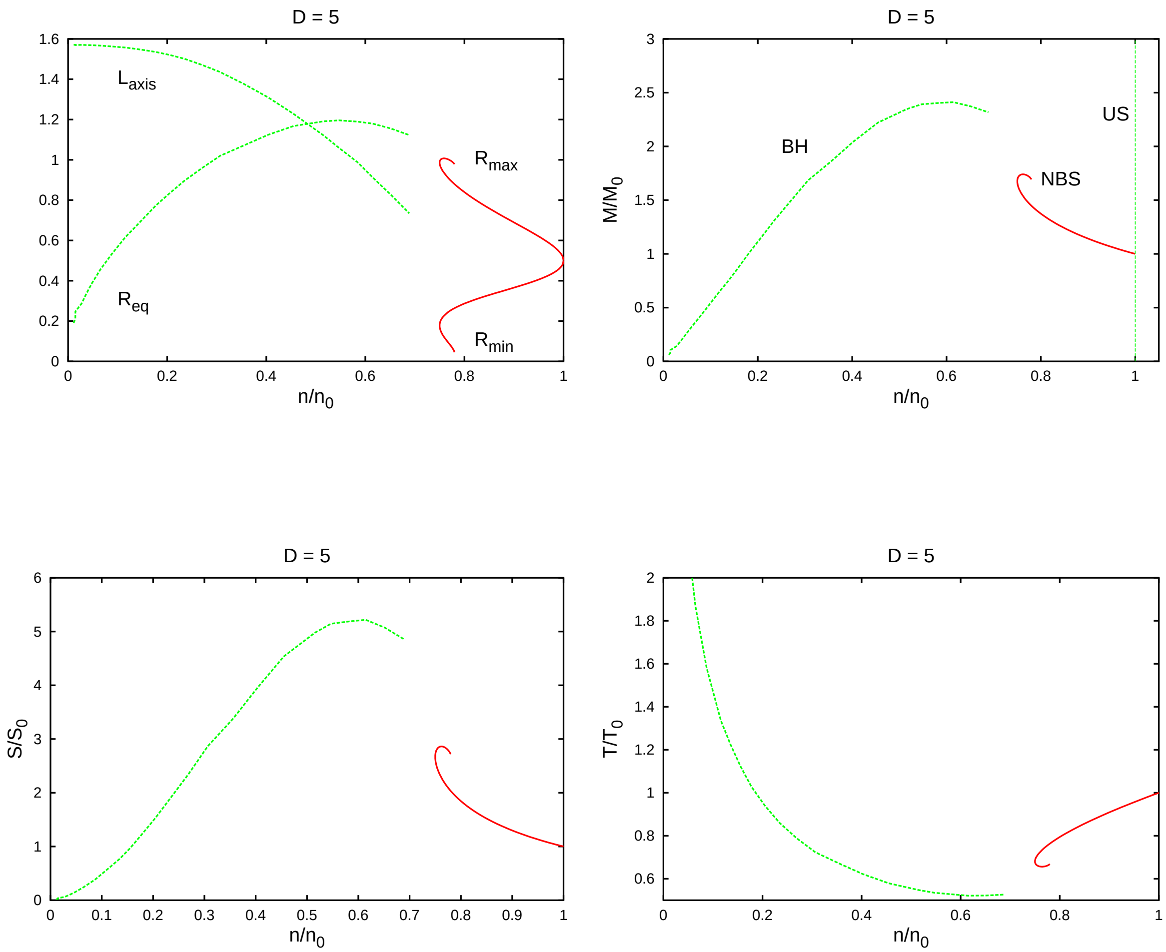
  and Smarr relation to obtain M**,** n

- R_{max} **(**R_{min}**) max (min) radius of** S^{D-3} **on the horizon**

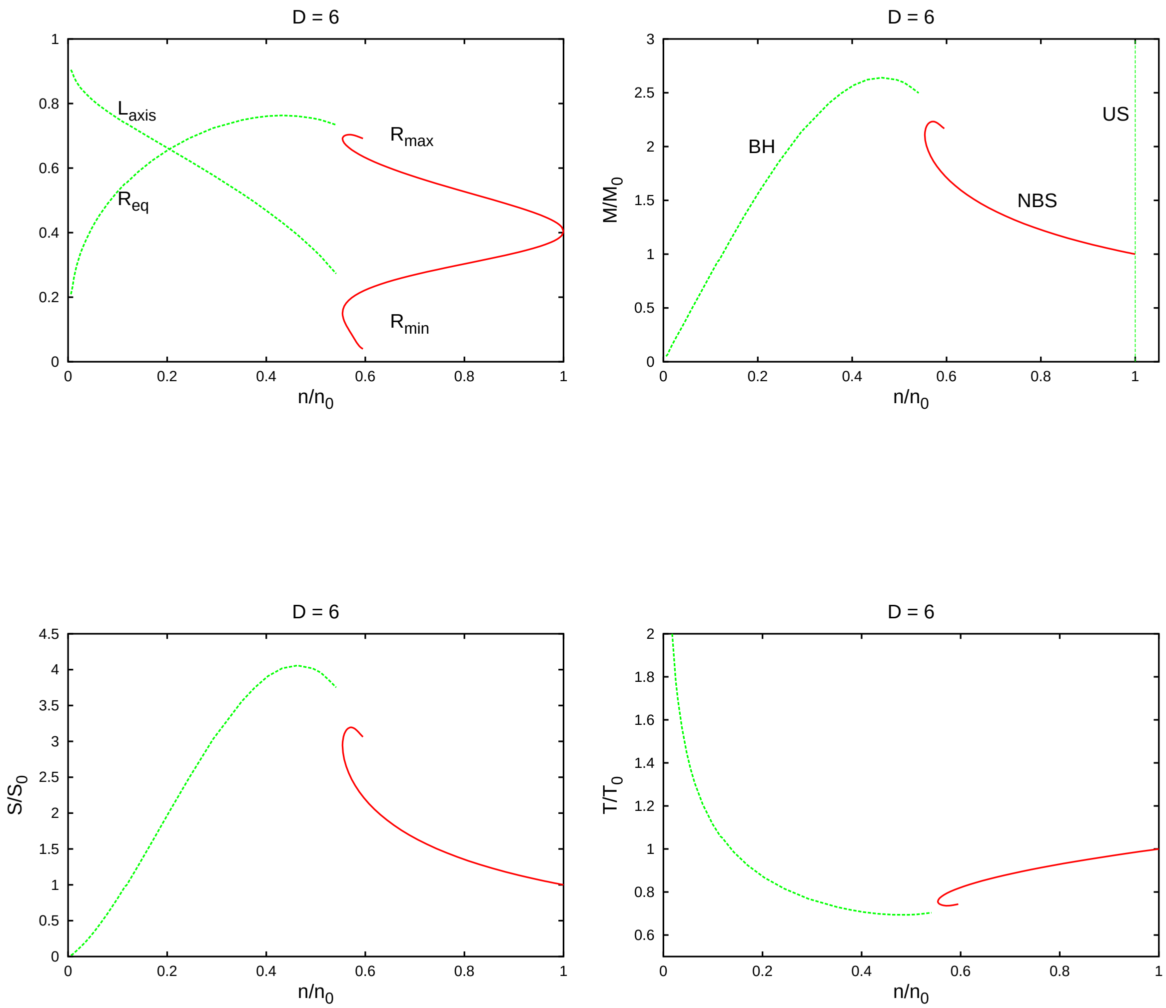
  L_{H} **proper length of the horizon in compact direction**



D = 5
D = 6
R_max
L_H/L
R_min
R_min, R_max, L_H/L
1/(1+λ)
D = 5
S/S_0
M/M_0
n/n_0
T/T_0
1/(1+λ)
D = 6
S/S_0
M/M_0
T/T_0
n/n_0
1/(1+λ)



black hole data from Kudoh and Wiseman, PRL**94**(2005)161102



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Einstein-Maxwell-dilaton Theory

- **Action**

$$I = \frac{1}{16\pi G}\int d^Dx\sqrt{-g}\left(R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{4}e^{-2a\phi}F^2\right)$$

$F = dA$, **ϕ dilaton, a dilaton coupling**

- **Field equations**

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{2}\ T_{\mu\nu},$$

$$\nabla^2\phi = -\frac{a}{2}e^{-2a\phi}F^2,$$

$$\partial_\mu(\sqrt{-g}e^{-2a\phi}F^{\mu\nu}) = 0.$$

- **Vacuum solution**

$$ds^2 = -V(x)dt^2 + h_{ij}(x)dx^idx^j$$

implies family of EMD solutions

$$\begin{aligned} ds^2 &= -V(\cosh^2\beta - \sinh^2\beta V)^{-2\alpha(D-3)}dt^2 \\ &\qquad +(\cosh^2\beta - \sinh^2\beta V)^{2\alpha}h_{ij}dx^idx^j, \end{aligned}$$

$$A_\mu = \sqrt{2(D-2)\alpha}\frac{\tanh\beta\ e^{a\phi_0}V}{\cosh^2\beta - \sinh^2\beta V}\delta_{\mu t},$$

$$\phi = \phi_0 - 2a(D-2)\alpha\log(\cosh^2\beta - \sinh^2\beta V),$$

$\alpha = (2a^2(D-2) + D - 3)^{-1}$, β, ϕ_0 **constant**

Einstein-Maxwell-dilaton Theory

- **Vacuum solution** $(M, \quad \mathcal{T}, \quad n, \quad S, \quad T)$

EMD solutions

$$\bar{M} = M(1 + 2(D-3-n)\alpha \sinh^2\beta),$$

$$\bar{\mathcal{T}} = \mathcal{T},$$

$$\bar{n} = \frac{n}{1 + 2(D-3-n)\alpha \sinh^2\beta}.$$

$$\bar{T} = T(\cosh\beta)^{-2\alpha(D-2)},$$

$$\bar{S} = S(\cosh\beta)^{2\alpha(D-2)},$$

- **Electric charge:**

$$Q_e = -\frac{M(D-3-n)}{(D-4)}\sqrt{\frac{\alpha}{2(D-2)}}$$

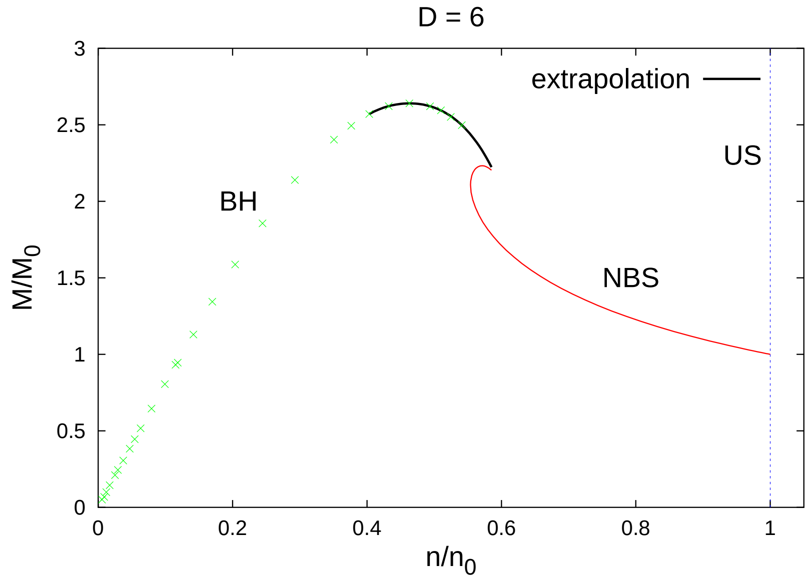
- **Dilaton charge:**

$$Q_d = -\frac{2aM}{D-4}(D-3-n)\alpha \sinh^2\beta$$

- **Smarr relation:**

$$\frac{D-3-n}{D-2}\bar{M} = \bar{T}\bar{S} - \frac{(D-3)(D-4)}{D-2}\Phi\tilde{Q}_e,$$

where $\tilde{Q}_e = \Omega_{D-3} L Q_e$**,** $\Phi = \sqrt{2(D-2)\alpha}\; e^{a\phi_0}\tanh\beta$



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Conclusions

- **$D=5$ and $D=6$ non-uniform black strings up to $\lambda \approx 9$**
- **(some) Evidence for topology changing transition**
- **Black strings in EMD theory from vacuum solutions**